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Cascade Performance with Accelerated or Decelerated Axial Velocity*

By Shigeo KUBOTA**

A theoretical method to estimate the effect of axial velocity change through a cascade was investigated. The change of axial velocity was reproduced by distributing sinks and sources within the blade passages, and the conclusions are set forth in some simple formulae. Some graphs for the numerical evaluation of the performance of NACA 65 series cascades were prepared, and several examples were compared with experimental data.

1. Introduction

The study of cascade performance with a changing axial velocity within the blade passages is important for the design of axial flow turbomachinery. In most of the multi-stage axial flow compressors and turbines, the magnitude of axial velocity varies considerably from the first to the last stage, because of the design requirements. Small axial velocity changes accumulate from each stage. Therefore, it is somewhat inexact to use the experimental data of cascade tests which are obtained under the condition of constant axial velocity, without any correction.

There is another reason why this problem is important. In the usual cascade wind tunnel test, owing to the development of a boundary layer on the surface of the side wall of the cascade, the axial velocity increases at the midspan through the cascade. In order to keep the axial velocity constant, the boundary layer must be sucked off in

some way, making the experimental technique more time consuming. Therefore, if the experimental data without any boundary layer removal can be corrected to the case of constant axial velocity, there is a certain possibility of reducing the difficulty of cascade testing very much.

N. Scholz⁽¹⁾, S. Katzoff⁽²⁾ and W.R. Hawthorne⁽³⁾ wanted to solve this problem and devised some equations to correct such an effect. However, since their treatment was based on a simple one-dimensional consideration, it seems to be insufficient to explain such a complicated phenomenon. J.R. Erwin and J.C. Emery⁽⁴⁾ proposed three empirical methods to correct the turning angle but, as the authors themselves said they didn't cover any of the experimental evidence. Later, T. Kawasaki⁽⁵⁾ carried out a method based on the two-dimensional treatment. In his method, he put a concentrated source at the middle point of the blade passage corresponding to the increment of the axial velocity. However, there is some doubt whether or not it is proper to express the effect of the axial velocity increment by such a concentrated singularity. The method here developed is based on the idea that sources

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are distributed in the whole domain of the blade passage, corresponding to the increase or decrease of the axial velocity. This assumption seems to be a more accurate expression of the phenomenon.

Generally speaking, there are many factors which would be related to the change of axial velocity. They are; hub ratios at the inlet and exit of the stages, accumulation of boundary layer thickness through stages, effect of compressibility, effect of secondary flow and leakage or extraction at any stage. In most cases, one might consider that all of these effects appear separately. The method here described can cover all of the effects mentioned above, except the effect of compressibility and secondary flow. The correction regarding these two factors must be taken into account in a proper way, if it is necessary.

2. Flat plate cascades

First consider the most simplified cascade, that is, a cascade composed of flat plates. In this theoretical consideration the following basic assumptions are made; the flow is steady, two-dimensional, non-viscous and incompressible. The cascade to be considered is composed of flat plates, whose chord length is c , spacing is s , stagger angle is γ as shown in Fig. 1. At the inlet of the cascade the flow is quite uniform, and its direction is changed by the cascade, flowing off to right. In the cascade, from $\xi = -(c/2)\cos\gamma$ to $\xi = (c/2)\cos\gamma$, the axial velocity of the flow is accelerated (or inversely, decelerated) from v_{a1} to v_{a2} .

In such a case our problem is to determine the performance of the cascade. That is, given the values of mean velocity direction measured from the chord line α , the magnitude V , and the changing condition of axial velocity within the blade passage $\Delta v_a(\xi)$, to calculate the value of the circulation around a blade Γ as a function of γ , $\lambda = s/c$ and $\Delta v_a(\xi)$. In this paper, variations of axial velocity were treated as a function of ξ only, and independent of η .

To solve this problem, we divide the flow into three elementary parts:

(i) Flow around the given cascade with a given mean velocity vector, without any change of axial velocity.

(ii) Flow consisting of a source and sink distribution only, corresponding to the given axial velocity change (the boundary condition is not satisfied).

(iii) Flow to cancel the normal velocities on the blade surface which were induced by the flow

Fig. 1 Flat plate cascade and velocity vector

Fig. 2 Relation between source distribution and axial velocity change

(ii), and an additional circulation to satisfy the Kutta-Joukowski's condition at trailing edge.

By superimposing the three elementary flows above, we can obtain the flow which satisfies all the prescribed conditions.

2.1 Flow (i)

The solution of Flow (i) is already well-known, and in this paper the conclusive equation only shall be given,

$$\Gamma = 4s\kappa \sin \alpha \sqrt{1 + 2\kappa^2 \cos 2\gamma + \kappa^4} \quad \dots\dots\dots (1)$$

where, κ is a parameter depending upon the values of the blade pitch-chord ratio λ , and the stagger angle γ . Although the functional relation $\kappa = f(\lambda, \gamma)$ is rather complicated, we can easily find it in the numerical tables⁽⁶⁾.

2.2 Flow (ii)

Next the relation between the source or sink distribution and the variation of axial velocity must be examined. As shown in Fig. 2, consider the source distribution spread over the belt-wise domain from $\xi = -(c/2)\cos\gamma$ to $\xi = (c/2)\cos\gamma$.

The strength of source per unit area is expressed by q , and it is a function of ξ only.

The velocity induced at an arbitrary point

(ξ_0, η_0) by this source distribution can be easily calculated by the law of Biot Savart, and is written as follows,

$$\Delta v_a = \frac{1}{\pi} \int_{-(c/2) \cos \gamma}^{\xi_0} q(\xi) \frac{\cos \theta}{r} d\xi d\gamma - \frac{1}{\pi} \int_{\xi_0}^{(c/2) \cos \gamma} q(\xi) \frac{\cos \theta}{r} d\xi d\gamma \quad \dots (2)$$

where

$$r = \sqrt{(\xi - \xi_0)^2 + (\eta - \eta_0)^2},$$

$$\cos \theta = \frac{\xi - \xi_0}{\sqrt{(\xi - \xi_0)^2 + (\eta - \eta_0)^2}}$$

Eq. (2) can be integrated at once, and the result is as follows :

$$\Delta v_a(\xi) = \int_{-(c/2) \cos \gamma}^{\xi} q(\xi) d\xi - \frac{1}{2} \int_{-(c/2) \cos \gamma}^{(c/2) \cos \gamma} q(\xi) d\xi$$

The second term on the right hand side of the above equation is one half of the fluid volume welled out per unit length of η -direction, and therefore, it is equal to $(v_{a2} - v_{a1})$. So we have,

$$\Delta v_a(\xi) = \int_{-(c/2) \cos \gamma}^{\xi} q(\xi) d\xi - \frac{v_{a2} - v_{a1}}{2} \quad \dots (3)$$

or, in differential form

$$\frac{d(\Delta v_a)}{d\xi} = q(\xi) \quad \dots (4)$$

Eqs. (3) and (4) show the conclusive relation between the source distribution and the axial velocity change. Whatever the change of axial velocity in ξ -direction may be we can now find the corresponding source distribution which satisfies the given axial velocity change, so long as the derivatives of Δv_a with respect to ξ have finite values.

2.3 Flow (iii)

Flow (i), of course, satisfies the boundary condition on the blade surface; however, Flow (ii) does not satisfy the boundary condition alone. Therefore, in order to cancel the normal velocity on the blade surface, Flow (iii) must now be considered. As illustrated in Fig. 3 the acceleration $\Delta v_a(\xi)$ of the axial velocity between the blade passages causes a normal velocity component on the blade surface, as follows :

$$\Delta v_n = \sin \gamma \cdot \Delta v_a$$

So, to cancel this normal velocity the following normal velocity shall be added on the blade surface,

$$\Delta v_n = -\sin \gamma \cdot \Delta v_a \quad \dots (5)$$

This means that the flow wells out on the upper (or lower) surface and sucks the same magnitude into the opposite surface. As a whole, the flow field can be considered as consisting of a doublet distribution as is schematically shown in the Figure. As can be seen easily from this figure, there occurs a flow from the upper surface to the lower surface surrounding the trailing edge T. This induces

Fig. 3 Illustration of flow (iii)

infinite velocity at T, and is inconsistent with the hypothesis of Kutta-Joukowski, so there occurs another circulation $\Delta \Gamma$ around the blade to cancel the infinite velocity at T. $\Delta \Gamma$ is nothing more than the change of circulation caused by axial velocity change, and what we want is to be able to calculate the value of $\Delta \Gamma$ as a function of the prescribed conditions.

Now we shall consider the present problem on the transformed imaginary plane of the complex number $\zeta = re^{i\theta}$.

The mapping function of flat plate cascade is already well known, and is written in the following equation.

$$z = \frac{s}{2\pi} \left[e^{-i\gamma} \log \frac{1+\kappa\zeta}{1-\kappa\zeta} + e^{i\gamma} \log \frac{\zeta+\kappa}{\zeta-\kappa} \right] \quad \dots (6)$$

where $z = x + iy$, $\zeta = re^{i\theta}$

By the law of continuity, the normal velocity component $q(\theta)$ on the unit circle is expressed as a function of the normal velocity component Δv_n on the blade surface as follows :

$$q(\theta) = \Delta v_n \frac{dx}{d\theta}$$

Substituting Eq. (5) into the above equation, we get,

$$q(\theta) = -\sin \gamma \Delta v_a(x) \frac{dx}{d\theta}$$

$dx/d\theta$ can be obtained by differentiating Eq. (6) with respect to θ , and the result is :

$$q(\theta) = \sin \gamma \Delta v_a(x) \times \frac{2s\kappa}{\pi} \frac{(1+\kappa^2) \cos \gamma \sin \theta - (1-\kappa^2) \sin \gamma \cos \theta}{1-2\kappa^2 \cos 2\theta + \kappa^4} \quad \dots (7)$$

By the above equation the distribution of sources (or sinks) on the unit circle, corresponding to the normal velocity on a blade, can be calculated as a function of the argument θ . Then the method to determine the value of additional circulation $\Delta \Gamma$ which cancels the tangential induced velocity at the trailing edge is already solved by Hudimoto and Hirose⁽⁶⁾, and applying this result we get the following equation (c. f. Appendix).

$$\Delta \Gamma = -\frac{2s\kappa(1-\kappa^4)}{\pi\sqrt{K}} \sin \gamma \times \int_{\theta_T}^{\theta_T+2\pi} \Delta v_a(x) \frac{1 + \cos(\theta_T - \theta)}{K'} d\theta \quad \dots (8)$$

where,

$$K = 1 + 2\kappa^2 \cos 2\gamma + \kappa^4, \quad K' = 1 - 2\kappa^2 \cos 2\theta + \kappa^4$$

Eq. (8) is the conclusive equation, which gives the value to correct the circulation around the blade due to the prescribed axial velocity change.

In the case of a linear axial velocity change in the x -direction, Eq. (3) can be expressed as follows:

$$\frac{\Delta v_a}{V} = \frac{x}{c} \frac{v_{a2} - v_{a1}}{V}$$

The definite integral, which is obtained by substituting the above equation in Eq. (8), is already solved by Hudimoto and Hirose⁽⁶⁾ also, and applying this solution we get the following formula.

$$\frac{\Delta \Gamma}{(v_{a1} - v_{a2})s} = \frac{2\lambda \sin \gamma}{\pi} \log \frac{1 + \kappa^2}{1 - \kappa^2} \dots \dots \dots (9)$$

By the assumption of linear axial velocity change, we can get such a concise conclusion. In the actual phenomenon, the change rate of axial velocity would be a very complicated one, however no one can imagine how it is, therefore for practical design purpose, the assumption of linear axial velocity change shall be accepted with reasonable accuracy.

Fig. 4 shows the result of a numerical calculation of the above equation, which gives the correction values for the circulation due to the change of axial velocity as a function of pitch-chord ratio λ and stagger angle γ .

Now, from the following figure, a concrete method can be suggested for correcting the velocity triangles in the case of accelerated or decelerated axial velocity to that of constant axial velocity. The practical drawing method is as seen in Fig. 5. Suppose the prescribed velocity triangle is as shown by arrows (solid lines) of the Figure. First, we draw the mean axial velocity line $m-m$.

Second, from the tops of both arrows, draw two parallel straight lines $L-L$ and $L'-L'$, which are inclined to the axial direction by an angle ω_f . ω_f has the following physical meaning.

Fig. 4 Correction of circulation around blades for axial velocity change

$$\tan \omega_f = \frac{\Delta \Gamma}{(v_{a1} - v_{a2})s}$$

The values of ω_f can easily be found in Fig. 4, if the values of pitch-chord ratio λ and stagger angle γ are known. Then, the two points of intersection of the lines $m-m$ and $L-L$, $m-m$ and $L'-L'$ correspond respectively to the velocity vectors at the inlet and exit of the cascade with a constant axial velocity. In Fig. 4, the most useful domain for practical purposes is shown by the shaded area. In this domain, $\tan \omega_f$ seems to vary from 0.3 to 0.9, and for the most practical purposes it will be enough to put $\tan \omega_f = 0.5$ for the usual axial flow compressor design.

The method above mentioned is exactly true for the case of keeping the vector mean velocity ahead and behind of cascade constant. Therefore, if the inlet velocity vector is to be kept constant, it is necessary to correct the turning angle correspondingly. However, in the case of the value of pitch-chord ratio not much exceeding 1, it can be allowed to consider that the value of correction is practically zero.

3. Cascade composed of arbitrarily shaped blades

To obtain the more practical solution, let's consider the flow through cascades composed of arbitrarily shaped blades. There are several methods to solve such a problem involving a small perturbation based on the solution of flat plate cascades. In this paper, one of the simplest approximation theories similar to the Hudimoto method⁽⁶⁾ shall be adopted. Since we require very small quantities in the final result, we shall consider only the first approximation as sufficient for our purposes.

3.1 General equations

Consider a cascade composed of an arbitrary blade shape such as Fig. 6. The whole blade shape is supposed to be prescribed by the following equation

$$y = f(x) \dots \dots \dots (10)$$

Fig. 5 Illustration of the method for correct velocity triangles

Now let's consider the source distribution corresponding to the axial velocity change from v_{a1} to v_{a2} . Restricting our problem to the case of uniformly distributed sources, the strength of source per unit area q must first be determined. In this case, defining the cross sectional area of the blade as A , the whole quantity of fluid Q welled out from the one passage between two adjacent blades is,

$$Q = q(cs \cos \gamma - A)$$

and this must coincide with the change of axial velocity before and after the cascade, therefore,

$$q = \frac{(v_{a2} - v_{a1})s}{cs \cos \gamma - A} \quad \dots\dots\dots (11)$$

Finally, using Eq. (4), the apparent change of axial velocity is given by the following equation.

$$\Delta v_a = \frac{v_{a2} - v_{a1}}{1 - B} \frac{x}{c} \quad \dots\dots\dots (12)$$

where,

$$B = \frac{A}{cs \cos \gamma}$$

B is the "blocking ratio" of the passage by the blade, and in the usual case of the axial flow compressor cascade, it has always small value around 0.1.

Then, the local apparent axial velocity change Δv_a at any point P on the blade surface brings a normal velocity component Δv_n to the blade surface at this point, and considering that the slope of camber line dy/dx is very small compared with one, v_n can be expressed as follows:

$$\Delta v_n = -\frac{dy}{dx} \cos \gamma \Delta v_a - \sin \gamma \Delta v_a \quad \dots\dots\dots (13)$$

In the above equation, the second term on the right hand side is the same as Eq. (5). Therefore, all we

Fig. 6 Cascade composed of arbitrarily shaped blade profile

have to do is to solve the first term only, and superimpose it onto the solution of a flat plate cascade. Now, considering the first term only, the radial velocity distribution around the unit circle becomes,

$$q(\theta) = \frac{dx}{d\theta} \Delta v_n = -\frac{d\left(\frac{y}{c}\right)}{d\theta} \frac{x}{c} \cos \gamma \quad \dots\dots\dots (14)$$

On the other hand, let's consider the flow which cancels the normal velocity around the unit circle. First, to satisfy the boundary condition, a quantity of fluid $q \cdot A$ must be sucked into the unit circle. Furthermore, considering that the quantity of fluid $q \cdot A$ must be sucked into the blade from the infinite distance in both directions perpendicular to the cascade axis, the following complex velocity potential function is given.

$$W = \frac{qA}{4\pi} \left\{ \log \frac{\zeta - \frac{1}{\kappa}}{\zeta - \kappa} + \log \frac{\zeta + \frac{1}{\kappa}}{\zeta + \kappa} \right\} + \sum_{n=1}^{\infty} \frac{E_n' + iF_n'}{\zeta'^n}$$

where,

$$\zeta' = \zeta e^{-i\theta}$$

Using Eq. (11) the above equation can be rewritten in the following form.

$$W = \frac{c(v_{a2} - v_{a1})}{4\pi} \frac{\lambda B}{1 - B} \left\{ \log \frac{\zeta - \frac{1}{\kappa}}{\zeta - \kappa} + \log \frac{\zeta + \frac{1}{\kappa}}{\zeta + \kappa} \right\} + c(v_{a2} - v_{a1}) \sum_{n=1}^{\infty} \frac{E_n + iF_n}{\zeta'^n} \quad \dots\dots\dots (15)$$

From the above equation, the radial velocity v_r and the tangential velocity v_θ on the unit circle become:

$$v_r = -\frac{c(v_{a2} - v_{a1})}{2\pi} \frac{\lambda B}{1 - B} - \frac{c(v_{a2} - v_{a1})}{\pi} \frac{\lambda B}{1 - B} \sum_{n=1}^{\infty} \kappa^{2n} \cos 2n\theta - c(v_{a2} - v_{a1}) \cdot \sum_{n=1}^{\infty} n(E_n \cos n\theta' + F_n \sin n\theta') \quad \dots\dots\dots (16 \cdot A)$$

$$v_\theta = -c(v_{a2} - v_{a1}) \sum_{n=1}^{\infty} n(E_n \sin n\theta' - F_n \cos n\theta') \quad \dots\dots\dots (16 \cdot B)$$

Therefore, setting Eq. (14) and Eq. (16·A) equal, we can determine the values of the Fourier Series E_n and F_n as follows:

$$\sum_{n=1}^{\infty} n(E_n \cos n\theta' + F_n \sin n\theta') = \frac{d\frac{y}{c}}{d\theta} \frac{x}{c} \cos \gamma \frac{1}{1 - B} - \frac{1}{2\pi} \frac{\lambda B}{1 - B} - \frac{\lambda B}{\pi(1 - B)} \sum_{n=1}^{\infty} \kappa^{2n} \cos 2n\theta \quad \dots\dots\dots (17)$$

Now, we can calculate the tangential velocity on

the unit circle using Eq. (16·B) and the determined values of E_n and F_n .

At the trailing edge,

$$(v_\theta)_{\theta'=0} = c(v_{a2} - v_{a1}) \sum_{n=1}^{\infty} n F_n \dots\dots\dots (18)$$

Therefore, determining the value of circulation $\Delta\Gamma$ so as to cancel the above induced velocity at the trailing edge, we obtain the following formula.

$$\frac{\Delta\Gamma}{(v_{a2} - v_{a1})s} = \frac{2\pi}{\lambda} \frac{1 - \kappa^4}{K} \sum_{n=1}^{\infty} n F_n \dots\dots\dots (19)$$

Now we can calculate the value of the circulation, which corresponds to the first term of Eq. (13). Therefore, superimposing this onto the solution of the flat plate cascade, we can calculate the correction value for the circulation in the case of an arbitrarily prescribed blade profile.

3·2 Calculation for NACA 65-series compressor cascades

Since the calculation for each case is a rather laborious one, for convenience it is preferable to make some numerical graphs for certain blade profile series. In this paper, numerical graphs for the rapid characteristic estimation of NACA 65-series compressor cascades were prepared. NACA 65-series blade profiles consist of a camber line and

$$\begin{aligned} & \sum_{n=1}^{\infty} n(E_n \cos n\theta' + F_n \sin n\theta') \\ &= C_{l0} \frac{d\left(\frac{y_c}{c}\right)_{C_{l0}=1.0}}{d\theta} \frac{x}{c} \cos \gamma + \frac{B}{1-B} \left\{ \frac{1}{B_0} \frac{d\left(\frac{y_t}{c}\right)_{t=t_0}}{d\theta} \frac{x}{c} \cos \gamma - \frac{\lambda}{2\pi} - \frac{\lambda}{2\pi} \sum_{n=1}^{\infty} \kappa^{2n} \cos 2n\theta \right\} \end{aligned}$$

Then, we divide the above equation into two parts as follows:

$$\begin{aligned} \sum_{n=1}^{\infty} n({}_c E_n \cos n\theta' + {}_c F_n \sin n\theta') &= \frac{C_{l0}}{1-B} \frac{d\left(\frac{y_c}{c}\right)_{C_{l0}=1.0}}{d\theta} \frac{x}{c} \cos \gamma \\ \sum_{n=1}^{\infty} n({}_t E_n \cos n\theta' + {}_t F_n \sin n\theta') &= \frac{B}{1-B} \left\{ \frac{1}{B_0} \frac{d\left(\frac{y_t}{c}\right)_{t=t_0}}{d\theta} \frac{x}{c} \cos \gamma - \frac{\lambda}{2\pi} - \frac{\lambda}{\pi} \sum_{n=1}^{\infty} \kappa^{2n} \cos 2n\theta \right\} \end{aligned}$$

Finally, putting

$$\begin{aligned} \tan \omega_c &= \frac{2\pi}{\lambda} \frac{1 - \kappa^4}{K} \sum_{n=1}^{\infty} n {}_c F_n \\ \tan \omega_t &= \frac{2\pi}{\lambda} \frac{1 - \kappa^4}{K} \sum_{n=1}^{\infty} n {}_t F_n \end{aligned}$$

we get the following conclusive equation

$$\frac{\Delta\Gamma}{(v_{a2} - v_{a1})s} = \tan \omega_c + \tan \omega_t$$

In Fig. 7, $\tan \omega_c$ and $\tan \omega_t$ are shown for the case of $C_{l0}=1.0$, $t=0$, and $C_{l0}=0$, $t=0.1$ respectively. It can be seen from the previous description that the solutions for all other combinations of C_{l0} and t can be easily found from this numerical graph.

4. Numerical examples

To confirm the validity of the theory, comparisons

thickness distribution, and both values change proportionally to some "designation" with the distribution remaining constant. The designation for camber line is C_{l0} , which expresses the lift coefficient in the case of an isolated wing. The designation for thickness is the "value of the maximum thickness", and in this paper, let us designate it temporarily t .

Now, as it can be seen from the above description, taking the chord line as the abscissa x , the ordinate y on the blade surface can be expressed by the following equation:

$$y = y_c \pm y_t$$

where, y_c represents the ordinate of the camber line, y_t represents the ordinate of thickness only, the + sign is taken for the upper surface and the - sign for the lower surface. Furthermore, we can write:

$$y = C_{l0}(y_c)_{C_{l0}=1.0} \pm \frac{t}{t_0}(y_t)_{t_0}$$

Hence,

$$\frac{dy}{dx} = C_{l0} \left(\frac{dy_c}{dx} \right)_{C_{l0}=1.0} + \frac{B}{B_0} \left(\frac{dy_t}{dx} \right)_{t=t_0} \dots\dots (20)$$

where B_0 is a blocking ratio corresponding to t_0 and t_0 is some standard thickness ratio.

Substituting Eq. (20) into Eq. (17) we can write,

Fig. 7 Effect of camber and thickness for NACA 65 series blades

were made with several sets of experimental data. All of the experimental data here used⁽⁷⁾⁽⁴⁾⁽⁸⁾ were collected from the publications on hand. All of the data were for NACA 65 series compressor cascades using low speed wind tunnels. In them, turning angles are plotted against attack angles for the cases of both a solid wall and a porous wall. In the latter case, a two-dimensional flow is supposed to be maintained at the middle of the span. For both cases, all data gave the rate of axial velocity change

v_{a2}/v_{a1} , before and after the cascades. Now starting with the value for a solid wall, what we want to do is to obtain the value of the turning angle for a two dimensional flow. In this case, the difference between solid wall and porous wall data consists in the effect of axial velocity change and secondary flow effect as described in the introduction. Therefore, correction must be made for the effect of

Fig. 8 Correction of turning angle by theory (example 1)

Fig. 9 Correction of turning angle by theory (example 2)

Fig. 10 Correction of turning angle by theory (example 3)

Fig. 11 Correction of turning angle by theory (example 4)

secondary flow also, and it was done by Otsuka's theory⁽⁹⁾.

In the first group of data⁽⁷⁾ (from Fig. 8 to Fig. 13), the value of $\Delta w_1/w_{1m}$ was estimated to be 0.03, according to the velocity distribution shown in the report. In all figures, thin solid line express the value for the porous wall, thin dotted lines show the value for the solid wall, while thick solid lines

show the results of calculations based on solid wall data.

Since the NACA porous wall data⁽⁴⁾ (Fig. 14) use a somewhat different value compared with the other report⁽⁶⁾, in this paper the value of the latter was adopted as the value of the porous wall experiment. The last data (Fig. 15) came from M. I. T.⁽¹⁰⁾ and in this case, as the data of a decelerated

Fig.12 Correction of turning angle by theory (example 5)

Fig.14 Correction of turning angle by theory (example 7)

Fig.13 Correction of turning angle by theory (example 6)

Fig.15 Correction of turning angle by theory (example 8)

case were also presented, they were used for comparison, and the result is given by the dot-dash line in the figures. In the decelerated case, since the secondary flow theory no longer has its meaning, the correction for the secondary flow was omitted. In both the NACA and MIT data, because of the lack of description of inlet velocity distribution, it was assumed that $\Delta w_1/w_{1m}=0.1$ temporarily.

Fig. 16 and Fig. 17 show another example. Fig. 16 shows the distribution of axial velocity and flow direction behind the inlet guide vane in an axial flow compressor, which is shown in Fig. 17. As seen in Fig. 16, axial velocity changes considerably through the inlet guide vane, therefore the turning angle estimated by the two dimensional cascade data (shown in solid line) does not coincide with the experimental value (shown in points).

Turning angle applying this theory (shown in dotted line) shows good agreement with experimental value.

5. Conclusions

In this paper, a general method was developed for the evaluation of cascade performance with accelerated or decelerated axial velocity. For the series of NACA 65 series compressor cascades especially, numerical graphs for practical use were presented. From the comparison of the theory with the data from several experiments, we see that good agreement exists between them. However, there still remains some discrepancy. On the one hand, it seems to depend upon the accuracy of the cascade testing technique itself: On the other hand, the more complicated factors such as the effect of viscosity might affect the performance, which is beyond the scope of the present treatment. Restricting the discussion to the procedure treated here, the accuracy of the calculation would be doubtful in the more highly cambered blade or in a narrower spacing than the present example, because of the invalidity of the simple conformal transformation function used. Fortunately, such a difficult case would not appear frequently in the practical design of axial flow compressors. This study has been performed during the author's sojourn in the Gas Turbine Laboratory of Massachusetts Institute of Technology, U. S. A. The author takes this opportunity to express many thanks to Prof. E. S. Taylor and all other staff members, who gave always encourage-

$$\Gamma = \frac{2s\kappa(1-\kappa^4)}{\pi\sqrt{K}} \int_{\theta_T}^{\theta_T+2\pi} \left(V \sin \alpha - V \cos \alpha \frac{dy}{dx} \right) \frac{1 + \cos(\theta_T - \theta)}{K'} d\theta$$

Therefore, in the case of zero attack angle,

$$\Gamma = -\frac{2s\kappa(1-\kappa^4)V}{\pi\sqrt{K}} \int_{\theta_T}^{\theta_T+2\pi} \frac{dy}{dx} \frac{1 + \cos(\theta_T - \theta)}{K'} d\theta$$

Fig. 16 Distribution of axial velocity and flow angle along radius behind of the inlet guide vane

Fig. 17 Axial flow compressor used

ments and advices. Prof. S. Otsuka of Nagoya University sent me a useful report and valuable advice for which the author is most thankful.

Appendix

Analogy with Thin Aerofoil Theory

According to the theory by Hudimoto and Hirose⁽⁶⁾, the circulation around the cambered blade of no thickness in uniform flow of velocity V and angle of attack α can be expressed approximately by the following equation.

Compared with the above equation and Eq. (8), there exists the following analogy.

$$\Delta v_a \sin \gamma = V \frac{dy}{dx}$$

Therefore, the circulation around flat plate cascade with axial velocity change, $\Delta v_a/V = F(x)$, is equal to that around cambered blade of no thickness in uniform flow of zero attack angle, whose camber line is expressed in the following form.

$$y = \sin \gamma \int F(x) dx$$

For example, if the rate of axial velocity change is uniform, the circulation around flat plate cascade is equal to that of the parabolic cambered blade in uniform flow.

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Discussion

H. TANAKA : (1) The author performed the calculation under condition of constant vector mean velocity at first, and then converted it to the condition of constant inlet velocity vector. In this connection, the author has explained that the existing flow angle would be scarcely affected by the change of inlet flow angle provided the value of pitch-chord ratio does not much exceed 1. However, according to the numerical examples already shown, as the majority of the examples show that the differences between porous and solid wall cascade data are of the order of half a degree, it seems to be inadequate to neglect the effect of inlet velocity vector variation.

(2) The author has not shown the form of mapping function in the case of arbitrarily shaped cascade but regarding Eq. (13), the author has explained that the second term is identical with the value of flat plate cascades. However, the mapping function for an arbitrarily shaped cascade is expected to take a different form from that of a flat plate cascade. Therefore, some correction factor must be added to the second term corresponding to the difference of the mapping function, and since dy/dx in the first term is a small quantity, it is considered to be essential to compare the order of magnitude with that of the correction factor for the second term. And in case the same mapping function is used for both the flat plate cascade and the arbitrarily shaped cascade, the accuracy of approximation to the unit circle is somewhat doubtful even if the pitch-chord ratio exceeds ratio 1.

In this case, the theory therefore, seems to be applicable only for the limited case. Furthermore, this applicable range is considered to be inconsistent with question (1) regarding the pitch-chord ratio.

(3) The author is supposed to have also done the correction for a secondary flow and, accordingly, it is desirable that the magnitude of correction for the secondary flow effect be shown in comparison with that for this theory.

(4) It is considered to be quite reasonable that the author deliberately neglected the correction for the exit flow angle owing to the change of inlet flow angle because the magnitude of that correction is supposed to be of the same order of the error of cascade test itself. However, for examples shown in Figs. 8, 10 and 13, as the differences of turning angle between porous and solid wall data are only of the order of half a degree, it is considered to be rather nonsense to make such correction of this theory itself. (Question (1) meant that, if the correction of this theory is worth applying to such examples as shown in Figs. 8, 10 and 13, it is considered necessary to make calibration for exit flow angle owing to the change of inlet flow angle too.)

(5) Regarding the applicable range of the theory mentioned in the latter part of question (2), the assumption described in question (1) is supposed to be inconsistent with the nature of approximation mentioned in question (2). Therefore, the author is required to show the applicable range of the theory concretely.

(6) As the author has already described that the order of magnitude of the corrected value for the secondary flow effect is the same as that of this theory, it is suggested that it would be useful to show the curve indicating the correction only for secondary flow effect shown in the examples.

Author's closure

(1) Although the description for the deviation of the exit flow angle owing to the variation of the inlet flow angle has been omitted in this paper, detailed examination of this problem is presented in the original Report (Cascade Performance with Accelerated or Decelerated Axial Velocity, MIT Gas Turbine Lab. Report, No. 56 (Sept. 1959)). Fig. 2 of the Report shows such effect, and from this figure, it can be understood that the deviation of the exit flow angle is about half a degree per five degrees of inlet flow angle variation. Therefore, in consideration of the fact that the cascade test itself contains errors of the same order, the calibration for such an effect has been deliberately neglected. In case it is necessary to make calibration, Fig. 2 of the Report should be available for such purpose.

(2) The purpose of this study is not to obtain a strict solution for a particular case, but to offer the general approximate solution within the range of practical design use. Therefore, mathematical procedures used, such as the form of mapping function, were simplified as much as possible. Accordingly, strictly speaking, the normal velocity components expressed by Eq. (13) are distributed along the surface of an arbitrarily shaped blade, but in this case, they are considered simply to distribute on the flat plate cascade consisting of the blade chord line itself.

It is, therefore, considered to be adequate that the ambiguity of the treatment is not due to the process of conformal mapping, but to such simplification of the boundary conditions.

However, such simplification of the boundary conditions is supposed to be generally admitted in the treatment of the so-called "thin aerofoil theory", and so the defect owing to such simplification does not present any peculiar problem of this theory but it is rather common nature of such theory. With reference to the question for the accuracy of approximation even in case of large pitch chord ratio, such accuracy results from the assumption of the said aerofoil theory, and, therefore, it is apparently admitted in general.

To conclude, there seems to exist, as pointed

out, some ambiguity about the accuracy of approximation in the theory, but such ambiguity is considered to be in the same magnitude as those of the general thin aerofoil theories.

(3) The correction for the secondary flow effect is shown in Fig. 4 of the prescribed original Report, and, generally speaking, in magnitude it is of the same order as that of this theory.

(4) Examples of the corrected value only for the secondary flow effect are shown with chain lines in Figs. 12 and 13. It is quite noticeable that in Fig. 12 the correction acts towards decreasing turning angles, but in Fig. 13 it acts towards increasing them, and accordingly the total correction value is apparently almost zero. However, such results can be understood for the first time only when all calculations have been completely finished, and no one could imagine about it beforehand.

It shall, therefore, be concluded that none can decide as to whether it is worth making correction or not, before making any calculation.

In order to verify the validity of the theory, the author adopted, as the examples here, only the cases, in which both porous wall and solid wall data are available. However, it must be emphasized that the essential purpose of this study is to estimate the performance of a cascade with axial velocity change by means of the experimental data for two dimensional flow only in hand or vice versa.

Therefore in such cases, it would be almost impossible to suppose beforehand the extent of the differences between the original and corrected values.

(5) Regarding the applicable limit of the theory, larger pitch chord ratio is suitable for the procedure of calculation, and accordingly, it might be concluded that there is no limitation towards larger pitch-chord ratio, and in such cases the exit flow angle could be corrected against inlet flow angle change, if necessary. For the smaller pitch-chord ratio, such correction deteriorates its accuracy, and the limitation for the practical design purpose would be about 0.8. On the other hand, apart from the problem of pitch-chord ratio, corrected values for highly cambered blades, whose C_{10} exceeds two, would have unsatisfactory accuracy for the sake of assumption.

(6) Examples are shown in Figs. 12 and 13.