Creep under Step-wise Temperature Cycling*

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In the present paper, the analytical relation of creep under temperature cycling to static creep under steady temperature is presented and is examined by the creep tests under step-wise temperature cycling on a low carbon steel, a 13 chromium steel and an AISI 304 type stainless steel.

It was found that temperature cycling does not give a serious influence on creep. The creep under cyclic temperature can be predicted by introducing the idea of the equivalent steady temperature for creep, which is determined from the data of usual creep tests at steady temperature combined with the prescribed program of temperature cycling. The analytical results are in good agreement with the experimental results under periodic stepped variation of temperature.

1. Introduction

In the preceding paper of the authors⁽¹⁾⁽²⁾, it was found that temperature variation is, as a matter of fact, an influencing factor on the process of creep, however, the influence of temperature history on creep is not so serious as to defy application in mechanics problems. Therefore, by using the principle of the "mechanical equation of state in solid", the creep curve under varying temperature can be predicted from the results of tests made at steady temperature.

In the present paper, the analytical relation of creep under temperature cycling to static creep under steady temperature is discussed and is examined by the creep tests under step-wise temperature cycling on a low carbon steel, a 13 chromium steel and an AISI 304 type stainless steel. It is intended to establish the conventional formulas to use for prediction of creep under cyclic temperature variation from the information of simple creep tests under steady temperature.

2. Material tested and testing apparatus

Chemical compositions and mechanical properties of material tested are shown in Tables 1 and 2, respectively. The diameter of specimen was 6 mm and the gauge length was 50 mm. The specimen of carbon steel was annealed after machining. Other

materials were heat treated before machining as shown in Table 1.

Apparatus for realizing temperature variation and one for measuring the elongation of the specimen were the same as those used in the previous study⁽¹⁾. Temperature cycle was successive heating at two temperature levels, i. e., a reference temperature T_0 and a temperature below or above it by definite degrees ΔT . Attention was also paid to completing every temperature change in very short time and to realizing the flat temperature path after a quick temperature change.

3. Analysis of creep under temperature cycling from the constant temperature creep test data

According to the mechanical equation of state in solid, the strain rate $d\epsilon/dt$ at any time t in the process of creep under a fixed stress σ is a function of strain ϵ and absolute temperature T , independent of the past history of the material;

$$d\epsilon/dt = f(\epsilon, T). \quad (1)$$

On the other hand, total strain ϵ in static creep before the beginning of tertiary stage is well expressed by the following equation;

$$\epsilon = \epsilon_0 + \beta t^n + \kappa t. \quad (\beta, n : \text{constants}) \quad (2)$$

Strain rate is obtained by differentiating Eq. (2) as

$$d\epsilon/dt = \beta n t^{n-1} + \kappa = u|_t + \kappa, \quad (3)$$

where $u|_t$ and κ are the transient creep rate and the steady state creep rate, respectively. The mechanical equation of state, in this case, for the strain hardening basis is obtained by eliminating t from the right hand sides of Eqs. (2) and (3). This formula is not a simple one and has to be solved by the

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numerical method. It can be reduced to a simple form in a special case where the creep strain is represented by an individual component of the transient stage creep and the steady state stage creep.

In the case of transient creep stage, Eqs. (2) and (3) are reduced to the following equations as the function of T and t , respectively;

$$\varepsilon = \varepsilon_0 + A_0 \cdot \exp(-K/T)t^n, \dots\dots\dots(4)$$

$$d\varepsilon/dt = A_0 \cdot n \cdot \exp(-K/T)t^{n-1}, \dots\dots\dots(5)$$

The instantaneous strain ε_0 is approximately constant under a fixed applied stress and n takes a constant value approximately, independent of stress and temperature. Thus, the mechanical equation of state in solid becomes

$$d\varepsilon/dt = A_0^{1/n} \cdot n \cdot \exp\left(-\frac{1}{n} \frac{K}{T}\right) (\varepsilon - \varepsilon_0)^{n-1/n}, \dots\dots\dots(6)$$

for the strain hardening basis.

Integrating Eq. (6), the following equation is obtained, provided that temperature T in this equation is represented by cyclic temperature $T(t)$, with period p of every cycle, and adopting the initial condition of $\varepsilon = \varepsilon_0$ at $t = 0$;

$$\begin{aligned} \varepsilon &= \varepsilon_0 + A_0 \cdot \left[\frac{t}{p} \int_0^p \exp\left\{-\frac{1}{n} \frac{K}{T(t)}\right\} dt \right]^n \\ &= \varepsilon_0 + A_0 I^n t^n, \dots\dots\dots(7) \end{aligned}$$

where

$$I = \frac{1}{p} \int_0^p \exp\left\{-\frac{1}{n} \frac{K}{T(t)}\right\} dt, \dots\dots\dots(8)$$

Eq. (7) yields to

$$\varepsilon = \varepsilon_0 + A_0 \cdot \exp(-K/T_{e,e})t^n, \dots\dots\dots(9)$$

where

$$T_{e,e} = -\frac{K}{n} / \ln I, \dots\dots\dots(10)$$

Eq. (9) shows that the strain in transient creep stage under the condition of cyclic temperature variation is equivalent to the strain in a static creep at the temperature equal to the equivalent steady temperature $T_{e,e}$, defined as Eq. (10). Therefore, the strain in transient creep stage under cyclic temperatures can be predicted from the static creep test data by calculating the equivalent steady temperature.

On the other hand, in the case of the combined stage of the transient creep and the steady state creep, the equivalent steady temperature defined for the transient creep stage was assumed to be extended to the stage of the steady state creep.

For most of the metallic materials, the relation of the transient creep rate u_t to the steady state creep rate κ represented as

$$u_t \cdot \kappa^{C_1} = C_2 t^{n-1}, \quad C_1 \cong \frac{1}{2}, \quad C_2 \cong 1 \sim 6, \dots\dots\dots(11)$$

has been verified by experiments as well as by analysis by the authors⁽⁸⁾. This relation holds independently of the magnitudes of applied stress and temperature.

From Eqs. (5) and (11) the temperature dependency of the steady state creep rate is represented as

Table 1 Chemical compositions (%) and the heat treatments of the materials

Symbols of the materials	C	Mn	Si	P	S	Cu	Ni	Cr	Heat treatment
S15C	0.15	0.43	0.26	0.013	0.007	—	—	—	900°C, 1 hr F.C.
SUS-2	0.16	0.50	0.46	0.023	0.009	—	—	13.11	970°C, O.Q., 760°C, T.
AISI 304	0.071	1.81	0.68	0.009	0.011	0.03	10.84	18.66	1100°C, W.Q.

Table 2 Mechanical properties of the material at room temperature

Symbols of the materials	Lower yield point kg/mm ²	Tensile strength kg/mm ²	Elongation %	Area contraction %	Hardness HVN(Load)
S15C	25.1	38.2	34	65.0	137 (50 gr)
SUS-2	52.5	71.7	20.2	66.2	260 (200 gr)
AISI 304	20.8	54.9	66.0	75.0	156 (5 kg)

Table 3 Condition of temperature cycling

Symbols of the materials	Reference temperature T_0 °C	Temperature difference from reference temperature ΔT °C	τ/p %			
S15C	450	+25	0,	—	33.3,	89.6, 100
	450	-150	—	8.3,	50.0,	91.7, 100
SUS-2	500	+50	0,	8.3,	33.3,	91.7, 100
	500	-100	—	2.1,	29.9,	91.7, 100
AISI 304	650	+50	0,	2.1,	33.3,	91.7, 100
	650	-50	—	2.1,	33.3,	91.7, 100

$$\kappa = (A_0 n / C_2)^{1/C_1} \cdot \exp\left(-\frac{1}{C_1} \frac{K}{T}\right) \quad (12)$$

Taking the procedure similar to the reduction of Eq. (7), we have

$$\kappa' = (A_0 n / C_2)^{1/C_1} \cdot I^{n/C_1} \quad (13)$$

as the creep rate at steady stage of creep under cyclic temperature. Comparing Eq. (12) and Eq. (13), we have the same equation as Eq. (10). From this consideration, therefore, it would be understood that the creep rate at steady stage of creep in the tests under cyclic temperatures is equivalent to the creep rate under a steady temperature of $T_{e.c.}$, the equivalent steady temperature.

Thus, for creep under periodic stepped variation of temperature, the creep strain that includes both components of transient and steady creep is expressed, by substituting Eqs. (9) and (13) into Eq. (2), as

$$\varepsilon = \varepsilon_0 + A_0 \cdot \exp\left(-\frac{K}{T_{e.c.}}\right) t^n + \left(\frac{A_0 n}{C_2}\right)^{1/C_1} \cdot \exp\left(-\frac{1}{C_1} \frac{K}{T_{e.c.}}\right) t, \quad (14)$$

where

$$1/T_{e.c.} = \frac{1}{T_0 + \Delta T} - \frac{n}{K} \left[\frac{\tau}{p} \cdot \exp\left\{-\frac{K}{n} \left/ \left(T_0 + \frac{T_0^2}{\Delta T} \right) \right\} + \left(1 - \frac{\tau}{p}\right) \right] \quad (15)$$

In this formula p is the period of temperature variation and τ is the time interval exposed to the reference temperature T_0 . It is obvious that for $\tau/p=1$, $T_{e.c.}=T_0$, being the case of creep under

steady temperature of T_0 , and for $\tau/p=0$, $T_{e.c.}=T_0 + \Delta T$, being the case of creep at a steady temperature of $T_0 + \Delta T$.

4. Experimental results and the discussion

The validity of the prediction of creep under periodic stepped variation of temperature from the data of creep tests at steady temperature was examined by experiments. Table 3 shows the condition of the temperature cycling. Figures from 1 to 3 show the creep curves on a low carbon steel, a 13 chromium steel and an AISI 304 stainless steel under the programs of temperature variation as shown in these figures, respectively. In these figures the abscissa shows the ratio of τ , the time interval exposed to the reference temperature T_0 , to the period p of every cycle. The circles in these figures are the experimental points and the full lines are drawn by the analysis based on the data of creep tests on the same materials at constant temperature.

In Fig. 1, the results on a low carbon steel, the analysis of creep curve is made for the transient stage of creep. It is known that the analytical curves are in good agreement with the experimental ones. The static creep tests for this material are performed under the temperature range from 300°C to 500°C and a tensile applied stress of 20.0 kg/mm². Fig. 2

Fig. 1 Comparison between both analytical and experimental results on creep of low carbon steel under periodically varying temperature

Fig. 2 Comparison between both analytical and experimental results on creep of 13 chromium steel under periodic variation of temperature

shows the case of a 13 chromium steel, where the analytical results including both components of transient and steady state creep also agree with the test results. The constant temperature creep tests, in this case, are carried out under the temperature range from 400°C to 500°C and the stress of 20.0 kg/mm². For the AISI 304 type stainless steel, the both results are also fairly close to each other as shown in Fig. 3, where the static creep tests are performed under the temperature range from 600°C to 700°C and a stress of 8.0 kg/mm².

5. Conclusion

From the present study the following are concluded; for creep under periodic stepped variation of temperature the creep strain that includes both components of transient and steady state creep is represented, by using the principle of the mechanical equation of state in solid, as

$$\epsilon = \epsilon_0 + A_0 \cdot \exp\left(-\frac{K}{T_{e.e.}}\right) t^n + \left(\frac{A_0 n}{C_2}\right)^{1/C_1} \cdot \exp\left(-\frac{1}{C_1} \frac{K}{T_{e.e.}}\right) t^n,$$

where

$$1/T_{e.e.} = \frac{K}{T_0 + \Delta T} - \frac{n}{K} \ln \left[\frac{\tau}{p} \times \exp\left\{-\frac{K}{n} \left/ \left(T_0 + \frac{T_0^2}{\Delta T}\right)\right\} + \left(1 - \frac{\tau}{p}\right)\right] \right]$$

In the formula p is the period of temperature variation and τ is the time interval exposed to the reference temperature T_0 . In the formula constants of A_0 , K , n , C_1 and C_2 are obtained from the data of creep tests at steady temperature, where C_1 takes the value of about 0.5 and C_2 is from 1 to 6 in most of the metallic materials. Equivalent steady temperature $T_{e.e.}$ is also obtained from the prescribed program of temperature cycling. The propriety of the present mode of prediction, that is, the adoption of the equivalent steady temperature for estimating creep under periodic variation of temperature was verified by the experiments on

Fig. 3 Comparison between both analytical and experimental results on creep of AISI 304 type stainless steel under periodic cycling of temperature

a low carbon steel, a 13 chromium steel and an AISI 304 type stainless steel.

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