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Heat Transfer of Slurry Flow with Internal Heat Generation*

(1st Report, Fully-Developed Laminar Heat
Transfer in a Circular Tube)

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Recently, the development of a homogeneous reactor which uses an aqueous slurry of uranium oxide or thorium oxide as a reactor fuel or a fertile material has been desired. In this case, the fluid is not a Newtonian fluid but rather a Bingham plastic, and the heat transfer of fluid flow with internal heat generation should be analyzed.

This paper presents the analytic solutions of the fully-developed heat transfer of a Bingham plastic with a uniform heat source in a laminar flow through a circular tube with uniform wall heat transfer beyond the entrance region of velocity and temperature. From the results shown in figures, it is clarified that the fluid-temperature distribution, the mixed-mean fluid temperature and Nusselt number are affected heavily both by the ratio of the yield stress to shearing stress at wall and by the wall heat transfer.

1. Introduction

The development of a homogeneous reactor which uses the suspensions of uranium oxide or thorium oxide in heavy water or light water as a reactor fuel or a fertile material has been recently desired. In such a reactor, there are many advantages: for example; high power density and high burnup of fuel are expected, fuel-preparation and reprocessing are simple, and continuous addition or removal of fuel can be accomplished since there is no solid fuel element. From the viewpoint of heat transfer, therefore, it is necessary to study the heat transfer of a non-Newtonian fluid with internal heat generation.

The purpose of this research is to clarify the temperature distribution of fluid and heat transfer at wall which are important factors for reactor design, and this paper presents the analytic solutions of temperature distribution of a laminar flow of a Bingham plastic with a uniform heat source through a circular tube with uniform heat transfer at wall, in the case where both temperature- and velocity-profile are fully developed.

Grigull⁽¹⁾ analyzed the fully-developed laminar

heat transfer of non-Newtonian fluid without an internal heat source. Poppendiek⁽²⁾ analyzed the fully-developed heat transfer both in the laminar and in the turbulent pipe flow of a Newtonian fluid with an internal heat source. On the other hand, Sparrow and Siegel⁽³⁾ obtained the analytic solutions of heat transfer in the entrance region of temperature in the laminar pipe flow of a Newtonian fluid with an internal heat source, and Schechter and Wissler⁽⁴⁾ extended the work of Sparrow et al. to a Bingham plastic. But, they treated only the case of a thermally insulated wall. Therefore, this first report deals with the fully-developed laminar heat transfer of a Bingham plastic both with internal heat generation and with wall heat transfer. The reason why the aqueous slurry of UO_2 or ThO_2 was assumed a Bingham plastic is that there are several reports⁽⁵⁾ of data of yield stress τ_y and viscosity μ_B of an aqueous slurry of UO_2 or ThO_2 .

2. Velocity distribution

The coordinates are shown in Fig. 1, where R

Fig. 1

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is the radius of a circular tube.

The shearing stress τ of a Bingham plastic is expressed by the following well-known equation:

$$\tau = \tau_y - \mu_B \frac{du}{dr} \quad (1)$$

where

τ : shearing stress

τ_y : yield stress

μ_B : viscosity of a Bingham plastic

u : velocity of fluid in the direction of x

According to experiments⁽⁵⁾, in the case of UO_2 aqueous slurry

$$\tau_y = k_1 \phi^4, \quad \mu_B = \mu \exp[k_2 \phi] \quad (2)$$

and in the case of ThO_2 aqueous slurry

$$\tau_y = k_3 \phi^3, \quad \mu_B = \mu \exp[k_4 \phi] \quad (3)$$

where

ϕ : volume fraction of solids in the suspension

μ : viscosity of water

k_1, k_2, k_3, k_4 : constant which depends on particle size

If the particle size of UO_2 or ThO_2 is equal to 1.4 microns,

$$\left. \begin{aligned} k_1 &= 150 \text{ lb/ft}^2, & k_2 &= 1.8 \\ k_3 &= 60 \text{ lb/ft}^2, & k_4 &= 0.8 \end{aligned} \right\} \quad (4)$$

Since the Bingham plastic has yield stress τ_y , the velocity profile of a fully-developed laminar pipe flow presents not a parabolic but a plug which means the uniform velocity in the core⁽⁶⁾. If the radius of the plug is r_y ,

$$\frac{\tau}{\tau_w} = \frac{r}{R} \quad (R \geq r \geq r_y) \quad \frac{\tau_y}{\tau_w} = \frac{r_y}{R} \equiv a \quad (5)$$

where

τ_w : shearing stress at pipe wall

r_y : radius of plug

a : specific radius of plug

From Eqs. (1) and (5), the velocity profile becomes

Fig. 2 Velocity distribution

$$0 \leq \frac{r}{R} \leq a :$$

$$u = \frac{R\tau_y}{\mu_B} \frac{(1-a)^2}{2a} \quad (6 \cdot a)$$

$$a \leq \frac{r}{R} \leq 1 :$$

$$u = \frac{R\tau_y}{\mu_B} \frac{1-2a+2a\left(\frac{r}{R}\right) - \left(\frac{r}{R}\right)^2}{2a} \quad (6 \cdot b)$$

Meanwhile, the mean velocity u_m is defined by

$$u_m = \frac{1}{\pi R^2} \left[\int_0^R u(2\pi r) dr \right]$$

Applying Eqs. (6) to the above equation,

$$u_m = \frac{R\tau_y}{\mu_B} \frac{a^4 - 4a + 3}{12a} \quad (7)$$

Fig. 2 shows the velocity profile u/u_m . The case of $a=0$ corresponds to a Newtonian fluid. It is clearly shown that there exists a plug in the Bingham plastic flow.

As mentioned above, both yield stress τ_y and viscosity μ_B are related to particle size and volume fraction. If we apply Eqs. (2), (3) and (4) to Eq. (7), the relationship between u_m/R and a is obtainable, which is illustrated in Fig. 3 for the cases of UO_2 -water slurry of 100°C and ThO_2 -water slurry of 100°C . From this figure, the specific radius a of the plug may be recognized as a key number of Bingham plastic flow.

Fig. 3 Example of a for UO_2 - and ThO_2 -aqueous slurry

3. Temperature distribution

The energy equation in steady state may be written as follows :

$$u \frac{\partial T}{\partial x} = \frac{\lambda}{c\gamma} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{W}{c\gamma} \quad \dots\dots\dots (8)$$

where

T : temperature

λ : thermal conductivity of fluid

c : specific heat of fluid

γ : specific weight of fluid

W : rate of internal heat generation per unit volume of fluid

If the temperature profile is fully developed, $\partial T / \partial x$ is independent of r , and

$$\frac{\partial T}{\partial x} = \frac{\partial T_m}{\partial x} \quad \dots\dots\dots (9)$$

where

T_m : mixed-mean fluid temperature

$$T_m = \frac{\int_0^R Tu(2\pi r)dr}{\pi R^2 u_m} \quad \dots\dots\dots (10)$$

And the following equation should be satisfied :

$$W\pi R^2 dx - q(2\pi r)dx = \pi R^2 u_m c \gamma \frac{\partial T_m}{\partial x} dx \quad \dots\dots\dots (11)$$

where

q : heat flux from fluid to outside ($q \geq 0$)

From Eq. (11),

$$\frac{\partial T_m}{\partial x} = \frac{W - \frac{2}{R}q}{u_m c \gamma} \quad \dots\dots\dots (12)$$

Putting Eqs. (12) and (9) into Eq. (8),

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) = \frac{W}{\lambda} \left(F \frac{u}{u_m} - 1 \right) \quad \dots\dots\dots (13)$$

where

$$F = 1 - \frac{2q}{WR} \quad \dots\dots\dots (14)$$

Accordingly, the thermally insulated wall corresponds to $F=1$. If not so, F is an arbitrary number between 1 and 0. Of course, F is negative if heat is transferred to the fluid from outside. When $F=0$, the mixed-mean temperature of the fluid is kept constant in the direction of x , and it is indicated

The temperature distributions calculated are as follows :

$0 \leq r' \leq a$:

$$\frac{T - T_w}{\frac{WR^2}{2\lambda}} = \frac{1}{2}(1 - r'^2) + \frac{3F(1-a)^2 r'^2}{a^4 - 4a + 3} - \frac{F}{12} \frac{13a^4 - 40a + 27}{a^4 - 4a + 3} + \frac{Fa^4 \ln a}{a^4 - 4a + 3} \quad \dots\dots\dots (15 \cdot a)$$

$a \leq r' \leq 1$:

$$\frac{T - T_w}{\frac{WR^2}{2\lambda}} = \frac{1}{2}(1 - r'^2) + \frac{F}{12} \frac{36(1-2a)r'^2 + 32ar'^3 - 9r'^4 - (27-40a)}{a^4 - 4a + 3} + \frac{Fa^4}{a^4 - 4a + 3} \ln r' \quad \dots\dots\dots (15 \cdot b)$$

Fig. 4 Temperature distribution

from Eq. (13) that temperature profile is independent of velocity profile u/u_m . Therefore, F may be recognized as a key number of heat transfer at wall.

The boundary conditions of fundamental differential Eq. (13) are

$$r=R : T = T_w \quad (T_w : \text{wall temperature})$$

$$-\lambda \left(\frac{dT}{dr} \right) = q$$

Since the velocity distributions are given by Eqs. (6·a) and (6·b), the analysis must be accomplished in two divided regions, i.e., region 1 ($0 \leq r/R \leq a$) and region 2 ($a \leq r/R \leq 1$). And the following conditions must be satisfied :

$$\frac{r}{R} = a : (T)_1 = (T)_2$$

$$\left(\frac{dT}{dr} \right)_1 = \left(\frac{dT}{dr} \right)_2$$

where the suffix shows the region.

Table 1

$r' = r/R$	$F=0$					$F=1/2$					$F=3/4$					$F=1$				
	Independent of a					$a=0$					$a=0.25$					$a=0.5$				
	$(T - T_w)/(WR^2/2\lambda)$					$a=0$					$a=0.25$					$a=0.5$				
0	0.5	0.125	0.1441	0.1732	0.2096	0.25	0.2096	0.1732	0.1441	0.125	0.0625	-0.0338	0.0099	0.0644	0.125	-0.2118	-0.1535	-0.0808	0	0
0.1	0.495	0.125	0.1433	0.1717	0.2076	0.2475	0.2076	0.1717	0.1433	0.125	-0.0325	-0.0325	0.0102	0.0638	0.12375	-0.2084	-0.1514	-0.0799	0	0
0.2	0.48	0.1248	0.1409	0.1673	0.2015	0.24	0.2015	0.1673	0.1409	0.125	-0.0285	-0.0285	0.0111	0.0622	0.12	-0.1981	-0.1453	-0.0771	0	0
0.25			0.1392								-0.0256					-0.1904				
0.3	0.455	0.1240	0.1370	0.1600	0.1913	0.2275	0.1913	0.1600	0.1370	0.125	-0.0220	-0.0220	0.0125	0.0594	0.11375	-0.1809	-0.1350	-0.0725	0	0
0.4	0.42	0.1218	0.1314	0.1497	0.1770	0.21	0.1770	0.1497	0.1314	0.105	-0.0129	-0.0129	0.0146	0.0555	0.105	-0.1573	-0.1206	-0.0660	0	0
0.5	0.375	0.1172	0.1235	0.1364	0.1587	0.1875	0.1587	0.1364	0.1235	0.08375	-0.0022	-0.0022	0.0173	0.0505	0.08375	-0.1406	-0.1020	-0.0576	0	0
0.6	0.32	0.1088	0.1124	0.1203	0.1363	0.16	0.1363	0.1203	0.1124	0.08	+0.0037	+0.0037	0.0204	0.0444	0.08	-0.1024	-0.0794	-0.0475	0	0
0.7	0.255	0.0950	0.0967	0.1005	0.1098	0.1275	0.1098	0.1005	0.0950	0.06375	0.0176	0.0176	0.0232	0.0372	0.06375	-0.0650	-0.0540	-0.0354	0	0
0.75											0.0207	0.0216	0.0234	0.0332		-0.0324	-0.0288	-0.0287	0	0
0.8	0.18	0.0738	0.0744	0.0756	0.0791	0.09	0.0791	0.0756	0.0744	0.045	0.0207	0.0216	0.0234	0.0332	0.045	-0.0324	-0.0288	-0.0287	0	0
0.9	0.095	0.0430	0.0431	0.0432	0.0437	0.0475	0.0437	0.0432	0.0431	0.02375	0.0170	0.0170	0.0172	0.0181	0.02375	-0.0090	-0.0086	-0.0076	0	0
1.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where $r' = r/R$

Fig. 4 illustrates the temperature distributions by using key numbers of F and a as parameter. It should be noted that the temperature distribution is heavily affected both by specific radius a and by wall heat transfer F , and the region where the fluid temperature is lower than the wall temperature appears under certain condition. The temperature profiles are tabulated in Table 1.

4. Mixed-mean temperature of fluid and Nusselt number

From Eqs. (10), (6·a), (6·b), (15·a), (15·b) and (7), the mixed-mean temperature T_m of fluid is obtainable:

$$\frac{T_m - T_w}{WR^2} = \frac{1}{a^4 - 4a + 3} \left[\frac{1}{10} (10 - 14a + 5a^4 - a^6) + \frac{F}{840(a^4 - 4a + 3)} (-3465 + 9792a - 6944a^2 - 1890a^4 + 2800a^5 - 1512a^6 + 3024a^7 - 1805a^8) + \frac{Fa^4}{12(a^4 - 4a + 3)} (12a^4 \ln a - 27 + 40a + 36a^2 - 72a^3 + 23a^4) \right] \dots (16)$$

In the above equation, if $a=0$ (Newtonian fluid)

$$\frac{T_m - T_w}{WR^2} = \frac{1}{24} (8 - 11F) \dots (17)$$

This is the same result as obtained by Poppendiek⁽²⁾ for the Newtonian fluid.

Fig. 5 Mixed-mean temperature of fluid

If $a=1$ (rod-like flow),

$$\frac{T_m - T_w}{WR^2/(2\lambda)} = \frac{1}{4}(1-F) \quad (18)$$

As shown in Fig. 5, the wall temperature T_w is equal to the mixed-mean temperature T_m when $F=8/11$ and $a=0$, but T_w is higher than T_m under a certain value of a if F is larger than $8/11$. When $F=1$, $T_m=T_w$ at $a=1$. Table 2 shows the numerical values of $(T_m - T_w)/(WR^2/2\lambda)$ for various numbers of F and a .

Heat transfer coefficient α is defined by the following equation :

$$\alpha = \frac{q}{T_m - T_w} \quad (19)$$

and the Nusselt number N_u is

$$N_u = \frac{\alpha(2R)}{\lambda} \quad (20)$$

Using Eq. (14),

$$N_u = \frac{2(1-F)}{\frac{T_m - T_w}{WR^2/(2\lambda)}} \quad (21)$$

Consequently, the Nusselt number can be calculated by using Table 2.

Fig. 6 shows the Nusselt number. The Nusselt number becomes 8 without dependence on F when $a=1$ as deduced from Eqs. (18) and (21). When $F=8/11$ and $a=0$, N_u tends to infinity since $T_m=T_w$ as shown in Fig. 5. When $F=3/4$, then $T_m=T_w$ at $a \approx 0.25$, $T_m < T_w$ for $0 \leq a < 0.25$ and $T_m > T_w$ for $0.25 < a \leq 1$. Hence, N_u is negative for $0 \leq a < 0.25$ and positive for $0.25 < a \leq 1$ according to the definition of Eq. (21). The dotted line drawn in Fig. 6 indicates that the Nusselt number N_u is negative according to Eq. (21), and arrows mean that N_u tends to infinity. When $F=1$, $N_u=0$ since the wall is thermally insulated. The chain line illustrates the results obtained by Grigull⁽¹⁾ in the case of $W=0$. When there is no internal heat generation, the Nusselt number is independent of F and $N_u=48/11$ for $a=0$ and $N_u=8$ for $a=1$.

5. Conclusion

The analytic solutions obtained are applied for various conditions of F and a as shown in figures. For example, since the specific radius a of a plug is obtainable from the particle size, volume fraction of solids and the mass flow rate through a given pipe as shown in Fig. 3, the temperature distribution and the mixed-mean fluid temperature corresponding to the rate of internal heat generation can be

Fig. 6 Nusselt number

Table 2 Numerical values of $\frac{T_m - T_w}{WR^2/2\lambda}$

	$F=0$	$F=1/4$	$F=1/2$	$F=8/11$	$F=3/4$	$F=1$
$a=0$	0.3333	0.2188	0.1042	0	-0.0104	-0.1250
$a=0.25$	0.3253	0.2171	0.1089	0.0105	0.0007	-0.1076
$a=0.5$	0.3103	0.2129	0.1156	0.0271	0.0182	-0.0791
$a=0.75$	0.2857	0.2039	0.1221	0.0477	0.0403	-0.0415
$a=1$	0.2500	0.1875	0.1250	0.0682	0.0625	0

calculated if the wall heat transfer is known. On the other hand, the cooling conditions at wall required to limit the temperature difference between fluid and wall can be calculated.

It should be noted that the temperature distribution of fluid is heavily affected by the particle size, solid concentration and cooling condition at wall if a slurry is used for nuclear reactor.

References

- (1) U. Grigull: *Chem. Ing. Techn.*, Vol. 28 (1956), p. 553.
- (2) H.F. Poppendiek: *Chem. Engng. Progr. Sympo. Ser.*, Vol. 50, No. 11 (1954), p. 93.
- (3) E.M. Sparrow and R. Siegel: *Nucl. Sci. Engng.*, Vol. 4 (1958), p. 239.
- (4) R.S. Schechter and E.H. Wissler: *Nucl. Sci. Engng.*, Vol. 6 (1959), p. 371.
- (5) J.A. Lane, H.G. MacPherson, and Frank Maslan: *Fluid Fuel Reactors* (1958), Addison-Wesley; *Nuclear Engineering*, Vol. 18, (1960), p. 104.
- (6) Y. Tomita: *Jour. Japan Soc. Mech. Engrs.*, Vol. 63, No. 503 (1960), p. 1596.