620.178.322.3-181.2:539.431.019

Some Studies on the Rotating Bending Fatigue of Large Mild Steel Specimens*

(Part 4, Effect of Size on Fatigue Strength of Cracked Specimen and Propagation Rate of Fatigue Crack)

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This paper is succeeded by No. 2 and No. 3 reports of the same title in which propagation of fatigue cracks on large smooth surface and fillet specimens has been dealt with. In this paper, the author has studied the following two problems:

(1) The fatigue strength of large smooth specimens having a diameter of 130 mm with fine cracks artificially produced beforehand by sea water corrosion.

(2) The influence of size on the fatigue strength of cracked specimens and propagation speed of fatigue cracks.

The test results have revealed that the fatigue limit of steel specimens, 130 mm in diameter with fine corrosion cracks on the surface is about $\pm 11.0 \text{ kg/mm}^2$. This limit coincides with the result assumed by studying the crack propagation of specimens, 120 mm in diameter. Analytical investigation of the propagation speed of fatigue cracks of $60 \phi \text{ mm}$ and $20 \phi \text{ mm}$ specimens has shown that the fatigue limit of these specimens are assumed to be about $\pm 12.0 \text{ kg/mm}^2$ and $\pm 15.0 \text{ kg/mm}^2$ respectively, when fine cracks may previously exist on their surfaces.

From the above results, it may be stated that the fatigue limit of smooth specimens having fine cracks on their surface will tend to increase as the diameter decreases.

On the other hand, from the analytical studies on the shape of shelly patterns on the fracture surface, the propagation of fatigue crack may be given approximately by the following formulae in connection with the diameter of specimen:

$$\sigma_0 \leq \nu_b \sigma_n < \sigma_0 + \sigma_s, \quad v_c = \frac{\nu_b \sigma_n - \sigma_0}{K + k/d^2}$$

where: v_c is propagation speed of fatigue crack μ/c , $\nu_b \sigma_n$: stress on the edge line of fracture surface kg/mm^2 , σ_n : stress on the specimen having no crack, ν_b : stress magnifying factor due to crack, σ_0 : fatigue limit of cracked specimen kg/mm^2 , σ_s : yielding point of the material kg/mm^2 , K and k : constants varying according to the properties of material and range of $\nu_b \sigma_n$, d : diameter of specimen mm.

Introduction

In the previous papers of the same title, the author has described the results of plain cylindrical test in Part 1⁽¹⁾, the propagation rate of fatigue crack on plain cylindrical specimens in Part 2⁽²⁾, and fatigue strength and propagation rate of crack

on shoulder-fillet specimens in Part 3⁽³⁾. He has reported also the results of fatigue tests on large specimens subjected to sea water corrosion⁽⁴⁾ and on the specimens fitted to flanges⁽⁵⁾, respectively.

From the results of the above-mentioned tests, it has been found that the ratio between fatigue strength of large specimens and that of small ones, which represents the size effects on the fatigue strength, is 0.80 to 0.90, 1.0, 2.4 and 0.57 to 0.66 for each specimen of the plain cylinder, shoulder-fillet,

* Received 20 th July, 1960.

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sea water corrosion and one fitted to flange, respectively.

Then, it has been recognized that the fatigue strength of specimens having fine cracks originated on the surface, which is subjected to low level of reversal stress under sea water corrosion or fretting corrosion, is more remarkably affected by the size of specimen than those of the plain cylindrical or shoulder-fillet specimens on which the fatigue crack appeared at the final stage of the tests.

In this paper the author describes the results of fatigue tests on large specimens with fine cracks, and studies the size effect on the propagation rate of fatigue cracks in plain cylindrical specimens 120, 60, and 20 mm in diameter. An analysis is also made on the mechanism of crack propagation to explain the difference of the above-mentioned size effects.

1. Fatigue tests on large specimens with fine cracks

1.1 Testing machine, test materials and specimen

The tests were carried out by using the large rotating bending fatigue machine⁽¹⁾ (7.5 t-m, 1 250 r.p.m.).

The specimens were machined from 0.2% carbon steel refined by 50 t acid open-hearth. Their mechanical properties and the chemical compositions will be tabulated in Table 2 of paragraph 2.1.

Fig.1 illustrates the shape and size of the specimen. The middle part of the specimen 128.08 to 133.85 mm in diameter was polished by emery paper to the surface roughness of $3\mu H_{\max}$ and the fine cracks were originated by sea water corrosion fatigue at this part of the specimen.

The crack was initiated in the same way as used for the sea water corrosion fatigue test, that is, the testing part of the specimen was covered by

a case made of brass plate and 3% salt water of two or three liters per one hour was poured on this part. This treatment was made under the rotating bending stress of 6 to 8 kg/mm² with 10^7 cycle endurance.

Since the surface subjected to sea water corrosion was covered by thick rust, the surface was at first polished by emery paper until the metal surface are covered with many corrosion pits and fine cracks. Then the surface was washed with hot water so as to remove the corrosive solution remaining in the pits and cracks.

The fatigue tests on the cracked specimen were carried out in the air similarly to the case of the tests on plain cylindrical ones.

1.2 Test results

The test results on the cracked specimens are summarized in Table 1. In this table, the reversal stress in A16 specimen with a uniform diameter is 7.5 kg/mm² and 6.6 kg/mm² on the driving and the loading side of the specimen respectively, while in the other specimens the stress distribution is uniform.

Fig.2 illustrates the S-N diagram of the test results in Table 1 and those of the previous tests on the plain cylindrical specimens⁽¹⁾ and specimens⁽⁴⁾ corroded by sea water.

It is revealed that the fatigue limit of the cracked specimen is 11.0 kg/mm² which is 1.5 kg/mm² lower than that of the sea water corrosion fatigue

Fig.1 The size of cracked specimen

Table 1 Results of tests on cracked specimens

Treatment to originate cracks			Fatigue tests on cracked specimen			Conditions after the test
Number of specimen	Reversal stress kg/mm ²	Numbers of reversals	Test number	Reversal stress kg/mm ²	Numbers of reversals	
A 16	6.6~7.5	17.2×10^6	16-1	10.0	10×10^6	not broken
			16-2	11.0	10×10^6	not broken
			16-3	12.0	10×10^6	not broken
			16-4	13.0	10×10^6	not broken
			16-5	14.0	10×10^6	not broken
			16-6	15.0	4.5×10^6	broken
A 17	8.0	10.0×10^6	17	13.0	5.6×10^6	broken
A 18	6.0	10.0×10^6	18	12.0	6.6×10^6	broken
A 19	8.0	10.0×10^6	19-1	10.7	30.0×10^6	not broken
			19-2	16.0	0.96×10^6	broken

(of which the fatigue strength at 3×10^7 cycles was 12.5 kg/mm^2), corresponding to 55% of the plain cylindrical one. In this figure, it can also be noted that the fatigue strength of A16 specimen subjected to a reversal stress with a step by step increase ascends remarkably. Such increase in fatigue strength may be due to the effect of under-stressing on the cracked specimen.

The surfaces of specimen subjected to the crack initiation treatment are shown in Figs. 3(a), (b). In these figures, many pits and cracks initiated at the pits are observed.

From the result of the sea water corrosion fatigue test, it was found that the mean depth of cracks which were observed on the subsurface of longitudinal section of the specimen was in proportion to the logarithmic value of the reversal numbers as given by Eq. (1).

$$l_m = C \log \frac{N}{N_c} \dots \dots \dots (1)$$

where,

l_m : the mean depth of cracks, in mm

C : a constant, $C = 0.86 \text{ mm}$ for the sea water

corrosion fatigue

N : numbers of stress reversals

N_c : the numbers of stress reversal at which cracks will be initiated, $N_c = 1.7 \times 10^6$ for the test

However, it seems that the mean depth of cracks is smaller than that given by Eq. (1), when the reversal stress is lower than 8 kg/mm^2 . Namely the mean depth of cracks in A17 specimen subjected to 8 kg/mm^2 of reversal stress is nearly equal to that

Fig. 2 S-N diagram on cracked specimen

(a) Surface of A17

(b) Surface of A18

Fig. 3

(a) Tip of crack on cracked specimen

(b) Tip of crack on corroded specimen

Fig. 4

given by Eq. (1), whereas in A16 specimen subjected to 6.6 to 7.5 kg/mm² of reversal stress it is a little smaller than that given by Eq. (1), and only few cracks are observed on the surface of A18 specimen subjected to 6.0 kg/mm² stresses as shown in Fig. 3(b).

In spite of unevenness of the mean depth of cracks as mentioned above, the fatigue strength of cracked specimen has been plotted as a line as shown in *S-N* curve. From this fact, it may be presumed that the decline of fatigue strength on cracked specimen is mainly caused by the notch effect and is not affected by the depth of crack.

Figs. 4(a), (b) illustrate the tip of cracks of the cracked specimen and that of sea water corroded one when the tests have been finished. It is found that the tip of the former is sharp comparing with that of the latter one.

Fig. 5 illustrates the fracture surface of A18 specimen. It is noted that the surface is very smooth as the reversal stress is low, and many plaits representing the joint of cracks are observed on the circumference of the fracture surface.

2. Test on the propagation rate of fatigue crack

The author has given the propagation rate of fatigue crack on large plain cylindrical specimens⁽²⁾ and shoulder-fillet ones⁽³⁾. In this paper, the size effect on the propagation rate of the crack is studied by using the test results on fatigue of plain cylindrical specimens with a diameter of 120, 60 and

20 mm, respectively.

2.1 Testing machine and specimen

The 120 mm ϕ specimens have been tested by the large rotating bending machine⁽¹⁾, the 60 mm ϕ by 2 t-m rotating uniform-bending machine, which was newly designed for a test of the crank shaft specimen, and the 20 mm ϕ one by 30 kg-m cantilever-type machine. The revolution speeds of these machines are 1 250, 1 800 and 3 000 r.p.m., respectively.

The material of the test specimens is the same as of the cracked specimen. However, their mechanical properties measured in 1960 differed from the data obtained when the material was supplied in 1956 (see Table 2). It can be seen that the latest results are 2 kg/mm² higher in yield stress and 1 kg/mm² lower in tensile strength than the previous ones.

Fig. 6 illustrates the shape and size of 60 mm ϕ and 20 mm ϕ specimens, (those of the 120 mm ϕ specimen were shown in the previous paper⁽¹⁾). The surfaces of these specimens were polished by emery paper to the surface roughness of $3\mu H_{\max}$ for the 120 mm ϕ specimen and $1\mu H_{\max}$ for 60 ϕ and 20 mm ϕ specimens.

2.2 Fatigue strength

The fatigue strength of 120 mm ϕ , 60 mm ϕ and 20 mm ϕ specimens as well as of 10 mm ϕ ones tested recently for comparison with the previous ones is plotted in *S-N* diagram of Fig. 7. Although the fatigue limit of 120 mm ϕ specimen is not obtained, the size effect on the fatigue strength of the plain cylindrical specimen is clearly observed in this figure.

Fig. 5 Fracture surface of A18 specimen

Fig. 6 Sizes of 60 and 20 mm ϕ specimens

Table 2 Mechanical properties and chemical compositions

Year of test	Yielding strength kg/mm ²	Tensile strength kg/mm ²	Elongation %	Reduction of area %	Breaking stress kg/mm ²	Chemical compositions %
1956	24.0~24.5	47.0~46.4	37.2~36.2	58.1~63.2	not tested	C, 0.22 Si, 0.23 Mn, 0.42 P, 0.028 S, 0.034
1960	25.7~27.1	45.5~46.0	39.1~41.1	57.8~62.4	83.1~90.2	

Table 3 tabulates the fatigue limits and size effect as expressed in terms of the ratio of the fatigue limit of the large specimen to that of 10 mm ϕ one. Also shown in the table are the results of 125 mm ϕ and 10 mm ϕ specimens tested previously.

The fatigue limit of 10 mm ϕ is 2 kg/mm 2 higher than that of one tested previously, and this value corresponds to that of the yield strength. It may be presumed that such an increase of fatigue strength and yield strength is caused by the aging of the material.

Fig. 7 S-N diagram of 120, 60, 20 and 10 mm ϕ specimens

2.3 Measurement of crack propagation rate

The measurement of the propagation rate of the fatigue crack has been carried out by analysing the shelly-patterns which were obtained in A20 of 120 mm ϕ specimen, in M2, M3, M6 of 60 mm ϕ specimens and MS1, 2, 4, 5, 6 of 20 mm ϕ ones. The fracture surface of A16 of cracked specimen was also used for the analysis as an example of the large specimen.

Table 4 shows the steps of stress change for each specimen. In this table, the reversal stress σ_n denotes the bending stress on the surface of specimen without taking into account the effect of the crack, ΔN the numbers of stress reversals corresponding to the stress, and the numerals in the column beneath the specimen number representing the order of the fringe in the shelly-pattern as shown in Fig. 8(a), (b), (c).

2.4 Analysis on the propagation rate of the fatigue crack

Details of the method of obtaining the relationship between the propagation rate of crack and the

Table 3 Fatigue limits and size effects

Diameter of specimen mm		10	20	60	125
1958	Fatigue limit kg/mm 2	21.7	—	—	20.0
	Size effect σ_{wd}/σ_{w10}	1.00	—	—	0.921
1960	Fatigue limit kg/mm 2	23.7	23.0	21.7	—
	Size effect σ_{wd}/σ_{w10}	1.00	0.98	0.916	—

Table 4 Sequences of load changing (σ_n kg/mm 2 , $\Delta N \times 10^3$)

A20-2	σ_n	ΔN	A16	σ_n	ΔN	M2	σ_n	ΔN	M3	σ_n	ΔN	M6-2	σ_n	ΔN
0	24.0	860.9	0	15.1	4 349.1	0	22.6	2 698.8	0	23.8	1 025.4	0	25.5	499.9
1	12.6	38.0	1	7.66	15.5	1	7.08	9.2	1	9.90	6.4	1	11.32	112.6
2	22.6	16.9	2	12.15	21.1	2	19.80	17.6	2	12.45	26.8	2	19.81	55.9
3	15.05	17.3	3	7.66	6.7	3	8.50	17.8	3	17.00	35.2	3	12.73	149.7
4	10.1	17.8	4	14.30	12.1	4	15.60	10.8	4	9.90	23.0	4	22.63	25.8
5	22.6	7.9	5	9.72	17.0	5	11.30	22.1	5	19.82	17.4	5	14.15	40.9
6	12.6	24.0	6	14.30	12.4	6	18.40	4.9	6	9.90	11.1	6	18.40	45.7
7	16.3	13.4	7	9.72	9.8	7	8.50	48.2	7	15.58	14.9	7	11.32	58.2
8	8.87	30.5	8	13.20	9.2	8	17.00	10.4	8	9.34	12.5	8	19.81	18.9
9	16.05	10.4	9	7.66	10.5	9	7.08	11.4	9	12.75	27.9	9	12.73	33.4
10	10.1	7.5	10	13.20	12.2	10	11.30	37.6	10	7.08	7.6	10	18.40	15.3
11	17.5	6.2	11	7.66	10.0	11	18.40	3.3	11	17.00	10.0	11	11.32	28.1
12	12.6	8.4	12	12.15	11.7	12	7.08	42.2	12	11.33	15.0	12	18.40	12.5

MS1	σ_n	ΔN	MS2	σ_n	ΔN	MS4	σ_n	ΔN	MS5	σ_n	ΔN	MS6	σ_n	ΔN
0	26.5	420.0	0	24.5	1 404.9	0	23.5	5 708.2	0	25.5	900.0	0	24.0	2 520.0
1	16.3	24.2	1	12.4	138.2	1	11.0	388.2	1	13.7	118.7	1	15.0	123.6
2	11.5	78.6	2	17.5	29.7	2	18.8	10.3	2	20.1	20.0	2	21.2	13.9
3	18.8	10.3	3	13.7	49.8	3	8.6	269.4	3	16.3	57.3	3	12.4	50.2
4	12.4	22.5	4	18.8	5.2	4	13.7	33.9	4	21.2	10.2	4	17.5	30.2
5	—	—	5	—	—	5	16.2	30.1	5	11.1	158.0	5	13.7	45.2
6	—	—	6	—	—	6	20.1	17.6	6	16.3	19.9	6	18.8	15.0
7	—	—	7	—	—	7	10.0	28.3	7	18.8	7.4	7	13.7	72.0

Fig.8 Fracture surfaces and sketches of shelly-patterns

reversal stress have been reported in Part 2⁽²⁾. As was described in the previous paper the stress along the edge line of the fracture surface is defined as the maximum tensile stress in rotating beam with a cross section as reduced by the fracture part from the original cross section of the specimen (see Fig. 9). The ratio of the edge line stress at any arbitrary point "b" to the nominal stress has been given by Eq. (2) of the report⁽²⁾.

$$\nu_b = \frac{y \cos \alpha'}{r i_z} + \frac{z \sin \alpha'}{r i_y} \dots \dots \dots (2)$$

where,

$$\tan \alpha' = \frac{i_z}{i_y} \cdot \frac{z}{y}$$

y, z : ordinate of point "b", $\overline{O_G Y}$ and $\overline{O_G Z}$ represent the principal axes of the cross section (see Fig. 9)

i_y, i_z : ratio of the moment of inertia of the cross section (with respect to $\overline{O_G Y}$ and $\overline{O_G Z}$) to that of the original section

r : radius of the specimen

On the other hand, the propagation rate of crack in a unit of micron meter per one stress cycle is represented as a value of the normal length between two adjacent fringes of shelly-pattern divided by the number of stress reversals spent for the crack propagation through this length.

Fig. 9

Figs. 10, 11 and 12 illustrate the relationship between the propagation rates of cracks (ν_c) and the edge line stresses ($\nu_b \sigma_n$) for 120 mmφ, 60 mmφ and 20 mmφ specimens, respectively. In these figures, the points connected by a line correspond to an edge line of which the fringe order in shelly-pattern is suffixed.

It is seen from these figures that the propagation rate of fatigue crack depends on the edge line stress and their relation is affected by the size of the specimen. The relationship between the rate ν_c and the stress $\nu_b \sigma_n$ may be approximated by either a straight or a quadratic curve as shown in these figures.

3. Considerations on the test results

Based upon the results obtained in the foregoing article and those of previous papers⁽²⁾⁽³⁾, the size effect on the crack propagation rate will be

Fig. 10 Propagating rate and edge line stress on 120 mm ϕ specimens

Fig. 11 Propagating rate and edge line stress on 60 mm ϕ specimens

Fig. 12 Propagating rate and edge line stress on 20 mm ϕ specimens

discussed in the following.

3.1 Relationship between propagation rate and stress at edge line

The relationship between the rate v_c and the stress $\nu_b \sigma_n$ is, in general, given by Eqs. (3a), (3b) and (3c).

$$v_c = (\nu_b \sigma_n - \sigma_0')^2 / A \quad (3a)$$

$$v_c = (\nu_b \sigma_n - \sigma_0) / B \quad (3b)$$

$$v_c = (\nu_b \sigma_n - \sigma_0) / C \quad (3c)$$

The constants A, B, C, σ_0' and σ_0 are given in Figs. 10, 11 and 12, and Eqs. (6a), (6b) in Part 2⁽²⁾ for plain cylindrical specimens, while those for shoulder-fillet specimens involve the stress gradient in sub-surface as shown by Eqs. (14a), (14b) of Part 3⁽³⁾.

Since the stress gradient was approximately given by Eq. (13) of Part 3, the diameter of a plain cylindrical specimen with the same gradient as that of shoulder fillet is given by Eq. (4).

$$\phi_r = \frac{\sigma_n}{r} (5\alpha_k - 4)$$

$$d' = \frac{d}{5\alpha_k - 4} \quad (4)$$

where,

ϕ_r : stress gradient in shoulder fillet

α_k : stress concentration factor at the fillet

d' : equivalent diameter

Consequently, it can be considered that the relationship between the propagation rate and the edge line stress in a shoulder-fillet specimen is

similar to that in a plain cylindrical specimen having the equivalent diameter.

Table 5 tabulates the constants of plain cylindrical specimen of respective size and shoulder-fillet specimens of the equivalent diameter.

Figs. 13 and 14 illustrate the relationship between the constants and the inverse of specimen diameter, $1/d$, and approximate formulas for these constants in terms of the inverse of diameter are also given.

By substituting these formulas into Eqs. (3a), (3b) and (3c), the propagation rate of fatigue crack is generally given by Eqs. (5a), (5b) and (5c). The applicable ranges of each equation are determined from the mean values of the test results in the same figures.

$$\sigma_s \leq \nu_b \sigma_n$$

$$v_c = \frac{(\nu_b \sigma_n - \sigma_0')^2}{K_1 + k_1/d^2} \quad (5a)$$

$$1.6\sigma_0 \leq \nu_b \sigma_n < \sigma_0 + \sigma_s$$

$$v_c = \frac{\nu_b \sigma_n - \sigma_0}{K_2 + k_2/d^2} \quad (5b)$$

$$\sigma_0 \leq \nu_b \sigma_n < 2\sigma_0$$

○ Material A ● Material B

Fig.13 Constant A and $1/d$

$$v_c = \frac{\nu_b \sigma_n - \sigma_0}{K_3 + k_3/d^2} \quad (5c)$$

where,

σ_s : yield strength of the test material, in kg/mm²

K_1, K_2, K_3 , and k_1, k_2, k_3 : constants depending on the material

Table 6 tabulates the values of K_1, K_2, K_3 , and k_1, k_2, k_3 for the test material A used for the plain cylindrical specimen and B for the shoulder-fillet one.

3.2 Fatigue strength of the cracked specimen

Since the fatigue strength of the 120 mm ϕ specimen having fine cracks on its surface is 11.0 kg/mm² (see paragraph 2.2) this coincides with the edge line stress σ_0 at which the crack is arrested in the 120 mm ϕ specimen.

It is presumed, therefore, that the stresses σ_0

Table 6 Constants K and k

Steel	K_1	K_2	K_3	k_1	k_2	k_3
A	490	24	70	140.2×10^4	7.0×10^4	4.52×10^4
B	305	13.3	—	147×10^4	4.54×10^4	—

○ Material A ● Material B

Fig.14 Constants B, C and $1/d$

Table 5 Constants A, B, C, σ_0' and σ_0 in respective size of specimen

Steel A, Plain cylindrical				Steel B, Plain cylindrical and shoulder-fillet				
Diameter	120 mm	60 mm	20 mm	125 mm, Pl. cyl.	135 mm, Shoulder-fillet			
Equivalent dia.	—	—	—	—	104 (R=100)	70 (R=40)	48.3 (R=25)	38.5 (R=15)
A	580	—	4 000	400	400	650	900	1 300
B	29	48	200	17	16.5	25	33	44
C	100	200	1 200	—	—	—	—	—
σ_0' kg/mm ²	8.0	—	12.0	7.0	7.0	7.0	7.0	7.0
σ_0 kg/mm ²	11.0	12.0	15.0	11.5	11.5	11.5	11.5	11.5

note; R, radius of fillet in mm.

tabulated in Table 5 approximate the fatigue limits of cracked specimens with respective diameters. Under this presumption, it is recognized that the fatigue limits of cracked specimens are remarkably affected by the size of specimen, e.g., the fatigue limit of 120 mm ϕ specimen is only 75% of that of 20 mm ϕ specimen.

On the other hand, it should be noted that the fatigue limit of cracked specimens is remarkably influenced by the under-stressing effect as shown in Fig. 2 in the case of A16 specimen. This can be easily seen from the relationship between the crack propagation rate and the stress at edge line. For instance, the stress level above which a crack starts to propagate in the specimen is higher than the limit σ_0 , when the stress is increased from the lower level. This is more pronounced in the smaller specimens.

3.3 Mechanism of fatigue crack propagation

If the fatigue crack tip is considered as a groove notch having a radius of curvature ρ (see Fig. 15), then the material in the vicinity of the tip will deform plastically even at a low level of stress. In this case, the volume of the plastically deformed material will depend upon the stress, and if the crack propagation rate is governed by the volume of a plastically deformed material, then it can be explained that the crack propagation rate is related to the stress level.

In Fig. 15, the stress distribution at the tip of crack is shown by the dotted line while the stress remains within the elastic range, whereas it is given by full line after the stress exceeds the elastic limit.

Since the stress gradient in a large specimen is not so steep as that in a small one, the volume of the plastically deformed material may increase in proportion to the size of specimens which, thereby, would influence the crack propagation rate as well.

However, the difference in the actual stress gradient in the tested specimen was not so pronounced as was observed in the difference of the crack propagation rate. When a crack approaches to point A in Fig. 15, the stress at the same point will increase gradually, and the material in the vicinity of point A would be strengthened by the coxing effect. Since the fatigue cracks propagate always through the affected zone, their propagation rate will be mainly governed by the intensity of the coxing-effect.

Although the mechanical details of the coxing-effect has not been made clear yet, it seems that the strengthening action may be more remarkable if the stress increases gradually at a low crack propagation rate. Hence, it can be assumed that

Fig. 15

the propagation rate of a crack accompanied by a small quantity of the plastically deformed material will be lowered by the coxing-effect, and vice versa. This would explain the phenomena of the size effect as observed in the relationship between the crack propagation rate and the stress at the edge line of fractured surface.

4. Conclusions

The results obtained in this series of tests will be summarized in the following items.

(1) The bending fatigue limit of the cracked specimen having 120, 60 and 20 mm diameter are 11.0, 12.0 and 15.0 kg/mm², respectively, and the size effect on the cracked specimen is remarkably large.

(2) The relationship between the propagation rate of fatigue crack and the stress at the edge line is generally given by the following equation.

$$\sigma_0 \leq \nu_b \sigma_n < \sigma_0 + \sigma_s$$

$$v_c = \frac{\nu_b \sigma_n - \sigma_0}{K + k/d^2}$$

where,

v_c : distance of the crack propagation per one cycle of reversal stress, in μ/c

σ_s : yield strength of the tested material, in kg/mm²

σ_n : reversal stress on the surface of specimen at fully removed point from the crack, in kg/mm²

ν_b : magnification factor of the edge line stress due to the crack

σ_0 : the lower limit of the stress maintaining the crack propagation in the specimen, in kg/mm²

K, k : constants depending on the material and the stress range

d : diameter of the specimen in mm

(3) The size effect on the propagation rate

of the fatigue crack can be explained by the coaxing-effect and the volume of plastically deformed material at the tip of the crack.

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Discussion

K. NISHIOKA : I should like to ask two questions.

(1) Does the $S-N$ curve for corrosion fatigue in salt water change according to the stress cycles per minute?

(2) What do you consider is the influence of inclination of $S-N$ curve on the rate of crack propagation which I have pointed out before^(a)?

H. NAKAMURA : (3) According to our previous experiments, the fatigue limits of cracked specimens (designated by σ_{wc}) were 10 kg/mm², where the material is SS41 and the diameter 10 mm, and 6 to 7 kg/mm² in case of the 0.35% C steel with 50 mm diameter. Your test results gave higher values of σ_{wc} ; 11, 12 and 15 kg/mm² in case of 120, 60 and 20 mm diameter, respectively.

It is doubtful whether the difference between the two would only depend upon the properties of material.

The value σ_{wc} seems to be raised by the effect of work-hardening and residual stress caused by the stress amplitude variation.

To find the value σ_{wc} or the propagation rate of crack, you introduced a method of alternately increasing and decreasing the stress amplitude, and we used the super-sonic inspector.

I wish to ask your opinion about the influence of the difference between the two on the value σ_{wc} .

Author's closure

(1) It has been reported by McAdam^(b) and Endo^(c) that the fatigue strengths of steels corroded by salt water have been influenced by the frequency of stress cycle. However, it should be noted that these test results have only dealt with the small specimens.

Although the cracks initiated by corrosion

fatigue decrease remarkably the sectional area of a small specimen, the cracks decrease only negligibly the sectional area of a large specimen, because the depth of the cracks in large specimen might be nearly equal to those of the small ones.

Therefore, it can be presumed that the fatigue strength of the small specimen subjected to salt water corrosion is governed by the rate of crack propagation, which is related with the frequency of stress cycle, while, that of the large specimen is not affected.

(2) It may be presumed that the inclination of $S-N$ curve would have some influence on the rate of crack propagation. However, the influence has not been observed in the author's test results, as the two kinds of materials used for the tests have a similar inclination (see the Part 1).

(3) The fatigue strength of the cracked specimen is generally represented by the nominal stress without taking into account the reduction of the cross sectional area due to the cracks. In the case of Mr. Nakamura's test results, it might be presumed that the depth of cracks was comparatively large and the reduction of sectional area of specimen could not be neglected. However, in the case of the author's tests, the depths of cracks were very small comparing with the size of specimen and the sectional area of specimen was not decreased.

The author presumes that the difference between the above two results can be explained as the effect of the crack depth and is not caused by the difference of the mechanical properties.

Concerning to the effect of the alternative increase and decrease of the stress, it would be considered that the fatigue strength will ascend due to the under-stressing effect when the stress is increased, but the fatigue strength will not be influenced when the stress is decreased. Therefore, it may be expected that the stress limit σ_0 maintaining the crack propagation will not be influenced by the stress changing, as the limit was obtained in the decreasing stage of the stress changing.

(a) M. Kawamoto and K. Nishioka: *Trans. Japan Soc. Mech. Engrs.*, Vol. 19, No. 83 (1953), p.86.

(b) McAdam: *Congress Intern. l'essai Matériaux* (1928), p.305.

(c) Endo and Miyao: *Trans. Japan Soc. Mech. Engrs.*, Vol.24, No.139, (1958), p.167.