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Studies on the Three-Dimensional Distribution of Rolling Pressure in Hot Rolling*

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In this paper a new analysis of pressure distribution in hot rolling is made.

For complexity of the problem, past investigations on the pressure distribution during the rolling are restricted to two-dimensional consideration. Then, the author develops a three-dimensional consideration of pressure distribution during the rolling by adopting the following two premises, namely the mean slipping angle is constant between the rolls and plate, and the rolling stresses states are expressed by partially approximating the yield criterion of Von Mises.

Consequently, the author can explain the pressure decrease toward the plate edge by taking the new factor of plate width.

1. Introduction

Many attempts have been made to obtain the satisfactory solution of rolling stresses during the hot and cold rolling, but most investigations^{(1)~(4)} remain within the two dimensional consideration, namely they were treated only without lateral spread.

In hot states, when the ratio of width to thickness of plate is small, we find that there exists a lateral spread. In such a case of course, we can not use the two-dimensional consideration already. Empirically, in the case the ratio of width to thickness of plate is smaller than about 15, we can not neglect the lateral spread during the rolling.

So, it is very important to develop a three-dimensional consideration of rolling pressure permitting the lateral spread.

2. List of symbols

Upper sign of Eq. (1) refers to the entry side of the arc of contact.

Lower sign of Eq. (1) refers to the exit side of the arc of contact.

$\varphi(x', y')$: auxiliary function given by Eq. (29)

p : stress acting from radial direction (compression is adopted +)

σ_x : horizontal stress ()

σ_y : lateral stress ()

k : mean yield stress obtained by simple compression

m_1, m_2 : stress expression defined by Eqs. (6) and

(7) respectively

τ_{10} : frictional stress component in the x -direction

τ_{20} : frictional stress component in the y -direction

θ : angular co-ordinate of the arc of contact

$-\sigma_f$: front tension acting on entry plane

$-\sigma_b$: buck tension acting on exit plane

l : projected length of arc of contact in the x -direction

R : roll radius

$2h_0, 2h_1$: thickness of plate before and after rolling

$2B_0$: width of plate before rolling

x : distance from the plane of exit

y : distance from the center line of plate

x' : non-dimensional independent variable equal to x/l

y' : non-dimensional independent variable equal to y/B_0

ζ : value defined by $\frac{h_1}{h_0 - h_1}$

C_0, C_1, C_2 : constants determined by approximation of yielding condition of Von Mises

D_0, D_1, D_2 : integral constant

3. Theoretical consideration

The rolled plate can be considered as consisting of thin vertical segments parallel to roll axes as shown in Fig. 1. During the rolling, the thin segment flows both in the rolling and the lateral direction between the plate and rolls as schematically shown in Fig. 2. Consequently, frictional stress τ has components in both directions, namely for the rolling direction

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has $\tau \cos \gamma$ and for the lateral direction has $\tau \sin \gamma$. But generally, slipping angle γ varies complicately between the contact surfaces. The author assumed mean slipping angle α between the rolls and plate, therefore frictional stress component τ_{10} for the rolling direction may be assumed $\tau \cos \alpha$ and for the lateral direction be assumed $\tau \sin \alpha$. However, we may assume τ as a constant value, because it is thought that sticking phenomenon will always occur over the whole contact surface in hot rolling. Then we can find that τ is equal to the maximum shearing stress, namely $\tau = k/2$ must be substantiated. Where k is the yield stress at the simple compression state. In this report, the following assumptions are adopted:

- (a) The vertical segment parallel to the roll axes remains parallel to the roll axes before and after rolling.
- (b) The frictional stress exerted by the rolls upon the rolled plate has the constant components in the rolling and lateral directions, namely ones given by $k/2 \cos \alpha$ and $k/2 \sin \alpha$ respectively.
- (c) Yield stress of the material remains constant during the pass which is equal to the mean yield stress.
- (d) Stress states are subjected to yield criterion of Von Mises which is approximated partially (discussed later).
- (e) Roll radius remains constant during the rolling.

Consider the equilibrium of stresses acting on the segment shown in Fig. 2. Equilibrium equations are given as follows for each x - and y -axis respectively.

$$h \frac{\partial \sigma_x}{\partial x} + \sigma_x \frac{\partial h}{\partial x} = p \tan \theta \mp \tau_{10} \quad (1)$$

$$h \frac{\partial \sigma_y}{\partial y} + \tau_{20} = 0 \quad (2)$$

where, $\tau_{10} = k/2 \cos \alpha$, $\tau_{20} = k/2 \sin \alpha$ and upper and lower signs can be used within the entry and exit side respectively. Moreover, we adopt the following geometrical approximations,

$$\frac{\partial h}{\partial x} = \tan \theta \mp \sin \theta = \frac{x}{R}, \quad h = h_1 + \frac{x^2}{2R} \quad (3)$$

Substituting Eq. (3) into Eqs. (1) and (2) results in,

$$\left(h_1 + \frac{x^2}{2R}\right) \frac{\partial \sigma_x}{\partial x} + \frac{x}{R} \sigma_x = \frac{x}{R} p \mp \tau_{10} \quad (4)$$

$$\left(h_1 + \frac{x^2}{2R}\right) \frac{\partial \sigma_y}{\partial y} + \tau_{20} = 0 \quad (5)$$

So, we introduce the following expressions,

$$p - \sigma_x = m_1 k \quad (6)$$

$$p - \sigma_y = m_2 k \quad (7)$$

Hence, m_1 , m_2 are functions of both x and y .

Fig. 1 Direction of frictional stress between the rolls and plate

Fig. 2 Stress state produced in thin segment of rolled plate

Differentiating Eq. (6) with respect to x under the assumption that k is constant and substituting it into Eq. (4) to eliminate σ_x , results in

$$\frac{1}{k} \frac{\partial p}{\partial x} = \frac{\partial m_1}{\partial x} + \frac{x}{R} \cdot \frac{m_1}{\left(h_1 + \frac{x^2}{2R}\right)} \mp \frac{\tau_{10}}{\left(h_1 + \frac{x^2}{2R}\right)} \quad (8)$$

Similarly, differentiating Eq. (7) with respect to y and substituting into Eq. (5) to eliminate σ_y , results in

$$\frac{1}{k} \frac{\partial p}{\partial y} = \frac{\partial m_2}{\partial y} - \frac{\tau_{20}}{\left(h_1 + \frac{x^2}{2R}\right)} \quad (9)$$

When we attend to Eq. (8) and (9), we find that these equations represent the pressure gradient toward X - and Y -directions respectively.

If we can know the relations between the m_1 , m_2 , x and y and the boundary conditions, we can easily obtain the pressure distributions over the arc of contact using Eqs. (8) and (9).

Now, we consider the yield condition that is proposed by Von Mises as follows,

$$(p - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + (\sigma_y - p)^2 = 2k^2 \quad (10)$$

Substituting Eqs. (6) and (7) into Eq. (10) yields,

$$m_1^2 - m_1 m_2 + m_2^2 = 1.0 \quad (11)$$

Eq. (11) is represented as an ellipse as shown in Fig. 3. Generally, rolling stresses vary within the first quadrant. Namely at the center of plate, they must satisfy the plane strain condition, therefore, $2\sigma_y = p + \sigma_x$ or $m_1 = 2m_2$ is substantiated.

These conditions correspond to point C in Fig. 3, but when a lateral spread is permitted, they must satisfy $2\sigma_y < p + \sigma_x$ or $m_1 < 2m_2$ relation.

This relation is satisfied along the arc CB. Otherwise, when tension is acting upon the plate to rolling directions, $2\sigma_y > p + \sigma_x$ or $m_1 > 2m_2$ must be satisfied. From the above consideration, we can find that they are enough to approximate the interval ACB on the ellipse by appropriate equation for rolling.

Here, we put this equation as follows :

$$m_1 = C_0 + C_1 m_2 + C_2 m_2^2 \dots \dots \dots (12)$$

Eliminate the stress from Eqs. (8) and (9) and using Eq. (12), we obtain the following differential equation,

$$2C_2 m_2 \frac{\partial^2 m_2}{\partial x \partial y} + 2C_2 \left(\frac{\partial m_2}{\partial x} \right) \left(\frac{\partial m_2}{\partial y} \right) + (C_1 - 1) \frac{\partial^2 m_2}{\partial x \partial y} + \frac{x}{R} \cdot \frac{C_1}{\left(h_1 + \frac{x^2}{2R} \right)} \frac{\partial m_2}{\partial y} \\ + \frac{x}{R} \cdot \frac{2C_2}{\left(h_1 + \frac{x^2}{2R} \right)} m_2 \frac{\partial m_2}{\partial y} - \frac{x}{R} \cdot \frac{1}{\left(h_1 + \frac{x'^2}{2R} \right)^2} \cdot \frac{\tau_{20}}{k} = 0 \dots \dots \dots (13)$$

Solution of Eq. (13) is given by,

$$m_2 = \frac{RB_0}{C_2 l^2} \cdot \frac{1}{D_0} \cdot \frac{\tau_{20}}{k} \ln(\zeta + x'^2) \cdot (\zeta + x'^2)^{-1/2} - \frac{1 + C_1}{2C_2} + (D_1 + D_0 y') (\zeta + x'^2)^{-1/2} \dots \dots \dots (14)$$

then, x' and y' are non-dimensional variables equal to x/l and y/B_0 respectively. By applying Eq. (14) to Eqs. (8) and (9) we obtain,

$$\frac{1}{k} \cdot \frac{\partial p}{\partial x'} = \frac{\partial}{\partial x'} \left[C_0 + C_1 \left\{ \frac{RB_0}{C_2 l^2} \cdot \frac{1}{D_0} \cdot \frac{\tau_{20}}{k} \ln(\zeta + x'^2) (\zeta + x'^2)^{-1/2} - \frac{1 + C_1}{2C_2} + (D_1 + D_0 y') (\zeta + x'^2)^{-1/2} \right\} \right. \\ \left. + C_2 \left\{ \frac{RB_0}{C_2 l^2} \cdot \frac{1}{D_0} \cdot \frac{\tau_{20}}{k} \ln(\zeta + x'^2) (\zeta + x'^2)^{-1/2} - \frac{1 + C_1}{2C_2} + (D_1 + D_0 y') (\zeta + x'^2)^{-1/2} \right\}^2 \right] \\ + \frac{2x'}{(\zeta + x'^2)} \left[C_0 + C_1 \left\{ \frac{RB_0}{C_2 l^2} \cdot \frac{1}{D_0} \cdot \frac{\tau_{20}}{k} \ln(\zeta + x'^2) (\zeta + x'^2)^{-1/2} - \frac{1 + C_1}{2C_2} + (D_1 + D_0 y') (\zeta + x'^2)^{-1/2} \right\} \right. \\ \left. + C_2 \left\{ \frac{RB_0}{C_2 l^2} \cdot \frac{1}{D_0} \cdot \frac{\tau_{20}}{k} \ln(\zeta + x'^2) (\zeta + x'^2)^{-1/2} - \frac{1 + C_1}{2C_2} + (D_1 + D_0 y') (\zeta + x'^2)^{-1/2} \right\}^2 \right] \\ \mp \frac{2R}{l} \cdot \frac{\tau_{10}}{k} \cdot \frac{1}{(\zeta + x'^2)} \dots \dots \dots (15)$$

$$\frac{1}{k} \cdot \frac{\partial p}{\partial y'} = D_0 (\zeta + x'^2)^{-1/2} - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} (\zeta + x'^2)^{-1} \dots \dots \dots (16)$$

As boundary conditions, we adopt the following three conditions in both entry and exit plane to make reference to Fig. 4.

(A) Lateral stress σ_y is equal to zero at the free edges, namely it is given as follows :

$$\text{at the entry plane, } \left(\frac{\sigma_y}{k} \right)_{x'=1.0, y'=1.0} = 0 \dots \dots \dots (17)$$

$$\text{at the exit plane, } \left(\frac{\sigma_y}{k} \right)_{x'=0, y'=1.0} = 0 \dots \dots \dots (18)$$

Fig. 3 Yield condition of Von Mises

Fig. 4 Co-ordinate expression of contact surface during the rolling

(B) Integrating values of horizontal stress σ_x on which they distribute along the entry and exit plane are equal to back and front tension, namely they are given as follows,

$$\text{at the entry plane, } \int_0^{1.0} \left(\frac{\sigma_x}{k} \right)_{x'=1.0} dy' = -\frac{\sigma_b}{k} \dots\dots\dots (19)$$

$$\text{at the exit plane, } \int_0^{1.0} \left(\frac{\sigma_x}{k} \right)_{x'=0} dy' = -\frac{\sigma_f}{k} \dots\dots\dots (20)$$

(C) As pressure distribution is symmetrical for the center of plate, then lateral deformation can not be produced there, so plane strain states must be substantiated, namely it is given as follows :

$$\text{at the entry plane, } [2\sigma_y = (\sigma_x + p)]_{\substack{x'=1.0 \\ y'=0}} \dots\dots\dots (21)$$

$$\text{at the exit plane, } [2\sigma_y = (\sigma_x + p)]_{\substack{x'=0 \\ y'=0}} \dots\dots\dots (22)$$

Using these conditions, we can easily obtain the pressure distribution over the arc of contact.

Considerations are made on the following two parts.

(1) Pressure distribution between the entry and neutral planes.

Integrating the Eq. (16) along the entry plane, we obtain

$$\frac{p}{k} = \left\{ D_0 (\zeta + 1)^{-1/2} - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} (\zeta + 1)^{-1} \right\} y' + D_2 \dots\dots\dots (23)$$

Here, remembering the Eqs. (6), (7), (12) and (14), we can find that other two stresses σ_x and σ_y are given by general equation already.

Combining the above mentioned boundary conditions and Eqs. (6), (7), (12) and (14), we can find the three unknown integrating constants which can be used within the entry side as follows,

$$D_2 = \frac{(2 - C_1) - \sqrt{(2 - C_1)^2 - 4C_0C_2}}{2C_2} + \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} (\zeta + 1)^{-1} \dots\dots\dots (24)$$

$$D_1 = (\zeta + 1)^{1/2} \left\{ D_2 - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} (\zeta + 1)^{-1} + \frac{1 + C_1}{2C_2} - \frac{1}{D_0} \cdot \frac{RB_0}{C_2 l^2} \cdot \frac{\tau_{20}}{k} (\zeta + 1)^{-1/2} \cdot \ln(\zeta + 1) \right\} \dots\dots\dots (25)$$

and another constant can be found as the value of following equation to be zero.

$$\begin{aligned} \frac{1}{2} \left\{ D_0 (\zeta + 1)^{-1/2} - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} (\zeta + 1)^{-1} \right\} + D_2 - C_0 - \frac{C_1}{2D_0} (\zeta + 1)^{1/2} \left[\left\{ D_2 - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} (\zeta + 1)^{-1} \right. \right. \\ \left. \left. + D_0 (\zeta + 1)^{-1/2} \right\}^2 - \left\{ D_2 - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} (\zeta + 1)^{-1} \right\}^2 \right] - \frac{C_2}{3D_0} (\zeta + 1)^{1/2} \left[\left\{ D_2 - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} (\zeta + 1)^{-1} \right. \right. \\ \left. \left. + D_0 (\zeta + 1)^{-1/2} \right\}^3 - \left\{ D_2 - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} (\zeta + 1)^{-1} \right\}^3 \right] + \frac{\sigma_b}{k} = 0 \dots\dots\dots (26) \end{aligned}$$

Substituting the above obtained constant into Eq. (23), we can find the pressure distribution along the entry plane as function of y' . Otherwise taking the upper sign and integrating the Eq. (15) along the constant line of $y' = y_i'$ results in,

$$\frac{p}{k} = \varphi(x', y_i') - \frac{2R}{l} \cdot \frac{\tau_{10}}{k} \cdot \frac{1}{\sqrt{\zeta}} \tan^{-1} \frac{x'}{\sqrt{\zeta}} + D_3 \dots\dots\dots (27)$$

here, constant D_3 can be found by using p/k given by Eq. (23) as boundary values. We represent this value as p_1/k , the resulting equation is given by,

$$\frac{p}{k} = \frac{p_1}{k} + \frac{2R}{l} \cdot \frac{\tau_{10}}{k} \cdot \frac{1}{\sqrt{\zeta}} \left(\tan^{-1} \frac{1}{\sqrt{\zeta}} - \tan^{-1} \frac{x'}{\sqrt{\zeta}} \right) + \{ \varphi(x', y_i') - \varphi(1, y_i') \} \dots\dots\dots (28)$$

then,

$$\begin{aligned} \varphi(x', y_i') = C_0 + C_1 \left\{ \frac{RB_0}{C_2 l^2} \cdot \frac{\tau_{20}}{k} \cdot \frac{1}{D_0} \ln(\zeta + x'^2) (\zeta + x'^2)^{-1/2} - \frac{1 + C_1}{2C_2} + (D_1 + D_0 y_i') (\zeta + x'^2)^{-1/2} \right\} \\ + C_2 \left\{ \frac{RB_0}{C_2 l^2} \cdot \frac{\tau_{20}}{k} \cdot \frac{1}{D_0} \ln(\zeta + x'^2) (\zeta + x'^2)^{-1/2} - \frac{1 + C_2}{2C_2} + (D_1 + D_0 y_i') (\zeta + x'^2)^{-1/2} \right\}^2 \\ + \left\{ C_0 - \frac{C_1(1 + C_1)}{2C_2} + \frac{(1 + C_1)^2}{4C_2} \right\} \ln(\zeta + x'^2) + 2 \left\{ \frac{2RB_0}{C_2 l^2} \cdot \frac{\tau_{20}}{k} \cdot \frac{1}{D_0} + (D_1 + D_0 y_i') \right\} (\zeta + x'^2)^{-1/2} \\ + 2C_2 \left\{ - \left(\frac{RB_0}{C_2 l^2} \cdot \frac{1}{D_0} \cdot \frac{\tau_{20}}{k} \right)^2 - \frac{1}{2} (D_1 + D_0 y_i')^2 - \frac{RB_0}{C_2 l^2} \cdot \frac{\tau_{20}}{k} \cdot \frac{1}{D_0} (D_1 + D_0 y_i') \right\} (\zeta + x'^2)^{-1} \end{aligned}$$

$$\begin{aligned}
& + \frac{2RB_0}{C_2 l^2} \cdot \frac{\tau_{20}}{k} \cdot \frac{1}{D_0} \ln(\zeta + x'^2) (\zeta + x'^2)^{-1/2} + 2C_2 \left\{ - \left(\frac{RB_0}{C_2 l^2} \cdot \frac{1}{D_0} \cdot \frac{\tau_{20}}{k} \right)^2 \right. \\
& \left. - \frac{RB_0}{C_2 l^2} \cdot \frac{\tau_{20}}{k} \cdot \frac{1}{D_0} (D_1 + D_0 y_i') \right\} \ln(\zeta + x'^2) (\zeta + x'^2)^{-1} - C_2 \left(\frac{RB_0}{C_2 l^2} \cdot \frac{1}{D_0} \cdot \frac{\tau_{20}}{k} \right)^2 \{ \ln(\zeta + x'^2) \}^2 (\zeta + x'^2)^{-1} \\
& \dots \dots \dots (29)
\end{aligned}$$

(2) Pressure distribution between the exit and neutral planes.

In the same way as the above mentioned method, integrating Eq. (16) along the exit plane, we obtain.

$$\frac{p}{k} = \left\{ D_0 \zeta^{-1/2} - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} \cdot \zeta^{-1} \right\} y' + D_2 \dots \dots \dots (30)$$

Combining the above mentioned boundary conditions and Eqs. (6), (7), (12) and (14), we can find the unknown integrating constant which can be used within the exit side as follows,

$$D_2 = \frac{(2 - C_1) - \sqrt{(2 - C_1)^2 - 4C_0 C_2}}{2C_2} + \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} \zeta^{-1} \dots \dots \dots (31)$$

$$D_1 = \zeta^{1/2} \left\{ D_0 - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} \zeta^{-1} + \frac{1 + C_1}{2C_2} - \frac{1}{D_0} \cdot \frac{RB_0}{C_2 l^2} \cdot \frac{\tau_{20}}{k} \zeta^{-1/2} \ln \zeta \right\} \dots \dots \dots (32)$$

and D_0 can be found as the value of following equation to be zero.

$$\begin{aligned}
& \frac{1}{2} \left\{ D_0 \zeta^{-1/2} - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} \zeta^{-1} \right\} + D_2 - C_0 - \frac{C_1}{2D_0} \zeta^{1/2} \left[\left\{ D_2 - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} \zeta^{-1} + D_0 \zeta^{-1/2} \right\}^2 \right. \\
& \left. - \left\{ D_2 - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} \zeta^{-1} \right\}^2 \right] - \frac{C_2}{3D_0} \zeta^{1/2} \left[\left\{ D_2 - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} \zeta^{-1} + D_0 \zeta^{-1/2} \right\}^3 \right. \\
& \left. - \left\{ D_2 - \frac{\tau_{20}}{k} \cdot \frac{2RB_0}{l^2} \zeta^{-1} \right\}^3 \right] + \frac{\sigma_f}{k} = 0 \dots \dots \dots (33)
\end{aligned}$$

Substituting the above obtained constant into Eq. (23), we can find the pressure distribution along the exit plane as function of y' . Otherwise, taking the lower sing and integrating the Eq. (15) along the constant line of $y' = y_i'$ results in,

$$\frac{p}{k} = \varphi(x', y_i') + \frac{2R}{l} \cdot \frac{\tau_{10}}{k} \cdot \frac{1}{\sqrt{\zeta}} \tan^{-1} \frac{x'}{\sqrt{\zeta}} + D_3 \dots \dots \dots (34)$$

Here, constant D_3 can be found by using p/k given by Eq. (30) as boundary values. We represent this value as p_2/k , the resulting equation being given by,

$$\frac{p}{k} = \frac{p_2}{k} + \frac{2R}{l} \cdot \frac{\tau_{10}}{k} \cdot \frac{1}{\sqrt{\zeta}} \tan^{-1} \frac{x'}{\sqrt{\zeta}} + \{ \varphi(x', y_i') - \varphi(0, y_i') \} \dots \dots \dots (35)$$

then, auxiliary function $\varphi(x', y_i')$ is given by Eq. (29) already, but we must remember that the constants D_0 , D_1 are different in each side given by Eqs. (25), (26) and (32), (33) respectively.

(3) Calculating example of rolling pressure.

We must preliminarily discuss the approximation of yield condition and mean slipping angle which are introduced in this consideration. Eq. (12) gives an approximate equation of yield condition in general form.

Here, we assume the constant as $C_0 = 1.0$, $C_1 = 0.60$ and $C_2 = -0.60$ respectively in Eq. (12), we obtain the equation in good agreement with Eq. (11) within the range ACB as shown in Fig. 5.

Next problem is to determine the mean slipping angle between the rolls and plate. We assume the rolling conditions as follows:

Roll diameter = 18.0 cm

Thickness of plate before and after rolling = 0.20 cm and 0.15 cm.

Plate width before rolling = 0.30 cm.

In this case, the mean slipping angle may be presumed in each entry and exit side about 5 and 4 degrees respectively.

Generally, the mean slipping angle can be determined experimentally with specimens which have arbitrary ratios of B_0/h_0 and $l/2h_0$. Namely we print the grid on the plate surface and measure the relative displacement over the arc of contact. Subsequently, we can estimate the mean value by dividing the arc of contact into halves.

The mean value in each case gives an approximate mean slipping angle in each entry and exit side respectively. Thus, the author calculates

Fig. 5 Approximation of yield condition

Fig. 6 Pressure distribution curve during the rolling

the pressure distribution between the plate and rolls as shown in Fig. 6.

Consequently, we can explain the pressure decrease toward the lateral direction.

4. Conclusion

It is quite difficult to obtain the exact solution of pressure distribution during the rolling, but by such an approximate consideration, we obtain good agreement with the experimental result with rough rolls as shown in Fig. 7. But, owing to the insufficiency of consideration, the calculated result shows more or less difference compared with the measured result⁽⁵⁾ approaching the edge of plate.

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— Lueg's measurement
 Calculated by new method
 (Mean yield stress k is adopted 40 kg/mm² from Lueg's experiment)

Fig. 7 Roll pressure distribution for rough rolls

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