Resistance of a Flow through an Annulus with an Inner Rotating Cylinder*

By Yutaka YAMADA**

In this paper, the resistance of a water flow through co-axial cylinders when the inner cylinder rotates, is studied experimentally. The resistances are measured for six gap sizes \(s/r_1 = 0.0136 \sim 0.0115\), \(s\): gap, \(r_1\): radius of inner cylinder) and for various combinations of axial and rotational flows.

Results obtained are as follows:
1. When the axial flow is laminar, the resistance of a flow is unaffected up to a certain rotating speed \((R_e: \text{critical rotating Reynolds number})\), but beyond this speed the flow resistance increases as the \(R_e\) increases.

2. The value of critical Taylor number \(R_{eT}(s/r_1)^{1/2}\) increases as the axial Reynolds number \(R_e = v_m s / \nu\) increases up to a certain value \((R_e = 600)\), beyond this value \(R_{eT}(s/r_1)^{1/2}\) decreases to zero with the increase of \(R_e\). When the axial flow is turbulent, such a critical rotating Reynolds number cannot be recognized.

3. When \(R_e > 10,000\), the resistance coefficient of a flow \(\lambda\) can be expressed by \(\lambda = 0.26 R_e^{-0.24} \times (1 + (7/8)(R_e/2R_{eT})^{2/3})^{5.38}\) even for small \(R_e\).

1. Introduction

In this paper, the writer describes the experimental results for the resistance of a water flow through an annulus, formed by two concentric cylinders, with the inner cylinder rotating and the outer cylinder stationary. Investigation of such a flow is important to study the labyrinth packing and fluid friction of a journal and others.

On the resistance of a flow through co-axial cylinders with a rotating inner cylinder, Suzuki(1) experimented the flow through the shorter cylinders in high Reynolds number. Cornish(2) measured the resistance of a flow through co-axial cylinders having fine clearance and Tomita(3) measured the resistance of a flow when the outer cylinder rotates.

When there is no axial flow and the speed of rotation of inner cylinder is increased from zero, the flow in the annulus is truly laminar until a critical speed of rotation is reached, at which Taylor vortices are formed. These vortices, which are ring-shaped around the rotationary axis, are shown schematically in Fig.1. G.I. Taylor(4) analyzed mathematically the critical speed of rotation, at which such vortices are formed and he confirmed them by his experiment. Taylor(4), Hagerty(5), Kaye and Elgar(6) observed that Taylor vortices keep their shape until the speed of rotation is increased from a critical value to a certain value. The formation of Taylor vortices is entirely different from occurrence of a turbulent flow(7). When the speed of rotation of inner cylinder is increased very much, the flow in the annulus becomes turbulent and vortices are superimposed on the turbulent fluctuation(8).

When the speed of rotation is increased furthermore, it seems that Taylor vortices are distorted and

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then disappear, and the flow becomes a purely turbulent flow. Kaye and Elgar indicated that there are four modes of a flow in co-axial rotating cylinder: purely laminar, purely turbulent, laminar plus Taylor vortices, turbulent plus Taylor vortices; and obtained the boundary between these modes by the use of a hot wire anemometer (see Fig. 2). Here, laminar plus Taylor vortices denote the flow in which there is no turbulent fluctuation, though there are Taylor vortices, and turbulent plus Taylor vortices denote the flow in which there are Taylor vortices and turbulent fluctuation.

Besides the above mentioned study, Goldstein, Page, Chandrasekhar investigated the stability of laminar flow between co-axial rotating cylinders.

In this report, the writer clarifies the character of resistance of a flow, at various Reynolds numbers and with the larger value of clearance than that of the others' experiments. That is to say, the resistance of a flow is obtained analytically and compared with the experimental results, furthermore, the stability of a laminar flow is also considered.

### 2. Experimental apparatus and method

The general arrangement of the apparatus is shown in Fig. 3. A pump A forces water from a reservoir to a head tank B, having an over flow pipe C, which regulates the water level in the tank. The water from the head tank B flows through a vertical pipe, down to the co-axial rotating cylinder. In order to get a water pressure higher than that possible with head tank apparatus, a pipe E is provided. F is a small generator used for revolution counter, and G is a valve to control the quantity of water. A commutator motor H, rotating in a range of 600–2 400 r.p.m., drives the inner cylinder, which rotates 90–5 000 r.p.m. with the aid of a step pulley. The temperature of water is measured as the mean reading of the inlet side thermometer I and the outlet side one I'.

The arrangement of the co-axial rotating cylinders is shown in Fig. 4. Inner cylinders, whose diameters are shown in Table 1, can be inserted co-axially into an outer cylinder. In Table 1, the value of $R_{cr}$ (theory) is the critical rotating Reynolds number calculated according to the theory of Taylor, which is concerned with the origin of Taylor vortex described in the following section. The value of $R_{cr}$ (theory) coincides with the values obtained by the torque measurement, described in the previous report.

Outer cylinder, whose inside diameter is 64.302 mm, is built by inserting a brass pipe in a cast iron pipe. Four pressure holes of 0.4 mm in diameter are arranged at 90° intervals on each cross section N₁, N₂, and N₃, as shown in Fig. 4. Each pressure hole on the same section is connected together with a groove, cut circumferentially in the brass pipe. The distance from the entrance of

![Fig. 2 Border line between various modes of flow](image)

![Fig. 3 General arrangement of apparatus](image)

![Fig. 4 Co-axial rotating cylinder used in this experiment](image)

### Table 1

<table>
<thead>
<tr>
<th>Sign of</th>
<th>Radius of inn. cyl. r₁ mm</th>
<th>Clearance s mm</th>
<th>Clearance ratio s/r₁</th>
<th>Crit. rot. $R_{cr}$ (theory)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>31.720</td>
<td>0.431</td>
<td>0.0136</td>
<td>355</td>
</tr>
<tr>
<td>B</td>
<td>31.533</td>
<td>0.618</td>
<td>0.0196</td>
<td>297</td>
</tr>
<tr>
<td>C</td>
<td>31.220</td>
<td>0.823</td>
<td>0.0296</td>
<td>243</td>
</tr>
<tr>
<td>D</td>
<td>30.695</td>
<td>1.556</td>
<td>0.0475</td>
<td>194</td>
</tr>
<tr>
<td>E</td>
<td>529.35</td>
<td>2.216</td>
<td>0.074</td>
<td>158</td>
</tr>
<tr>
<td>F</td>
<td>528.636</td>
<td>3.315</td>
<td>0.115</td>
<td>130</td>
</tr>
</tbody>
</table>
cylinder to the section N₁ is 90 mm and to the section N₃ is 190 mm.

In most experiments of this paper, pressure drop is measured between N₁ and N₂, but when the clearance of annulus is large, pressure drop is often measured between N₂ and N₃, to eliminate the effect of inlet length. When there is no axial flow, it is very difficult to remove the pressure difference between two pressure holes, which is caused by rotation of inner cylinder. Occurrence of pressure difference Δp₀ mentioned above, is caused chiefly by difference of the shape of the pressure hole entrance, since the variations of clearance, size and eccentricity between outer and inner cylinder are very small in this apparatus. The writer makes Δp₀ very small by adjusting the hole entrance. However, when the value of clearance is narrower than 0.923 mm, there is Δp₀ in some degree, and so the value of pressure loss has to be corrected by Δp₀.

The following nomenclature are used in this paper.

\[ r₁ : \text{radius of inner cylinder} \]
\[ r₂ : \text{radius of outer cylinder} \]
\[ s = r₂ - r₁ : \text{clearance} \]
\[ l : \text{distance between pressure holes} \]
\[ vₐ : \text{mean velocity of axial flow} \]
\[ h : \text{loss of head} \]
\[ Δp = p₁ - p₂ : \text{pressure difference between pressure holes} \]
\[ γ : \text{weight of fluid per unit volume} \]
\[ g : \text{acceleration of gravity} \]
\[ ν : \text{kinematic viscosity} \]
\[ λ : \text{resistance coefficient} \]
\[ ω : \text{angular velocity of inner cylinder} \]
\[ u₁ = r₁ω : \text{peripheral velocity of inner cylinder} \]
\[ v₂ = (vₐ + (kh₁/2))^1/2 : \text{resultant velocity} \]
\[ Rₐ = νₛ/r₁ : \text{axial Reynolds number} \]
\[ Rₘ = u₁/s/ν : \text{rotating number} \]
\[ Rₕ = v₂/s/ν : \text{resultant Reynolds number} \]
\[ Rₚ = (s/r₁)1/2 : \text{Taylor number} \]

The resistance coefficient is defined as

\[ h = Δp/γ = λ·l/2s·vₐ^2/2g \] (1)

3. Resistance of a flow through a stationary annulus

When the ratio of radius of inner cylinder to that of outer cylinder is nearly equal to one, the value of λ can be indicated only as function of Reynolds number for both laminar and turbulent flow, independently of the clearance. In our previous experiments(19), the resistance coefficient λ exactly coincides with the theoretical value

\[ λ = 45/Rₐ \] (2)

for a laminar flow, and the empirical formula of λ for a turbulent flow are expressed as

\[ λ = 0.26Rₐ^{-0.54} \] (3)

The experiments are performed at first when the inner cylinder is stationary.

Marks O in Figs.7-12 and ? in Figs.11 and 12 show the experimental results for this case. From these figures, we can see that the values of λ agree well with Eqs.(2) and (3) for a laminar and a turbulent flow respectively.

In the transient region from laminar to turbulent flow, however, the value of λ depends on the position of pressure holes, as shown in Figs.11 and 12. The marks O show the results of pressure drop between N₁ and N₂, and marks ? show that between N₂ and N₃. The value of λ of marks O is larger than that of marks ?. This means the effect of inlet length, that is, the flow which does not develop yet to the terminal velocity profile enters the measurement section.

When the inner cylinder rotates at over some speeds, however, the value of λ obtained by using the pressure hole near the entrance side N₁ is coincident with that of N₂, even though the clearance is large.

4. Resistance of a flow through rotating co-axial cylinders

4-1 Theoretical consideration

The loss of head for rotating co-axial cylinders can be expressed by Eq.(1), similar to that for stationary cylinders, where the value of vₐ shows the axial mean velocity. The value of λ depends on the clearance size, radius and rotating speed of inner cylinder and it can be considered as a function of the three dimensionless variables, namely

\[ λ = f(Rₐ, s/r₁, Rₘ) \]

It is clear that the resistance of a flow through the co-axial rotating cylinders depends on the axial component of the shearing stress which acts on the wall. The value of λ are estimated under this idea. For the simplicity, the secondary flow and the curvature of cylinder wall are neglected and further the flow characteristics are assumed to be similar to those in a turbulent flow through pipes or in boundary layers. Then we can assume the velocity distribution for turbulent flow as

\[ v = v₀(2y/s)^{1/2} \] (4)

where v is the velocity at distance y from the wall,
\[ v₀ \] is max. velocity which occurs at the midpoint of the gap, and s is a constant. The resistance coefficient λ is assumed as

\[ λ = cRₐ^{-α} \] (5)

where the values of c and α are constants.

From Eq.(5), the shearing stress on the wall of stationary cylinders can be expressed as
\[ \tau_s = \frac{1}{8} \rho C (\frac{v_{E}}{\nu}) \frac{v_{E}^{2}}{v_{E}} \]  

(10)

Then the relation between \( \lambda \) and \( \tau_s \) for co-axial rotating cylinders is

\[ \tau_s = \frac{\lambda \rho v_{E}^{2}}{8} \]  

(11)

Therefore, from Eqs. (9), (10), (11)

\[ \lambda = cR \frac{v_{E}}{v_{E}^{2}} \]  

(12)

\[ = cR \frac{v_{E}}{R} \]  

(13)

\[ = cR \frac{1}{1 + h} \left( \frac{R}{2R} \right)^{(1-\alpha)/2} \]  

where \( R = v_{E}/\nu \) is a resultant Reynolds number.

In the case of a laminar flow, it is evident that the axial velocity distribution is independent of the peripheral velocity distribution, and then the axial component of shearing stress is independent of the peripheral one, according to the Navier-Stokes equation. For a laminar flow, therefore, the value of \( \lambda \) is independent of the value of \( R \), and the value of \( \lambda \) is equal to the theoretical value for stationary cylinders even if the inner cylinder rotates. This fact may be obtained from Eq. (12). The theoretical value of \( \lambda \) for stationary cylinders is \( 48/R \). Hence from Eq. (5), \( c = 48, \alpha = 1 \). Substituting these values into Eq. (12), the value of \( \lambda \) is \( 48/R \), which is the value for stationary cylinders.

For a turbulent flow, on the other hand, the empirical formula of \( \lambda \) is expressed by Eq. (3) in the case of a simple axial flow. Then \( c = 0.26, \alpha = 0.24 \). In the range of Reynolds number of our experiments, the law of \( 1/7 \) power may be applicable to the velocity distribution, so that in Eq. (6) \( n = 7 \), hence \( h = 7/8 \). Substituting these values into Eq. (12)

\[ \lambda = 0.26 R \frac{1}{1 + h} \left( \frac{R}{2R} \right)^{(1-\alpha)/2} \]  

(14)

Assuming the law of \( 1/7 \) power in the velocity distribution, we can obtain a purely axial flow by using Blasius' equation for the flow resistance in...
the pipe.

\[ \lambda = 0.27 \frac{R_e}{R_s}^{-1/4} \] .................................(14)

Then Substituting the value \( c = 0.27 \), \( \alpha = 1/4 \) and \( k = 7/8 \) into the Eq. (12), we have

\[ \lambda = 0.27 \frac{R_e}{R_s}^{-1/4} \left[ 1 + \left( \frac{7}{8} \right) \left( \frac{R_e}{2R_s} \right)^{1/2} \right] \] ...........................(15)

The value of \( \lambda \) measured for stationary co-axial cylinders, are somewhat larger than those in Eq. (14), but in good agreement with Eq. (3), so that the value computed by Eq. (13) will be compared with the experimental results in the next section.

Relations between \( \lambda \) and \( R_s \) for various values of \( R_w \) calculated from Eq. (13), are shown in Fig. 6. It is seen easily thereby that the curves for \( R_w \gg R_s \) run parallel with the curve for a laminar flow. On the other hand, if we use \( R_w/R_s \) as parameter, the \( \lambda - R_s \) curves run parallel with the curve of a turbulent flow for \( R_w = 0 \). From Eq. (12)

\[ \lambda \frac{R_s}{R_w} = cR_e^{1/4} \] .................................(16)

therefore, the value of \( \lambda \cdot R_s/R_w \) can be indicated only as a function of the resultant Reynolds number \( R_e \), this function has the same form as Eq. (5). Assuming the velocity distribution mentioned above, \( k = 7/8 \). Substituting the velocity into the Eq. (9), we have

\[ v_E = \left[ v_w^2 + \left( \frac{7}{8} \right) \left( \frac{R_e}{2} \right)^{1/2} \right] \] .................................(17)

\[ R_w = \left[ R_s^2 + \left( \frac{7}{8} \right) \left( \frac{R_e}{2} \right)^{1/2} \right] \]

4.2 Experimental results

In our experiments, six clearances within 0.431 ~ 3.315 mm as shown in Table 1 are used. When clearance \( s = 0.431 \) mm, experimental results are as shown in Fig. 7 in which \( \lambda \) is the ordinate, \( R_s \) is the abscissa and \( R_w \) is a parameter. For small magnitudes of \( R_w \), the value of \( \lambda \) is independent of \( R_w \) and shows good agreement with the theoretical value of a laminar flow, but beyond a certain value of \( R_w \) the value of \( \lambda \) increases sharply and the tendency of \( \lambda \cdot R_s \) curves is similar to that in Fig. 6.

When there is no axial flow, the value of \( R_w \) at which Taylor vortices occur is about 355 as seen in Table 1. We carried out experiments when the inner cylinder rotated, and the values of \( R_s \) were larger than 500, as shown in Fig. 7 therefore larger than \( R_w \) described in Table 1.

For \( R_w = 1000 \), the value of \( \lambda \) is in good agreement with the theoretical value \( 48/R_w \) within the range of \( R_s = 130 \sim 940 \) and the flow state is laminar. When \( R_s \) is small, an increasing of \( \lambda \) from the theoretical value is due to the transition from laminar to laminar + Taylor vortex flow. When \( R_s \) is large, on the other hand, \( \lambda \) increases due to the transition to a purely turbulent flow (see Fig. 2).

When \( R_s \) is smaller than 50, the value of \( \lambda \) is slightly smaller than the theoretical value of \( \lambda \) for the laminar flow. The curves of \( R_w = 1300 \) and 1500 have a similar tendency to that of \( R_w = 1000 \), and the range of \( R_s \) in which the flow keeps laminar state decreases with an increase in \( R_w \). When \( R_w \) is larger than 1800, the flow cannot keep laminar state.

When the clearance \( s \) is comparatively large, i.e. \( s = 0.618 \sim 3.315 \) mm, the
The relation between $\lambda$ and $R_e$ for various $R_w$ is shown in Figs. 8~12. In these cases, we obtain also the results similar to Fig. 7. From these experiments, we can see that when $R_w$ is large, the value of $\lambda$ can be expressed by Eq. (13) even for small $R_e$ (see Fig. 6). In Figs. 7~12, we adopted $R_w$ as a parameter, but if we take $R_w/R_e$ the curves of $\lambda$ vs. $R_w/R_e$ run approximately parallel to the curve of a turbulent flow for $R_w=0$. As an example, from Fig. 9 where $s=0.923$ mm, we obtain Fig. 13 by such a method.

As described above, the theoretical value of $\lambda(R_e/R_e^*)$ is equal to $cR_e^{0.5}$. When we define $R_e^*$ as the Eq. (17) and take $\lambda - R_w/R_e$ as the ordinate, $R_e^*$ as the abscissa, $R_w/R_e$ as the parameter, we obtain Fig. 14 where $s=0.923$ mm, as an example. From this figure, we can see that the value of $\lambda - R_w/R_e$ is larger than the theoretical value 0.26 $R_e^{0.24}$ or 0.27$R_e^{0.24}$. This is probably due to the fact that the value of $\lambda$ becomes larger than theoretical value owing to occurrence of Taylor vortices between co-axial cylinders. Asanuma obtained the following empirical formula from Cornish's experiment.

$$\lambda = 0.27R_e^{0.24}\left[1 + 0.5\left(\frac{R_w}{R_e}\right)^{3.9}\right]$$

When we define $R_e^*$ as follows

$$R_e^* = \left[|R_e| + 0.5R_e^*\right]^{1/2}$$

from Eq. (18) we have

$$\frac{R_w}{R_e} = 0.27R_e^{0.24}$$

In Fig. 15 where $s=0.923$ mm, $R_e^*$ is defined by the Eq. (19).

The value of $k$, used to define $\nu_{e}$ or $R_e^*$, is 7/8, if we accept the assumption described above. But $k$ is considered to be a correction factor to convert the velocity of the moving wall to the mean velocity of the pipe flow which has the same velocity profile near the stationary wall, and hence the same shearing stress on the wall, as on the moving wall. In Asanuma's empirical formula, the value of $k$ is equal to $\sqrt{2}$, this value is much greater than the writers' value.

The relationship between $\lambda$ and $R_w$ for various values of $R_e$ and $s/r_1$ is shown in Fig. 16, together with broken lines calculated from the Eq. (13) for comparison. When $R_w$ is large, the value of $\lambda$ is in good agreement with the value computed by Eq. (13), and for the small values of both $R_e$ and $R_w$, the lines are almost parallel to the abscissa. These parallel lines indicate that the flow is laminar. When $R_e$ is smaller than about 1 000, and $R_w$ is large beyond a certain value, the value of $\lambda$ increases sharply and varies extremely with the magnitude of $s/r_1$. For instance, in the case of $R_e=250$ the value of $\lambda$ increases with an increase in $s/r_1$, for $R_w<2 000$, but for $R_w>2 000$ $\lambda$ decreases with an increase in $s/r_1$. The value
of $R_w$ at which $\lambda - R_w$ curves cross each other, increases with an increase in $R_w$. In the neighborhood of the crossing points, the value of $\lambda$ decreases sometimes with an increase in $R_w$.

In the case of curves for $R_w = 2.500$, such a phenomenon that the value of $\lambda$ sharply increases as the $R_w$ increases beyond a certain value, as described above, cannot be recognized. The value of $\lambda$ increases gradually with an increase of $R_w$ in like manner as the broken line, and the magnitude of $\lambda$ is larger than the value of the broken line.

In Fig. 16, marks $+$ and $\times$ show the experimental results by Cornish and Suzuki respectively. Both experiments are carried out for smaller magnitudes of $s/r_1$ than that of our experiment, and the values of $\lambda$ obtained from these experiments show good agreement with the values of our experiment considering the effect of $s/r_1$. When both values of $R_w$ and $R_e$ are small, the variation of $\lambda$ for several values of $s/r_1$ is very large as shown in Fig. 16. Cornish mentioned that the value of $\lambda$ is independent of $s/r_1$. This statement is based on the fact that his experiments are carried out in a narrow range of the small magnitude of $s/r_1$, i.e., $s/r_1 = 0.0086 \sim 0.0059$.

Tomita obtained the relation between pressure drop, discharge and rotating speed, using co-axial cylinders with a very wide clearance between a stationary inner cylinder and a rotating outer cylinder, i.e., $s/r_1 = 1$ (outer cylinder only) $\sim 0.178$. The data of Tomita for $s/r_1 = 0.178$ are rewritten by our dimensionless expression assuming the temperature of water is 15°C, and are shown in Fig. 17, together with the curves given by Eq. (13) and our experimental curves for $s/r_1 = 0.115$ for comparison. In this figure, the value of $\lambda$ for rotating outer
5. Stability of a laminar flow between rotating co-axial cylinders

Taylor\(^{(1)}\) obtained the speeds of rotation at which the laminar flow breaks down and the Taylor vortices occur in a narrow annulus between rotating co-axial cylinders, for the case of a zero axial flow. When the inner cylinder rotates and outer cylinder is stationary, the critical angular velocity is expressed by the following formula

\[ \omega_c^2 = \frac{\pi^2 L^2 (r_1 + r_2)}{2P s^3 r_2^2} \]  

where

\[ P = 0.0571(1 - 0.652s/r_1) + 0.00056(1 - 0.652s/r_1)^{-1} \]

If we denote the critical rotating Reynolds number, \( r_1 \omega_0 s^2 / \nu \), by \( R_{cw} \), we get from Eq. (21)

\[ R_{cw}(s/r_1)^{1/2} = \frac{\pi^2}{P u / T}(1 + s/2r_1)^{1/2} \]  

In Table 1, the values of \( R_{cw} \) are computed by this equation. When \( s/r_1 \) is nearly equal to 1, the critical Taylor number \( R_{cw}(s/r_1)^{1/2} \) is equal to 41.1.

The boundary lines, at which the value of \( \lambda \) increases sharply from the theoretical value for the laminar flow when \( s/r_1 = 0.013 \), \( 0.0196 \), and \( 0.0296 \), are shown in Fig. 18, where the ordinate is the critical Taylor number \( R_{cw}(s/r_1)^{1/2} \) and the abscissa is the critical axial Reynolds number \( R_w \). And the boundary lines obtained by the other authors are also indicated in the figure for comparison.

By these three experiments we can see that the critical Taylor number \( R_{cw}(s/r_1)^{1/2} \) increases with an increase in \( R_w \) up to the \( R_w \) of about 600 and with an increase in \( s/r_1 \). In this range, these lines show the border line between the laminar region and the laminar-Taylor vortices mentioned in Fig. 2. When \( R_w \) increases beyond this value, the value of \( R_{cw}(s/r_1)^{1/2} \) decreases rather with an increase in \( R_w \). Therefore, these lines show the border between laminar and turbulent flow.

The values by Cornish\(^{(2)}\) are determined by his measurement of pressure drop in the same manner as these experiments and show a similar tendency to that of our experiments considering the effect of \( s/r_1 \). When \( R_w \) is very small, however, the value of \( R_{cw}(s/r_1)^{1/2} \) is greater than the theoretical value.
by Taylor, because the estimation of the value of $R_{ue}$ from measurement of $\lambda$ is impossible when $R_e$ is very small, because the occurrence of Taylor vortices not always makes the flow resistance so large as seen in Figs. 7~8. The experimental data by Fage\textsuperscript{(19)}, shown in Fig.18, are also obtained through the measurement of pressure drop. Extending the line to $R_e=0$, the line passes the theoretical value of Taylor, but the tendency and the magnitude of the line are generally quite different from ours and other experimental results. It may be because of his experimental error. Goldstein\textsuperscript{(9)} analyzed mathematically the stability of a flow through rotating co-axial cylinders. The value obtained by him is confined within a range of small $R_e$, so it is impossible to compare it with our experimental results. Kaye and Elgar\textsuperscript{(10)} obtained the border lines for two large clearance sizes of annuli by their observation of velocity fluctuation, using the hot wire anemometer. The value of $R_{ue}(s/r_1)^{1/2}$ obtained by them coincides with Taylors' theoretical value for $R_e=0$. When $R_e$ increases from zero, the values of $R_{ue}(s/r_1)^{1/2}$ for large clearance are nearly same as our results, but the values for small clearance are nearly equal to the values of Cornish.

6. Conclusion

The flow resistance of water through the co-axial cylinders when the inner cylinder rotates is studied experimentally. Experimental results are summarized as follows.

1) When $R_e > 10,000$, the coefficient of flow resistance $\lambda$ is almost independent of clearance ratio $s/r_1$ and shows a good agreement with theoretical value (Eq. (13)). Namely, when $R_e$ is constant and $R_e$ varies, the value of $\lambda$ decreases and approaches gradually to the value of a turbulent flow for stationary co-axial cylinders with an increase in $R_e$. On the other hand, in the range of small $R_e$, $\lambda$ becomes extremely larger than that for stationary co-axial cylinders. The $\lambda-R_e$ curves run parallel to the theoretical line of a laminar flow for stationary co-axial cylinders.

2) When the axial flow is laminar, the value of $\lambda$ is unaffected up to a certain value of $R_{ue}(R_{ue})$, but beyond this value $\lambda$ increases sharply with an increase in $R_e$. The critical Taylor number $R_{ue}(s/r_1)^{1/2}$ increases up to the value of about 600 with an increase in $R_e$ and $s/r_1$. While the axial flow is turbulent, such $R_{ue}$ cannot be recognized.

3) When the value of $R_e$ is a little larger than $R_{ue}$, the value of $\lambda$ increases extremely as $s/r_1$ increases.

4) When $R_e$ is very small, the value of $R_{ue}$ cannot be obtained by means of pressure drop measurement.

5) We defined the resultant Reynolds number and by using this number we can discuss the theoretical formulas and experimental results.

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References