Fundamental Research on the Gear Lubrication*
(2nd Report, Lubrication Performance on Gear-Tooth Surface)

By Tokio Sasaki**, Kenjiro Okamura***, Tadataka Konishi****, and Yoshinobu Nishizawa****

For the purpose of improving lubricating performance on the gear-tooth surface, the authors investigated the relationships of lubrication properties to the driving conditions and the lubricating methods with back-to-back type gear testing machine with which the frictional moment and the oil-film thickness between gear-teeth were measured.

The lubricating characteristics have been classified by the values of the dimensionless parameter $\frac{\tau v_i}{P}$ as follows: boundary lubrication $\leq 5 \times 10^{-4} \leq$ semi-fluid lubrication $\leq 5 \times 10^{-4} \leq$ fluid lubrication. Owing to the effects of the discontinuous contact, the dynamic load, the variation of frictional direction and the surface roughness on the gear-tooth surface, the frictional coefficient is larger and the oil-film thickness is smaller than those obtained respectively on the roller surface which was considered in the equivalent condition to the gear-tooth surface. The stirring resistance in the oil-bath lubrication is much influenced by the oil viscosity, the rotating speed, and the oil amount.

1. Introduction

The importance of gear lubrication has been recognized with an increasing use of various kinds of gears under severe conditions. The use of rationally lubricated gears is enabled only by investigating relationships between the lubrication characteristics and the driving conditions of gears. From this point of view, the fundamental characteristics of lubrication for the contact surface of lubricated rollers accompanied with sliding were experimentally investigated in a wide range of driving, designing and lubricating conditions. A dimensionless parameter, $\frac{\tau v_i}{P}$, was induced, and the lubrication characteristics were classified into three regions: fluid lubrication, semi-fluid lubrication, and boundary lubrication(1).

In the case of the actual gear-tooth surface the discontinuity of contact appears to have much influence upon the oil-film formation(2). Therefore, the lubrication characteristics of the roller surface are not always the same as those of the actual gear-tooth surface.

In this paper, the experimental investigation on the lubrication characteristics of the gear-tooth surface were carried out with the back-to-back type gear testing machine. This paper is concerned with calculation method for frictional coefficient on the gear-tooth surface for this special experiment, the comparison between both lubrication characteristics of the gear-tooth surface and the roller-surface, and the lubricating methods for gears.

2. Back-to-back type gear testing machine

In the back-to-back type gear testing machine, the driving power is very small, compared with the circulating power. Various types of back-to-back

![Fig. 1 Construction of the main part of the testing machine](image-url)
type gear testing machine have been used since Lewis' first study for the measurement of durability, efficiency, dynamic load, noise, lubrication, and so on.

The testing machine used in this investigation contains main bearings in which variation of friction is set as little as possible in order to improve the measuring accuracy of frictional moment. With this testing machine the thickness of oil film formed between the contacting gear-tooth surfaces can be measured. The construction of the main part of the testing machine is shown in Fig. 1. Four gears are concentrated in one place. The testing machine is so designed that the bearing load due to the circulation torque is small and all gears can be driven under the same lubricating condition. The frictional moment including one at the bearings of the main shafts (1) and (2) is measured with a torque meter (1). The brush (1) is used to measure the electric resistance between the contacting gear-tooth surfaces, which is related to the oil-film thickness between the surfaces. The main shaft (2) is electrically insulated from the other part of the testing machine with the insulating cylinder (1). The main ball bearings are adequately pre-loaded and lubricated with oil mist. The gears are loaded by the coil spring (3), which has no eccentricity to the rotation of the main shaft (2) at the position twisted under the testing condition.

The capacity of the testing machine is as follows:

Maximum contact pressure \( p_{\text{max}} = 85 \text{ kg/mm}^2 \)
Maximum rotating speed of the main shaft \( N = 3500 \text{ r.p.m.} \)
Capacity of torque meter \( M_{\text{max}} = 5 \text{ kg m} \)

In order to avoid the influence of accuracy of gear-tooth surface upon test results, the following attentions were paid. Gear material used was Ni-Cr steel. The gear-tooth surface was carburized, quenched, and ground. The accuracy of gear-tooth surface was checked.

The main dimensions of gears tested were as follows:
Pressure angle : \( \alpha = 20^\circ \), Module \( m = 4 \), Contact width \( B = 10 \text{ mm} \), Effective mating length \( l_e = 21.6 \text{ mm} \), Relative radius of curvature \( \rho = 7.9 \) to 10.3 mm Tooth form : Involute standard gear-tooth of ordinary depth, Number of gear tooth \( z = 30 \), gear ratio \( i = 1 \), Mating rate \( n_s = 1.83 \), Specific sliding \( S = 0 \) to 0.95, Lubricants used were straight turbine oil \# 90 and engine oil \# 50.

3. Calculation of frictional coefficient on the gear-tooth surface

The usual method of measurement of consumption power is not satisfactory for investigation of the lubrication characteristics of gears, comparing both test results obtained by the back-to-back type gear testing machine and the roller-testing equipment. Therefore, it is necessary to calculate the frictional coefficient on the lubricated gear-tooth surface.

In Fig. 1, two pairs of mating gears are \( (a_1, a_2) \) and \( (b_1, b_2) \). The number of mating points in this case is three or four as shown in Fig. 2. In other words, there are two cases that one or two pairs of gear-teeth are mating at the same time for \( (a_1, a_2) \) gears and \( (b_1, b_2) \) gears. There is discrepancy between the mating positions for \( (a_1, a_2) \) gears and \( (b_1, b_2) \) gears. Supposing that the discrepancy is equal to the half of the normal pitch \( t_n \) as shown in Fig.3, the number of mating points for all test gears is three or four as shown in Fig. 2 on the basis of the action line of \( (a_1, a_2) \) gears. Therefore, the amount measured with the back-to-back type gear testing machine is the total frictional moment under the condition of three or four contact points.

It was already reported in the previous paper that frictional coefficient \( \mu \) on the rolling contact surface of rollers is much affected by oil viscosity \( \zeta \), rolling speed \( v_r \), contact load per unit width \( P \), and specific sliding \( S \), and that it is little affected by relative radius of curvature \( \rho \). In practical cases,
the movement of mating position is accompanied with variations of rolling speed, specific sliding, and contact load. Therefore, the test by back-to-back type gear testing machine cannot be simply compared with the roller test even at the same condition of \( \eta _{0}, \eta /P \) and \( S \).

In order to compare both tests, the average frictional coefficient, the average contact load, and the average rolling speed were adopted in this paper.

The moment \( M_\theta \) measured with the torque meter is expressed as follows:

\[
M_\theta = M_4 + M_5 + M_6
\]

(1)

where

\( M_4 = \text{moment of } O_4 \text{-shaft produced by } (a_1, a_2) \text{ gears} \)

\( M_5 = \text{moment of } O_5 \text{-shaft produced by } (b_1, b_2) \text{ gears} \)

\( M_6 = \text{frictional moment at the bearings between the test gears and the torque meter} \)

The forces acting on the gear-tooth surfaces are shown in Fig. 4. In involute gears, \( P_n \), the normal force acting vertically on the gear-tooth surface is expressed by circumferential force, \( P_n \):

\[
P_n = P_n / \cos \alpha, \quad P_n = T/r
\]

(2)

where

\( T = \text{twisting moment given} \)

\( r = \text{radius of pitch circle} \)

The frictional force acting on the gear-tooth surface \( F = \mu P_n \) changes its direction at the pitch point. Therefore, the reaction force caused by mated gears at the bearing is expressed for one pair of mated gears as follows:

\[
P_r = P_n \tan \alpha \cdot \mu P_n \cos \alpha
\]

(3)

The influence of forces introduced above upon the moment measured with the torque meter will be discussed.

Since angular-contact ball bearings used are adequately pre-loaded, the variation of the frictional loss of the bearing is very small. The initial frictional loss can be measured under the non-mating condition (free rotation). However, in case that both pairs of \((a_1, a_2)\) and \((b_1, b_2)\) gears are mated, the reaction force at the bearings is twice greater than that given by Eq. (3). The amount of increase in frictional loss at the bearings due to this load is considered in the following. Supposing that bearing positions are supporting points, as shown in Fig. 5 (a), the amount of increase in frictional loss at the bearings is expressed:

\[
\Delta M_\theta = \mu \Delta P_r \cdot (a + b) \cdot d_1 \cdot d_2 \cdot \cos \beta
\]

(4)

where

\( d_1 \) and \( d_2 \) = diameters of the internal race ways at contact points with balls for ball bearings 1 and 2

\( \mu_\theta = \text{frictional coefficient for both bearings (supposing that it is not varied with load variation)} \)

\( \beta = \text{angle between the contact line of balls on race ways and the central section} \)

But, this expression is not sufficient because pre-load is given to the ball bearings. In order to measure \( \Delta M_\theta \), as shown in Fig. 5 (b), ball bearings 3 are installed at the position of gear setting and the load corresponding to \( 2P_r \) is imposed outside the bearings. The frictional loss of the bearings 3 is measured by the balance-beam method. Subtracting this value from the value indicated on the torque meter, \( \Delta M_\theta \) is obtained as shown in Table 1. It is found that \( \Delta M_\theta \) is not affected by rotational speed.

\( \Delta M_\theta \) is negligibly small for the purpose of measuring the average frictional coefficient because the frictional loss of the bearing \( M_\theta \) in the case of non-mating is nearly equal to 2.5 kg cm and the moment measured during the gear mating test is 10 to 30 kg cm. Therefore, the frictional loss of the bearing in the case of non-mating is considered

\begin{table}[h]
\centering
\caption{Measured values of \( \Delta M_\theta \)}
\begin{tabular}{|c|c|}
\hline
\( 2P_r \text{ kg} \) & \( \Delta M_\theta \text{ kg cm} \) \\
\hline
20,9 & 0.51 \\
40,8 & 0.47 \\
59,8 & 0.28 \\
80,5 & 0.31 \\
\hline
\end{tabular}
\end{table}
to be $M_0$.

$M_0$ and $M_0'$ are expressed as follows:

$M_0 = T \cdot \rho_0 F$, $M_0' = - T \cdot \rho_0' F$ \hspace{1cm} (5)

$\rho_0$ and $\rho_0'$ are radii of curvature at the mating points for $(a_0, b_0)$ gears, and depend upon the positions of the mating points. The moment due to friction on gear-tooth surfaces is given as follows:

$M = M_0 - M_0' = \mp \rho_0 F \pm \rho_0' F$ \hspace{1cm} (6)

$M$ is positive if this moment is in the same direction as that of rotation of $O_y$-shaft. The plus and minus signs in Eq. (6) change at the pitch point.

In order that gears I and II can mate at both points A and B successively, as shown in Fig. 3, the total amounts of wear of gears I and II should be equal at both points A and B. The amount of wear $w_{A1}$ of gear I at the point A is expressed:

$w_{A1} = K_1 \mu_1 P_a v_{ad} |P_{A1}|$ \hspace{1cm} (7)

where

$K_1$ = constant depending upon the material of gear

$\mu_1$ = frictional coefficient at point A

$P_a$ = normal force at point A

$v_{ad}$ = sliding speed at point A

Supposing that the same material is used for both gears I and II, the total amount of wear of both gears at point A is expressed as follows:

$w_A = w_{A1} + w_{A2} = K_1 \mu_1 P_a v_{ad} |P_A|$ \hspace{1cm} (8)

$\rho_A = \rho_{A1} \rho_{A2} / (\rho_{A1} + \rho_{A2})$

Considering the average frictional coefficient, the following expression is deduced, since $w_A$ is equal to $w_B$.

$(P_a v_{a1} / |P_a|) = (P_a v_{a1} / |P_a|)_B \hspace{1cm} (9)$

$P_{A1} + P_{B1} = P_a$ \hspace{1cm} (10)

Putting

$(P_a v_{a1} / |P_a|)_B = \kappa.$

Eqs. (9) and (10) are expressed as follows:

$P_{A1} = \frac{1}{1 - \kappa} P_a, P_{B1} = \frac{\kappa}{1 - \kappa} P_a$ \hspace{1cm} (11)

$\kappa = \left(1 - \frac{2 \alpha - 1}{x(1-x)} \right) / \left(1 - \frac{2 \alpha - 1}{x(1-x)} \right)_B$

Therefore, the normal force $P_a$ at a contact point is expressed for an entire region of gear mating $(x_1 \sim x_2)$ as follows:

$P_a = \frac{CT}{r \cos \alpha}$ \hspace{1cm} (12)

$C = \frac{1}{1 - \kappa}$ $x_1 \leq x \leq x_1'$

$C = 1$ $x_1' \leq x \leq x_2'$

$C = \frac{\kappa}{1 - \kappa}$ $x_2' \leq x \leq x_2$

where

$x_1$ = Starting point of mating

$x_2$ = End point of mating

$x_1'$ and $x_2'$ = Boundary points changing from mating of one pair to two pairs of gear-teeth and vice versa

The value of $C$ in gears used is shown in Fig. 6. The moment due to frictional force $F$ on gear-tooth surfaces of $(a_1, a_2)$ gears is expressed:

$M_{P_a} = \sum \rho_a F P_a = \phi_{P_a} / r \cos \alpha$ \hspace{1cm} (13)

Fig. 6 Variation of load distribution

Fig. 7 Variation of $\phi_{P_a}$ with $x$

Fig. 8 Variation of $\phi_{P}$ with $x$
\[ \phi_{fa} = \frac{1}{1+\kappa} (-\rho_{2a} + \kappa \rho_{2a} b), \text{ for mating of two pairs of gear-teeth} \]
\[ = \pm \rho_{2a}, \text{ for mating of one pair of gear-teeth} \]

Fig. 7 shows the value of \( \phi_{fa} \).

The frictional moment for \((b_a, b_{a2})\) gears is obtained in the same procedure as for \((a_1, a_{2a})\) gears.

Considering an angle of 2\( \alpha \) between both action lines of \((a_1, a_{2a})\) and \((b_a, b_{a2})\) gears and a discrepancy of \( t_a/2 \) between the mating positions for \((a_1, a_{2a})\) and \((b_a, b_{a2})\) gears, the moment \( M \) caused by the friction on gear-tooth surfaces is given as sum of \( M_{fa} \) and \( M_{fa} \) obtained above.

\[ M = \phi_{fa} \mu \frac{T}{r \cos \alpha} \]  \hspace{1cm} (14)

\( \phi_{fa} \) varies with the movement of mating positions as shown in Fig. 8; that is, it is found that \( \phi_{fa} \) varies with a period of \( x_a' - x_a \). It is considered from this that the measured value of \( M \) also varies periodically. However, the value of \( M \) indicated in the meter gives the mean value \( \bar{M} \) in the mating test of gears rotating with a circumferential speed of about 10 m/sec.

Putting

\[ \bar{M} = \bar{\phi}_{fa} \mu \frac{T}{r \cos \alpha} \]  \hspace{1cm} (15)

\[ \bar{\phi}_{fa} = 2 \phi_{fa} = \frac{2}{x_a' - x_a} \int_{x_a}^{x_a'} \phi_{fa} dx \]  \hspace{1cm} (16)

This can be calculated from Fig. 8.

The calculation of Eq. (16) yields

\[ \bar{\phi}_{fa} = -0.746 \text{ cm} \]

Therefore, the following expression is obtained for the average frictional coefficient on the gear-tooth surface in this experiment.

\[ \mu = \frac{\bar{M} r \cos \alpha}{\bar{\phi}_{fa} T} = 7.56 \bar{M} \]  \hspace{1cm} (17)

Using the mean value of \( C \) in Fig. 6, the mean normal load is deduced as follows:

\[ \bar{P}_n = 0.607T/r \cos \alpha \]  \hspace{1cm} (18)

4. Relationship between driving conditions and the lubrication performance

The relationship between driving conditions and the lubrication performance in the actual running of gears in oil-bath lubrication was investigated in the range of the twisting moment and the rolling speed indicated in the following:

\[ T = 100 \text{ to } 760 \text{ kg cm}, \hspace{0.5cm} v_r = 2.6 \text{ to } 8.4 \text{ m/sec} \]

Oil used was turbine oil #90 and its viscosity was 0.0072 to 0.0095 kg sec/cm². Turbine oil of 400 ml was charged at the start of the experiment, and by this the gear-tooth at the lowest position was submerged until the dedendum. The stirring resistance by oil-bath lubrication was calibrated and its effect on the test results was taken away.

The relation between the electric resistance and the oil-film thickness was obtained by the calibrating apparatus. In this mating test, however, there are three or four mating points as shown in Fig. 2, and it appears that the oil-film thickness at these points is not same. The relation between the electric resistance \( R \) measured by the meter and the electric resistance \( R_i \) related to the oil film at mating points is expressed:

\[ 1/R = \sum 1/R_i \]

Using the average oil-film thickness \( \bar{h}_o \) as in the case of frictional coefficient, the following expression is deduced since the average number of mating points is 3.3.

\[ \bar{R}_i = 3.3 R \]  \hspace{1cm} (19)

Fig. 9 shows an example of the frictional moment on the gear-tooth surface \( \bar{M} \) and the oil-film thickness \( \bar{h}_o \) in relation to the circumferential force \( P_n \). It is found from this figure that

![Fig. 9 Variations of average frictional moment \( \bar{M} \) and average oil-film thickness \( \bar{h}_o \) with circumferential force \( P_n \) under various rotating speeds](image)

![Fig. 10 Relation between frictional coefficient \( \mu \) and dimensionless parameter \( \eta_{O} / \bar{P}_n \) for the gear mating test](image)
friction moment increases with an increase in circumferential force almost in linear relationship having 45° inclination on the bi-logarithmic graph. On the other hand, the average oil film thickness decreases with an increase in circumferential force in exponential relationship.

Fig. 10 shows the relation between $\mu$ and $\eta_{nr}/P_s$ for the gear mating test. The relation between $\mu$ and $\eta_{nr}/P$ for the roller test is shown in a broken line for comparison. All data points in Fig. 10 are concerned with the combined conditions of three kinds of average normal load and four kinds of rolling speed. The points are scattered, but it appears that the relation between $\mu$ and $\eta_{nr}/P_s$ in the mating test is represented with two lines as shown in the figure. The value of $\mu$ is almost constant, (i.e. about 0.09), for $\eta_{nr}/P_s$ below $5 \times 10^{-6}$, and it decreases with an increase in $\eta_{nr}/P_s$ in linear relationship for $\eta_{nr}/P_s$ above $5 \times 10^{-4}$.

A broken line in Fig. 10 is a typical curve of $(\eta_{nr}/P_0) \cdot \mu$ for the experimental result of the roller test for various contact loads, rolling speeds, specific slidings, and oil viscosities. It is found that the critical values for the roller test at which the lubrication characteristics change from boundary lubrication to semi-fluid lubrication, and to fluid lubrication are $5 \times 10^{-7}$ and $5 \times 10^{-3}$, respectively. On the other hand, the lubrication characteristics in the gear mating test are classified as follows:

- Boundary lubrication $\eta_{nr}/P_s < 5 \times 10^{-4}$
- Semi-fluid lubrication $5 \times 10^{-4} < \eta_{nr}/P_s < 5 \times 10^{-4}$
- Fluid lubrication $5 \times 10^{-4} < \eta_{nr}/P_s$

The critical values for $\eta_{nr}/P_s$ in the gear mating test appears to be ten times greater than those in the roller test.

In the region of boundary lubrication, the value of $\mu$ in the gear mating test is about 1.8 times greater than that in the roller test. In the region of semi-fluid lubrication, frictional coefficient decreases with an increase in $\eta_{nr}/P_s$ in almost same tendency for both gear mating test and roller test, but the value of $\mu$ in the gear mating test is nearly three times greater than that in the roller test for the same value of $\eta_{nr}/P_s$. It is concluded from this result that the lubrication characteristics are improved in gears as well as in rollers with a large value for $\eta_{nr}/P_s$. The value of $\mu$ in gears is generally greater than that in rollers, and this value in gears becomes 0.05, which is the value for $\mu$ in the region of boundary lubrication for rollers, when the value of $\eta_{nr}/P_s$ increases up to $1 \times 10^{-4}$. Such great difference between both lubrication characteristics in gear mating and roller tests is explained in the following.

The condition of contact between lubricating surfaces is continuous in rollers, while discontinuous in gears. The normal load changes discontinuously with variation of the number of mating gear-teeth. The frictional force changes its direction at the pitch-point. The roughness of contact surfaces affects the lubrication characteristics. The contact surfaces were super-finished in the case of the roller test, and its roughness was less than 0.2 $\mu$, while that of gear-tooth surfaces, which are ground, is 2 to 3 $\mu$. In order to investigate the effect of surface roughness on the lubrication characteristics, surfaces which were just produced with turning by a lathe were used for the roller test. It was found that the broken line in Fig. 10 in this case moves in a parallel shape upward in an increasing direction of $\mu$.

The relation between $\mu$ and surface roughness $H_{\text{max}}$ in this case is shown in Fig. 11. It is seen that $\mu$ varies considerably in the range for small values of $H_{\text{max}}$ including 2 to 3 $\mu$, which correspond to the actual surface roughness in gear-tooth. The value of $\mu$ in this range is 1.5 to 1.8 times greater than that for the super-finished surfaces shown in double circles. Therefore, it is considered that the
surface roughness is a main cause for the difference between values for $\mu$ in both gear mating and roller tests.

It is supposed that fluid lubrication in gears may be produced in the range of $\frac{\eta v_r}{P_a}$ above $5 \times 10^{-4}$, from Fig. 10. It is considered that the rolling speed is required to be above 25 m/sec, in order to produce fluid lubrication in the case of gears, for an oil with a comparatively high viscosity of 0.1 kg/sec/m² (such as engine oil #50 at 17°C) and a comparatively light load of 50 kg/cm. Since the roughness of gear-tooth surface less than 1 $\mu$m is difficult to be produced in the practical manufacturing, perfect fluid lubrication can not be realized in the practice.

Fig. 12 shows an example of the relation between $\frac{\eta v_r}{P_a}$ and lubricating oil-film thickness in the gear mating test. It was already found that the oil-film is generated in the same mechanism for semi-fluid lubrication as for fluid lubrication. The oil film thickness is little influenced with specific sliding, but it increases proportionally with the relative radius of curvature of gear-tooth surfaces. An experimental result for the roller test is also shown in Fig. 12 and was compared with the result for gear mating test. In this roller test, relative radius of curvature for rollers was 10.5 mm (cf. relative radius of curvature for gears was 8 to 10.3 mm).

The average oil film thickness $h_0$ increases with $\frac{\eta v_r}{P_a}$ in almost linear relationship for both test, but the rate of increase is rather smaller for the gear mating test than for the roller test. Therefore, the amounts of $h_0$ are much different in the both tests for large values of $\frac{\eta v_r}{P_a}$. A great difference of the frictional coefficient in both tests is considered to be caused by the oil film thickness which is about 1 $\mu$m in the case of super-finished surfaces with roughness of about 0.1 $\mu$m and below 1 $\mu$m in the case of gear-tooth surfaces with roughness of 2 to 3 $\mu$m. It appears from Fig. 12 that the amount of $h_0$ is almost same in the case of boundary lubrication for both tests.

It is concluded that the discontinuity of contact, the variation in load, the inversion of frictional direction, and the difference in surface roughness worsen the lubrication characteristics in the case of the gear-tooth surface, compared with the roller surface.

5. Relationship between the lubricating methods and the lubrication performance

In order to discuss the relation between the lubricating methods and the lubrication performance, the stirring resistance in the oil-bath lubrication was investigated. The frictional coefficient and the oil film thickness between the lubricated gear-tooth surfaces in the case of oil-bath lubrication, oil-jet lubrication, and oil-mist lubrication were measured under the following driving conditions:

$T=500$ to 1200 kg cm, $P_a=54$ to 150 kg/cm²
$N=880$ to 2250 r.p.m., $v_r=1.9$ to 4.8 m/sec
$\eta=0.004$ to 0.08 kg sec/m²

Figs. 13 and 14 show examples of experimental results on the stirring resistance. Since four gears are used in the gear mating test discussed in this paper, the whole amount of stirring resistance in oil-bath lubrication is twice greater than that in Fig. 13.

Fig. 13 shows the stirring resistance in relation to oil viscosity $\eta$ and oil temperature $t$. In this case a circumferential speed of gears was 5.9 m/sec. It is found from Fig. 13 (a) that the stirring resistance depends upon oil viscosity and not upon kind of oil. In the range of $\eta$ below 0.05 kg/sec/m² the value of $M_s$ remains comparatively small, but in the range of $\eta$ above 0.05 kg/sec/m² it increases considerably. The relation between $M_s$ and $t$ is shown in Fig. 13 (b) for engine oil #50. $M_s$ decreases considerably with an increase in the initial temperature, but the rate of the decrease becomes small in temperature range above 20°C. In the actual oil-bath lubrication, however, oil
viscosity decreases due to the temperature rise caused by the stirring resistance, and it tends to a constant value when the caloric and the radiating values are balanced.

Table 2 Summary of experimental results for three kinds of lubricating methods

<table>
<thead>
<tr>
<th>Lub. method (Oil amount)</th>
<th>$v$ m/sec</th>
<th>$F_n$ kg/cm</th>
<th>$\eta_{v}/F_n$</th>
<th>$\mu$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil-bath lubrication (500 ml/m)</td>
<td>2.69 (1 250)</td>
<td>54</td>
<td>1.78 x 10^-6</td>
<td>0.066</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>92</td>
<td>1.01</td>
<td>0.075</td>
<td>0.7</td>
<td></td>
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<tr>
<td></td>
<td>130</td>
<td>0.73</td>
<td>0.866</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.72 (1 730)</td>
<td>54</td>
<td>2.78</td>
<td>0.007</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>92</td>
<td>1.58</td>
<td>0.064</td>
<td>0.8</td>
<td></td>
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<tr>
<td></td>
<td>130</td>
<td>0.96</td>
<td>0.075</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.84 (2 250)</td>
<td>54</td>
<td>3.01</td>
<td>0.059</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>92</td>
<td>1.77</td>
<td>0.084</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>130</td>
<td>1.20</td>
<td>0.078</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>Oil-jet lubrication (800 ml/min)</td>
<td>2.69 (1 250)</td>
<td>54</td>
<td>1.88</td>
<td>0.059</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>92</td>
<td>1.08</td>
<td>0.069</td>
<td>0.7</td>
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<tr>
<td></td>
<td>130</td>
<td>0.82</td>
<td>0.081</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.72 (1 730)</td>
<td>54</td>
<td>2.67</td>
<td>0.068</td>
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<td>0.075</td>
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<td>4.84 (2 250)</td>
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<td>1.90</td>
<td>0.087</td>
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<td>130</td>
<td>1.35</td>
<td>0.073</td>
<td>0.6</td>
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<tr>
<td>Oil-mist lubrication (0.5 ml/min)</td>
<td>2.69 (1 250)</td>
<td>54</td>
<td>1.77</td>
<td>0.096</td>
<td>0.2</td>
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<tr>
<td></td>
<td>92</td>
<td>1.05</td>
<td>0.146</td>
<td>0.1</td>
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<tr>
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<td>130</td>
<td>0.75</td>
<td>0.177</td>
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<tr>
<td></td>
<td>3.72 (1 730)</td>
<td>54</td>
<td>2.61</td>
<td>0.077</td>
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<tr>
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<td>92</td>
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<td>0.106</td>
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<tr>
<td></td>
<td>4.84 (2 250)</td>
<td>54</td>
<td>2.69</td>
<td>0.085</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>92</td>
<td>1.77</td>
<td>0.130</td>
<td>0.2</td>
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</tr>
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</table>

Fig. 14 shows $M_e$ in relation to the rotational speed $N$ and the oil amount $Q$. It is found that the rotational speed has little influence upon $M_e$ for oil amount less than 500 ml (in other words, the gear-tooth at the lowest position is submerged until a little inside of the dedendum). $M_e$ increases remarkably with an increase in $Q$ for any oil viscosity and rotational speed. The amount of oil sticking to gear-tooth surfaces does not change and does not improve the lubrication performance, even if $Q$ increases. The excess oil increases power loss, decreases oil viscosity, and forms bubbles. Therefore, an oil amount which submerges the gear-tooth at the lowest position until the dedendum is considered to be suitable for a charged oil.

Table 2 shows a summary of experimental results for the three kinds of lubricating methods for the turbine oil # 90 ($7=0.0035$ to 0.0040 kg sec/m²). Either frictional coefficient or oil film thickness is almost same for both oil-bath lubrication and oil-jet lubrication. However, in the case of oil-mist lubrication the oil-film thickness is less than half and the frictional coefficient is about 1.5 times greater than in the case of oil-bath lubrication and oil-jet lubrication. The rotational air flow along the circumference of gears prevents the oil mist from its effective lubrication on gear-tooth surfaces, and the oil amount insufficient for oil film formation decreases $\delta_0$ and increases $\mu$.

Therefore, a special supplying device for oil mist for gears is required.

6. Conclusions

The experiments performed with the back-to-back type gear testing machine lead to the following conclusions.

(1) The equation expressing frictional coefficient on gear-tooth surfaces was deduced. With this equation frictional coefficient was calculated from the measured values for rotational moment produced with the back-to-back type gear testing machine in which mating gears contact three or four points at the same time.

(2) Introducing a dimensionless parameter $\eta_{v}/F_n$, the lubrication characteristics for the gear-tooth surface was classified as follows:

- Boundary lubrication $\eta_{v}/F_n<5x10^{-4}$
- Semi-fluid lubrication $5x10^{-4}<\eta_{v}/F_n<5x10^{-4}$
- Fluid lubrication $5x10^{-4}<\eta_{v}/F_n$

(3) The oil film is formed on the gear-tooth surface in the region of semi-fluid lubrication with the same principle as in the region of fluid lubrication.

(4) Comparing the experimental results for the gear mating test and the roller test, the
discontinuity of contact, the variation in load, the inversion of frictional direction, and the difference in surface roughness worsen the lubrication characteristics in the case of the gear-tooth surface.

(5) The stirring resistance in oil-bath lubrication was clarified in relation to oil viscosity (i.e. temperature), rotational speed, and oil amount (i.e. submerged depth).

(6) Oil-bath lubrication, oil-jet lubrication, and oil-mist lubrication were compared for the gear lubrication methods, and their effects on the lubrication characteristics were discussed.

References


Discussion

**Y. Goto** : (1) It is found from the experimental results in the transmission that the coefficient of rolling friction with sliding for the bearing steel (SUJ-2) varies considerably with variation in temperature. How is considered the influence of temperature on the frictional coefficient in your study?

(2) The results as shown in Appendix Fig. 1 were obtained in the experiment mentioned above. The relation between the frictional coefficient and the contact pressure shows a different tendency with temperature in the figure. Have such results as shown in Appendix Fig. 1 been obtained in your experiment? How do you think that the influences of transmitting power and contact temperature upon the transmitting efficiency? Do you think that the results shown in Appendix Fig. 1 can be well explained with a relation of $\dot{z}N/P \propto \mu$?

**T. Ueno**: (3) What is the basis of the formula (7)? How certain is it? Are the test gears tip-relieved? How do you think about the efficiency of the elastic tooth deformation for the load distribution?

(4) What will be the data shown in Figs. 9, 10, 11, and 14, if they are expressed by efficiency or % to the allowable load carrying capacity of the test gears?

(5) The electric current through the contact point is changing off and on with very high frequency, and it seems the contact point does not have a constant resistance. How is it in your study?

Authors’ closure

(1) The answer will be given regarding this question from the standpoint of the relation between oil viscosity and frictional characteristics. It was already described in many papers that oil viscosity has much influence upon the coefficient of rolling friction accompanied with sliding.

Fig. 10 shows the value of $\mu$ obtained for the following conditions:

- $F_n = 30$ to $130$ kg/cm, $v_r = 1.8$ to $4.8$ m/sec
- $\eta = 0.003$ to $0.15$ kg sec/m$^2$

An example of the effect of oil viscosity upon $\mu$ is shown in Appendix Fig. 2. It should be noted that the figure shows the experimental results for boundary lubrication and semi-fluid lubrication.

The experimental results on the roller surface with sliding might be suitable for this question. This refers to Fig. 10, *Bulletin of JSME*, Vol. 4, No. 14 (May, 1961), p. 387.

(2) With variation in oil temperature, the oil viscosity varies, hence the frictional coefficient and the transmitting efficiency vary. Since temp-

![Append-Fig. 1 Relation between frictional coefficient and contact pressure](image1)

![Append-Fig. 2 Variation of $\mu$ with $\eta$](image2)

![Append-Fig. 3 An example of typical relation $zN/P=\mu$](image3)
perature alone is indicated and not oil viscosity in Appendix-Fig.1, it is difficult to say if the experimental results similar to Appendix-Fig.1 can be obtained in the authors’ test. Regarding the fact that diagrams shown in Appendix-Fig.1 can be well explained with a relation of \( zN/P - \mu \), it is not clear if the present question is limited only to the region of fluid lubrication or includes other regions such as semi-fluid lubrication and boundary lubrication as shown in Appendix-Fig.3. It seems that two diagrams for frictional coefficient shown in Appendix-Fig.1 differ too much for temperature difference of 40°C if other conditions are same. It is considered that viscosity of oil film at contact point differs from that of supply oil due to a large variation in contact load. It might occur that the relation between the contact load and for different temperatures is completely reversed if the lubrication characteristics change.

It is suggested that a better result will be obtained if data is arranged in relation to \( zN/P - \mu \).

(3) Eq. (7) is deduced as follows: It is supposed that the continuity of mating and the wear of gears depend on the frictional work on gear-tooth surfaces, the number of mating points, and the gear material. The frictional work is expressed \( \Delta A = \gamma A (F_o dA) \). The wear amount of gear-tooth is proportional to the frictional work per unit area.

\[ w = K_o \gamma A (dA/ds) \]

Eq. (7) is derived from this equation.

The test gears were not tip-relieved.

It is considered that elastic deformation of gear-tooth has much influence upon the load distribution as the discussers points out. This will be discussed in successive reports.

(4) Figs. 9 and 10 show many data points for various loads, speeds and oil viscosities. The rate of average frictional moment to the total torque is expressed as follows:

\[ G_f = \bar{M}/(\bar{P} \gamma + \bar{M}) = \bar{M}/T \times 100\% \]

Substituting Eq. (17) into this,

\[ G_f = \mu T(7.56(T + \frac{\mu T}{7.56}) = \mu T \times 100\% \]

The following table is obtained from this equation and Fig. 9.

<table>
<thead>
<tr>
<th>( P ) kg</th>
<th>( \bar{M} ) kg cm</th>
<th>( \bar{P} \gamma ) kg cm</th>
<th>( G_f ) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.65</td>
<td>60</td>
<td>1.08</td>
</tr>
<tr>
<td>20</td>
<td>1.30</td>
<td>120</td>
<td>1.08</td>
</tr>
<tr>
<td>100</td>
<td>7.0</td>
<td>600</td>
<td>1.16</td>
</tr>
</tbody>
</table>

From Fig. 10

\( \bar{\mu} = 0.08, \hspace{0.5cm} G_f = 1.06\% \)

Therefore, the average efficiency of gears is 99 to 98%. Since it is found from Fig. 8 that \((\bar{\phi} \gamma)_{\text{max}}\) is four times greater than \(\bar{\phi} \gamma\), the minimum of average efficiency of gears is about 95%.

The maximum and minimum values of \( M_s \) in this experiment were 2.5 to 0.5 kg cm, respectively. The stirring horse power is expressed: \( N_s = \pi n M_s / 2250 \). Therefore, the maximum and minimum stirring horse power are 0.08 and 0.006 HP, respectively. It is difficult to compare \( M_s \) with the allowable load carrying capacity because there is no coincidence between factors influencing \( M_s \) and factors influencing the allowable power.

(5) An example of electric contact resistance in the roller test measured with the oscillograph is shown in Fig. 8, Bulletin of JSME, Vol. 4, No. 14, p. 386. It is supposed that a variation of electric resistance is much larger in the gear mating test than in the roller test because of discontinuity of contact dynamic load, and others. However, a value of electric current through the contact point was comparatively stable. In the case of measuring the electric contact resistance of oil film, it is important to note the amount of the voltage charged. Too high voltage charged causes a variation in measured value. A suitable voltage was selected in this study.