621. 67. 01:532. 595. 2

Water Hammer Analysis for Pumps in Parallel Operation*

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In pumping stations several pumps having different characteristics often are installed in parallel and operated partially or totally depending on the demand for water, in order to maintain better efficiency of the plant. This paper presents a method of analysis of water hammer in parallel operation of different pumps.

As one example for this analysis the parallel operation of two pumps is chosen, and the transient phenomena of each pump following power failure are calculated. By these calculations a result as follows has been confirmed: When power failure occurs in parallel operation of two different pumps and there is a certain difference in the speed drop of each pump, the discharge of one pump, of which the speed reduces more slowly than the speed of the other pump, does not decrease but increases for a short time after power failure.

1. Introduction

There are many literatures on the analysis and experiment of water hammer in the operation of a one-pump-unit. For calculation of water hammer of pumps in parallel operation, when each of them has the same characteristics, it is possible to substitute them with one pump. There also are, however, many pumping stations where several different pumps are installed in parallel and operated partially or totally depending on the demand for water, in order to maintain better efficiency of the plant. The analysis of water hammer in this case is described in this paper.

2. Method of analysis

As one example for this analysis the parallel operation of two pumps is chosen. Those two pumps are named A and B respectively. The pipe line is schematically shown in Fig. 1. Delivery pipes of pump A and pump B (branch pipes) are con-

Fig. 1 Schematic view of pipe line

nected to the main pipe at point C. The length of the two branch pipes here will be considered very short compared with the total length of the delivery pipe line. Velocity head and friction loss in the pipe line are neglected.

2.1 Nomenclature

The following nomenclature is used in the paper:

- L : length of pipe line, m
- A : mean cross-sectional area of pipe line, m^2
- a : velocity of pressure wave, m/sec
- V : mean velocity of water in pipe line, m/sec
- H : head at C, m
- H_p : pumping head, m
- Q : discharge in main pipe at C, m^3/sec
- Q_A : discharge of pump A, m^3/sec
- Q_B : discharge of pump B, m^3/sec
- N : pump speed, r. p. m.
- M : pump torque, kgw m
- H_0 : head loss across discharge valve, m

$$\xi = \frac{H - H_0}{H_n}$$

Subscript 0 denotes a value just prior to the start of a transient, and subscript n denotes rated values

$$\begin{aligned} h &= H/H_n \\ h_{pA} &= H_{pA}/H_n, & h_{pB} &= H_{pB}/H_n \\ h_{vA} &= H_{vA}/H_n, & h_{vB} &= H_{vB}/H_n \\ q &= Q/Q_n, & q_A &= Q_A/Q_{nA}, & q_B &= Q_B/Q_{nB} \\ n_A &= N_A/N_{nA}, & n_B &= N_B/N_{nB} \\ m_A &= M_A/M_{nA}, & m_B &= M_B/M_{nB} \end{aligned}$$

Subscripts A and B indicate respectively pump A and pump B.

* Received 30th November, 1960.

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$\rho = \frac{aV_n}{2gH_n}$: a pipe-line constant

GD^2 : flywheel effect of rotating parts ($=4Ig$),
kgw m²

I : moment of inertia of rotating parts, kgw
m sec²

g : acceleration due to gravity, m/sec²

V , Q , H_p , N and M are taken as positive for values in pumping.

2.2 Analysis

The water hammer for pumps in parallel operation is calculated, under conditions arising from parallel operation, by using the principle of the known methods⁽¹⁾ of water hammer analysis for a one-pump-unit.

Pressure variation at point C is caused from velocity variation in the pipe line subsequent to power failure:

This pressure variation is expressed by

$$\xi_z = 2\rho(q_z - q_0) - 2\sum \xi_{z-1} \dots (1)$$

Subscript z denotes a value at a time z counted with a time interval $2L/a$ after the start of a transient.

Total discharge of the two pumps is equal to the discharge at C in the main pipe

$$\begin{aligned} Q_z &= Q_{zA} + Q_{zB} \\ \frac{Q_z}{Q_n} &= \frac{Q_{zA}}{Q_{nA}} \frac{Q_{nA}}{Q_n} + \frac{Q_{zB}}{Q_{nB}} \frac{Q_{nB}}{Q_n} \\ q_z &= q_{zA} \frac{Q_{nA}}{Q_n} + q_{zB} \frac{Q_{nB}}{Q_n} \dots (2) \end{aligned}$$

If α_v denotes a coefficient of head loss across the valve, then

$$h_{vA} = \alpha_{vA} q_{zA}^2, \quad h_{vB} = \alpha_{vB} q_{zB}^2$$

and relations between pumping head h_{pz} and head at C h_z are as follows

$$h_{pzA} = h_z \pm \alpha_{vA} q_{zA}^2 \dots (3)$$

$$h_{pzB} = h_z \pm \alpha_{vB} q_{zB}^2 \dots (4)$$

The positive sign is used for flow in the pumping direction, and the minus sign for reverse flow. The relation between the usual coefficient of head loss ζ_v and the coefficient α_v is

$$\begin{aligned} H_{vA} &= \zeta_{vA} \frac{V_A^2}{2g} \\ h_{vA} &= \zeta_{vA} \frac{1}{2gH_n} \left(\frac{Q_{nA}}{A_A} q_{zA} \right)^2 = \alpha_{vA} q_{zA}^2 \\ \alpha_{vA} &= \frac{Q_{nA}^2}{2gA_A^2 H_n} \zeta_{vA}, \end{aligned}$$

$$\text{similarly } \alpha_{vB} = \frac{Q_{nB}^2}{2gA_B^2 H_n} \zeta_{vB}$$

From the inertia equation of rotating parts the pump speed and pump torque have the following relations:

$$n_{aA} - n_{bA} = \frac{60gM_{nA}}{\pi(GD^2)_A N_{nA}} (m_{aA} + m_{bA}) \Delta t \dots (5)$$

$$n_{aB} - n_{bB} = \frac{60gM_{nB}}{\pi(GD^2)_B N_{nB}} (m_{aB} + m_{bB}) \Delta t \dots (6)$$

Subscripts a and b denote values at the beginning and end of a short time interval Δt

h_{pzA} , q_{zA} , n_{zA} , m_{zA} and h_{pzB} , q_{zB} , n_{zB} , m_{zB} change depending on the characteristics of pump A and pump B respectively:

$$\begin{aligned} h_{pzA} &= F_{1A}(q_{zA}, n_{zA}) \dots (7) \\ m_{zA} &= F_{2A}(q_{zA}, n_{zA}) \dots (8) \end{aligned}$$

Complete characteristics of pump A

$$\begin{aligned} h_{pzB} &= F_{1B}(q_{zB}, n_{zB}) \dots (9) \\ m_{zB} &= F_{2B}(q_{zB}, n_{zB}) \dots (10) \end{aligned}$$

Complete characteristics of pump B

These characteristics are obtained by experiments and shown in diagrams.

By using Eqs. (1)~(10) the transient phenomena following power failure are calculated at every given short time interval Δt : Ten unknown variables q_z , q_{zA} , q_{zB} , h_z , h_{pzA} , h_{pzB} , m_{zA} , m_{zB} , n_{zA} and n_{zB} are obtained by solving simultaneously ten equations. In this calculation the trial and error method is used, namely, whether an assumed value of an arbitrary variable among the ten variables satisfies simultaneous equations or not.

The method of analysis described in this paper can be applied to water hammer for three or more pumps in parallel operation.

3. Examples

3.1 Specifications of pumps

(a) Pump A

Type Double suction 2-stage pump (diameter of discharge pipe: 1500 mm)
Discharge 474 m³/min
Total head 211 m
Speed 500 r.p.m.
Motor power (rated) 18600 kW
Specific speed $N_s = 234$ (r.p.m., m³/min, m)

(b) Pump B

Type Single suction 2-stage pump (diameter of discharge pipe: 1300 mm)
Discharge 237 m³/min
Total head 211 m
Speed 600 r.p.m.
Motor power (rated) 9280 kW
Specific speed $N_s = 280$ (r.p.m., m³/min, m)

3.2 Data for calculation

$N_{nA} = 500$ r.p.m., $N_{nB} = 600$ r.p.m.
 $H_{nA} = H_{nB} = 211$ m
 $Q_{nA} = 7.9$ m³/sec ($=474$ m³/min),
 $Q_{nB} = 3.95$ m³/sec ($=237$ m³/min)
 $Q_n = Q_{nA} + Q_{nB} = 11.85$ m³/sec

$$A_A = \frac{\pi}{4} \times 1.8^2 \text{ m}^2, \quad A_B = \frac{\pi}{4} \times 1.6^2 \text{ m}^2,$$

$$A = \frac{\pi}{4} \times 2.5^2 \text{ m}^2$$

Table 1 Summary of hydraulic transients, for two pumps shutdown in parallel operation, with valve closure

$2L/a$	h_z %	ξ_z %	$2\sum_{i=1}^z \xi_{z-1}$ %	ξ_z^+ %	$q_z - 100$ %	s mm	ζ	α_{vA}	q_{zA} %	h_{vA} %	$h_{p_{zA}}$ %	m_{zA} %	$m_{zA} + m_{0A}$ %	$n_{zA} - n_{0A}$ %	n_{zA} %	α_{vB}	q_{zB} %	h_{vB} %	$h_{p_{zB}}$ %	m_{zB} %	$m_{zB} + m_{0B}$ %	$n_{zB} - n_{0B}$ %	n_{zB} %
0	100	0	0	0	0	0.0	0.00	0.0×100	100	0.0	100	100			100	0.0×100	100	0.0	100	100			100
1/4	85	-15	0	-15	-12.3	18.1	0.00	0.0	108.0	0.0	85	96	196	3.94	96.06	0.0	47.1	0.0	85	60	160	16.40	83.60
1/2	72	-28	0	-28	-23	36.2	0.00	0.0	112.0	0.0	72	89.5	185.5	3.75	92.31	0.0	7.0	0.0	72	39.5	99.5	9.71	73.89
3/4	65	-35	0	-35	-28.7	54.3	0.03	0.0	111.0	0.0	65	84	173.5	3.49	88.82	0.0	-8.1	0.0	65	36.5	76	7.42	66.47
1	58	-42	0	-42	-34.5	72.5	0.16	0.0	107.8	0.0	58	77	161	3.24	85.58	0.0	-19.1	0.0	58	36.5	73	7.13	59.34
1 1/4	64	-36	-30	-66	-54.1	90.6	0.55	0.1	90.7	0.1	64.1	68	145	2.91	82.67	0.1	-42.5	0.0	64	54.5	91	8.88	50.46
1 1/2	68.5	-31.5	-56	-87.5	-71.7	108.7	1.39	0.3	72.2	0.2	68.7	60.5	128.5	2.58	80.09	0.1	-59.5	0.0	68.5	70	124.5	12.15	38.31
1 3/4	68.5	-31.5	-70	-101.5	-83.2	126.9	3.01	0.7	60.0	0.3	68.8	53.5	114	2.29	77.80	0.3	-69.6	0.1	68.4	79	149	14.54	23.77
2	67	-33	-84	-117	-95.9	144.9	5.68	1.3	44.0	0.3	67.3	44.5	98	1.97	75.83	0.5	-75.7	0.3	66.7	81	160	15.61	8.16
2 1/4	66	-34	-102	-136	-111.5	163.0	10.1	2.4	23.3	0.1	66.1	33	77.5	1.56	74.27	0.9	-81.1	0.6	65.4	84.5	165.5	16.15	-7.99
2 1/2	65.5	-34.5	-119	-153.5	-125.8	181.2	17.6	4.1	4.8	0.0	65.5	27	60	1.21	73.06	1.6	-87.0	1.2	64.3	83	167.5	16.35	-24.34
2 3/4	65.5	-34.5	-133	-167.5	-137.3	199.3	31.9	7.4	-10.6	0.1	65.4	25	52	1.05	72.01	3.0	-90.7	2.5	63.0	79	162	15.81	-40.15
3	70	-30	-150	-180	-147.5	217.4	59.0	13.7	-24.3	0.8	69.2	28.5	53.5	1.08	70.93	5.5	-93.9	4.8	65.2	73	152	14.84	-54.99
3 1/4	79	-21	-170	-191	-156.6	235.5	113	26.3	-36.3	3.5	75.5	36	64.5	1.30	69.63	10.5	-97.2	9.9	69.1	67.5	140.5	13.71	-68.70
3 1/2	90	-10	-188	-198	-162.3	251.0	208	48.5	-45.0	9.8	80.2	42.5	78.5	1.58	68.05	19.4	-96.9	18.2	71.8	56.5	124	12.10	-80.80
3 3/4	98.5	-1.5	-202	-203.5	-166.9	256.0	258	60.1	-51.9	16.2	82.3	48.5	91	1.83	66.22	24.1	-96.9	22.6	75.9	47.5	104	10.15	-90.95
4	105	5	-210	-205	-168.1	261.1	326	76.0	-55.0	23.0	82.0	51	99.5	2.00	64.22	30.4	-94.3	27.0	78.0	36	83.5	8.15	-99.10
4 1/4	109.5	9.5	-212	-202.5	-166.0	266.2	439	102.3	-55.0	30.9	78.6	49.5	100.5	2.02	62.20	41.0	-88.0	31.7	77.8	22	58	5.66	-104.76
4 1/2	110.3	10.3	-208	-197.7	-162.1	271.3	565	131.6	-52.9	36.8	73.5	46	95.5	1.92	60.28	52.7	-80.5	34.1	76.2	10.5	32.5	3.17	-107.93
4 3/4	113	13	-205	-192	-157.4	276.3	785	182.9	-50.0	45.7	67.3	40	86	1.73	58.55	73.2	-72.2	38.2	74.8	2	12.5	1.22	-109.15
5	114.7	14.7	-200	-185.3	-151.9	281.4	1080	251.6	-45.8	52.8	61.9	36	66	1.53	57.02	100.8	-64.1	41.4	73.3	-4	-2	-0.20	-108.95
5 1/4	115.7	15.7	-193	-177.3	-145.4	286.5	1580	368.1	-40.5	60.4	55.3	30	76	1.33	55.69	147.4	-55.2	44.9	70.8	-8	-12	-1.17	-107.78
5 1/2	119.2	19.2	-187.4	-168.2	-137.9	291.6	2570	598.8	-34.2	70.0	49.2	24.5	54.5	1.10	54.59	239.8	-45.3	49.2	70.0	-13.5	-21.5	-2.10	-105.68
5 3/4	117	17	-179	-162	-132.8	296.6	3450	803.9	-30.0	72.4	44.6	22	46.5	0.93	53.66	321.9	-38.4	47.4	69.6	-20	-33.5	-3.27	-102.41

$$V_n = \frac{Q_n}{A} = \frac{11.85}{\frac{\pi}{4} \times 2.5^2} = 2.41 \text{ m/sec}$$

$$M_{nA} = 36\,200 \text{ kgw m}, \quad M_{nB} = 15\,100 \text{ kgw m}$$

$$L = 605 \text{ m}, \quad a = 1\,045 \text{ m/sec}, \quad \frac{2L}{a} = 1.16 \text{ sec}$$

$$(GD^2)_A = 196\,000 \text{ kgw m}^2, \quad (GD^2)_B = 14\,000 \text{ kgw m}^2$$

$$2\rho = \frac{a V_n}{g H_n} = \frac{1\,045 \times 2.41}{9.8 \times 211} = 1.22$$

$$\alpha_{vA} = \frac{Q_{nA}^2}{2g A_A^2 H_n} \zeta_{vA} = 2.33 \times 10^{-3} \zeta_v,$$

$$\alpha_{vB} = \frac{Q_{nB}^2}{2g A_B^2 H_n} \zeta_{vB} = 9.33 \times 10^{-4} \zeta_v,$$

Since discharge valves used here are similar to each other, ζ_{vA} and ζ_{vB} are considered equal and represented by ζ_v . The coefficient of head loss ζ_v is shown in Fig. 2.

Fig. 3 (a) and Fig. 3 (b) show respectively the complete characteristics of pump A and pump B.

3.3 Results of calculation

Hydraulic transients, for pumps shutdown, with and without valve closure have been calculated. These results are shown in Fig. 4 and Fig. 5. Procedure of calculation with valve closure is

summarized in Table 1 and the results are shown on complete characteristic curves in Fig. 3 (a) and Fig. 3 (b).

Figs. 3~5 show that the discharge of pump A

Fig. 2 Coefficient of head loss ζ_v across the valve

Fig. 3 (a) Hydraulic transients, for pump A shutdown, with valve closure

Fig. 4 Time history of hydraulic transients, for two pumps shutdown in parallel operation, with valve closure

Fig. 3 (b) Hydraulic transients, for pump B shutdown, with valve closure

Fig. 5 Time history of hydraulic transients, for two pumps shutdown in parallel operation, without valve closure

increases for a short time immediately after power failure and then decreases. This is because pressure in the pipe line decreases due to a rapid drop in the speed of pump B, while pump A still has a fairly high head due to a relatively slow deceleration in the speed of pump A.

4. Conclusion

A method of analysis of hydraulic transients following power failure in parallel operation of different pumps has been described.

By examples described in this paper a result as follows has been confirmed: When power failure occurs in parallel operation of the two different pumps and there is a certain difference in the speed drop of each pump, the discharge of one pump, of which the speed reduces more slowly than the speed of the other pump, does not decrease but increases for a short time after power failure.

The author wishes to acknowledge the advice of Dr. T. Kobori and laborious calculations of Messrs. A. Yamauchi, A. Kojima and H. Shimodaira.

Reference

- (1) For instance, R.M. Peabody: *Trans. ASME*, Vol. 61, No.2 (1939), p. 117.

Discussion

S. SAITŌ: (1) The performances of a pump receiving the water hammer action which is occurring in the pipe line toward the impeller from the exit-valve and the reflected wave of the main wave which returns to the impeller part through the valve opening at some instant of valve-closing, can be considered to be somewhat different from that of the pump not receiving such water hammer. Moreover, the difference will increase when large pumps with different characteristics are under parallel-running. Therefore, it seems necessary that the effect of the water hammer action upon the pump-characteristics should be considered.

(2) As the present calculation is an approximate one, it is desirable that the results are compared in future with the experimental results in order to confirm its accuracy.

Author's closure

The author wishes to thank Professor Saitō for his discussion.

(1) Pump characteristics at each instant during transients are considered, in a strict sense, to be different from characteristics in a steady state. However, in the case of one pump, the comparison between the calculation and the experiment shows that it will be practically valid to assume that the characteristics during transients are equal to the characteristics in a steady state.^(a)

(2) In this paper the following have been assumed: 1) The velocity head may be neglected, 2) The length of branch pipes and suction pipes may be neglected in the case where their length is negligibly shorter than the total length of the pipe line. These assumptions seem to be valid for practical use, since the results of calculations and experiments of one-pump-unit operations coincide within a relatively small error. The author has not yet found opportunity to make experiments for parallel operation, and hopes to do so in the nearest future.

(a) R.T. Knapp: *Trans. ASME*, Vol. 59 (1937), p. 683.