$$\alpha_p \doteq 1 + \frac{1}{5} \sqrt{\frac{R}{10}}$$

(3) Both the velocity distribution at any radial station and the pressure distribution in any nozzle can be determined only by R . The pressure loss coefficient ξ of any nozzle is also determined only by R . However, as two regions exist, one of the different formulas must be applied to evaluate ξ , according to the combination of R and α_2 .

(4) The radial velocity u_0 at the central core of the clearance of a nozzle increases temporarily at the entrance and then decreases with the flow. When R is about 50, u_0 is held nearly constant in

the inlet region.

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531. 717. 082. 36:532. 525

Studies on the Characteristics of Radial-Flow Nozzles*

(2nd Report, Theoretical Analysis of Inward Flow)

By Tatsuya HAGIWARA**

The inward flow characteristics of radial-flow nozzles are studied by the similar procedure to the 1st Report. As a parameter, $R = Vh^2/\nu r_2$ should be used to calculate velocity distributions in the clearance, pressure losses, etc., where r_2 is the outer radius of a nozzle and V is a radial mean velocity at r_2 . When R is over 5.42, boundary layers cannot fill up the clearance, and if R is less than 5.42, the velocity distribution in the clearance becomes perfectly parabolic, that is, Poiseuille flow will be established. In either case, the thickness of the boundary layer reduces to zero at the center of the nozzle. Therefore, the pressure loss of the nozzle should be expressed by one of the three different equations according to the combination of R and the diameter ratio of the nozzle.

1. Introduction

A theoretical analysis was carried out of the outward flow in a radial-flow nozzle, and the thickness of the boundary layer, the velocity and the pressure distribution, and the pressure flow characteristics were made clear in the 1st Report⁽¹⁾. The same procedure as for the theoretical analysis about the inward flow is performed in this report. The assumptions established in the previous report and hence the equation of the boundary layer are applicable in this case as it was.

The inward flow of a radial-flow nozzle has been rarely utilized in practice and it seems that the

theoretical analysis for this case has not been done up to this time. As mentioned later, it is necessary to clarify the characteristics of the inward flow in order to evaluate the characteristics of pneumatic micrometers with push-pull function⁽²⁾ and V-shape characteristics⁽³⁾ used in the measurement of vibration.

Nomenclatures are almost the same but the radius ratios are based on the outer radius r_2 , because the flow of fluid starts from the outer radius. The thickness of the boundary layer decreases, after it fills up the space because of converging flow, then a few nomenclatures are newly defined as follows.

α : radius ratio based on r_2

r_p' : radial station where the Poiseuille flow ends

($\alpha_p' = r_p'/r_2$)

V : mean radial velocity at the entrance

* Received 21th June, 1961.

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2. Solutions of equation

2.1 Thickness of boundary layer

Taking the minus sign in Eq. (19) in the 1st Report, we have

$$\left. \begin{aligned} \frac{d^2v}{dx^2} + \frac{1}{x} \frac{dv}{dx} + \left\{ 1 - \frac{(a/2)^2}{x^2} \right\} v &= 0 \\ X = \frac{a}{2b} + \frac{x}{bv} \frac{dv}{dx}, \quad x = 2\sqrt{\frac{b}{R}} \alpha \end{aligned} \right\} \dots\dots\dots (1)$$

and similarly to the 1st Report,

$$\left. \begin{aligned} X &= \frac{1}{10} \left[\frac{48-7\eta}{3-\eta} \eta + 48 \log_e \left(1 - \frac{\eta}{3} \right) \right] \\ R &= \frac{Qh}{2\pi r_2^2 \nu}, \quad \alpha = \frac{r}{r_2} \end{aligned} \right\} \dots\dots\dots (2)$$

Eq. (1) is the Bessel's differential equation, hence, noting that a is an odd integer, the solution is

$$X = \frac{a}{2b} + \frac{x}{b} \frac{DJ'_{a/2}(x) + J'_{-a/2}(x)}{DJ_{a/2}(x) + J_{-a/2}(x)} \dots\dots\dots (3)$$

where D is an integration constant. Simplifying Eq. (3) by the reduction formulas of the Bessel function, we have

$$X = \frac{x}{b} \frac{DJ_{3/2}(x) - J_{-3/2}(x)}{DJ_{5/2}(x) + J_{-5/2}(x)}, \quad x = c\alpha, \quad c = 2\sqrt{\frac{b}{R}} \dots\dots\dots (4)$$

The Bessel function $J_{\pm a/2}(x)$ is expressed by the trigonometrical function, when a is an odd integer. Then Eq. (4) becomes

$$X = \frac{1}{b} \left\{ \frac{3}{c^2 \alpha^2} + \frac{1}{c\alpha \cot(c\alpha + K) - 1} \right\}^{-1} \dots\dots\dots (4)'$$

where $D = \cot K$ and K is an arbitrary constant.

As the fluid flows from the circumference to the center of a nozzle, α becomes gradually less than 1 with the flow, hence the initial condition must be

$$X=0 \text{ or } \eta=0 \text{ at } \alpha=1.$$

Then the equation for the constant K becomes

$$\tan(K+c) = c \dots\dots\dots (5)$$

and from Eq. (4)', we have

$$X = \frac{1}{b} \left\{ \frac{3}{c^2 \alpha^2} + \frac{c - \tan c(1-\alpha)}{(c^2 \alpha + 1) \tan c(1-\alpha) - c(1-\alpha)} \right\}^{-1} \dots\dots\dots (6)$$

This solution is similar to the solution for the outward flow, but it is expressed by tangents instead of hyperbolic tangents. Similarly to the previous report, the relation between η and α is obtained with R as a parameter, as shown in Fig. 1. When R is small, the boundary layer increases its thickness gradually, and finally a Poiseuille flow appears at α_p like in the outward flow. On the other hand, when R is large, the thickness of the boundary layer increases temporarily in the vicinity of the entrance, but at a certain position, it begins to decrease. As shown in Fig. 1 by a chain line, the maximum of η

Fig. 1 Thicknesses of boundary layers

takes place in the range $\alpha=0.740 \sim 0.654$. The critical value of R for these two phenomena is 5.42. In this case, the maximum value of η becomes just unity.

When R is less than 5.42, though the boundary layer fills up the clearance for a while, it begins to decrease its thickness from a certain position, since mean velocity becomes larger, hence the pressure decreases in the direction of flow. To determine this position, named α_p' , taking the minus sign in Eq. (18) of the 1st Report and putting the condition $d\eta/d\alpha=0$ at $\eta=1$, i.e. $dx/d\alpha=0$ at $X=X_1$ into this equation, we have

$$\alpha_p' = \frac{1}{2} \sqrt{(aX_1 - bX_1^2)R} \dots\dots\dots (7)$$

where X_1 is the value of X for $\eta=1$, and is 0.10374. Putting numerical values, $a=5$ and $b=18.9$, into Eq. (7), we have

$$\alpha_p' = 0.2808\sqrt{R} \text{ or } x_p' = 2\sqrt{\frac{b}{R}} \alpha_p' = 2.441$$

Behaviour of the flow beyond α_p' can be defined by Eq. (4)' with the initial condition of $X=X_1$ at $\alpha=\alpha_p'$. The solution is

$$\left. \begin{aligned} X &= \frac{1}{b} \left\{ \frac{3}{c^2 \alpha^2} \right. \\ &\quad \left. + \frac{\theta - \tan c(\alpha_p' - \alpha)}{(\theta c \alpha_p' + 1) \tan c(\alpha_p' - \alpha) - (\theta - c \alpha_p')} \right\}^{-1} \\ \theta &\equiv \tan(K + c \alpha_p') = \frac{(2 - bX_1) \sqrt{b(5 - bX_1)X_1}}{2 + 4bX_1 - b^2X_1^2} \end{aligned} \right\} \dots\dots\dots (8)$$

Putting numerical value X_1 and b into Eq. (8), θ becomes 0.01599. Thus the right halves of curves with $R < 5.42$ in Fig. 1 are obtained from Eq. (8); α_p and α_p' are the function of R only as shown in Fig. 2. When $R > 5.42$ both α_p and α_p' disappear

and the maximum values of η come out at the positions shown by a broken line in Fig. 2.

To discuss the behaviour of a flow, it is convenient to distinguish the flow into three regions after the manner of the previous report.

I-region: The boundary layer does not fill up the clearance but is increasing or decreasing its thickness. If R is more than 5.42, all flows exist in this region, and even R is less than 5.42, a flow may exist here depending on the value of α .

II-region: In this region, a Poiseuille flow exists and this region can be realized with the specified combinations of R and α .

III-region: From the point of view that a boundary layer is decreasing its thickness, this region is the same as the I-region, but the Poiseuille flow must have been once established in this region. When $R < 5.42$, a flow may appear in I-, II- and III-region successively according to the values of α .

2.2 Velocity distribution

The velocity distribution in the clearance is determined by the η only, regardless of the region. A flow has a uniform radial velocity V at the circumference of a nozzle, and the flow velocity at the central core of the clearance u_0 is

Fig. 2 Values of α_p and α_p'

Fig. 3 Velocities at the central core

$$\frac{u_0}{V} = \frac{3}{\alpha(3-\eta)} \quad \dots\dots\dots (9)$$

Fig. 3 shows u_0 with R as a parameter, and the curve noted $\eta=1$ represents the case of the Poiseuille flow. Discontinuity appears when η becomes unity because of the simplified assumption for the velocity distribution in a boundary layer. The velocity profiles are determined by the following equation

$$\frac{u}{V} = \frac{12}{\alpha(3-\eta)} \frac{z}{\eta h} \left(1 - \frac{z}{\eta h}\right) \quad \dots\dots\dots (10)$$

The velocity profiles calculated for three different R are shown in Fig. 4. In the case of $R=5.42$, the boundary layer fills up the space at $\alpha=0.654$, and then begins to decrease its thickness immediately. When $R=20$, the thickness of the boundary layer reaches to its maximum value ($\eta_{\max}=0.59$) at $\alpha=0.725$. The velocities in the central core are always increasing, but the velocity in the neighbourhood of the wall is once retarded and then accelerated.

2.3 Pressure loss and pressure distribution

Neglecting the pressure loss at the inlet, the static pressure at a position in the I-region is given as

$$p_2 - p = \frac{\rho}{2} V^2 \left\{ \frac{3}{(3-\eta)\alpha} \right\}^2 \quad \dots\dots\dots (11)$$

where p_2 is the static gauge pressure of fluid staying outside of the nozzle. Then, the equation for the pressure loss coefficient there is

$$\xi_I = \left\{ \frac{3}{(3-\eta)\alpha} \right\}^2 \quad \dots\dots\dots (12)$$

Fig. 4 Velocity distributions in the clearance

In general, as a nozzle is surrounded by the atmosphere, it is rational to put p_2 zero. Hence, p becomes always a negative value and is equal to $-\xi(\rho V^2/2)$.

In the II-region, adding the pressure losses due to the Poiseuille flow to what developed by the region boundary α_p , the total pressure loss is obtained and the pressure loss coefficient for the II-region is given as follows

$$\left. \begin{aligned} \xi_{II} &= -\frac{p}{(\rho/2)V^2} = \frac{54}{35\alpha^2} + \frac{24}{R} \log_e \frac{1}{\alpha} + f_{II(R)} \\ f_{II(R)} &= \frac{99}{140\alpha_p^2} - \frac{24}{R} \log_e \frac{1}{\alpha_p} \end{aligned} \right\} \dots\dots\dots (13)$$

This equation has a similar form to Eq. (30) in the 1st Report but α_p and α are fractions of unity; $f_{II(R)}$ is a function of R only and becomes zero at $R=4.32$. In other words, when $R>4.32$, the actual pressure loss becomes less than what obtained under the assumption that a Poiseuille flow was existing all along. The reason for this phenomenon is that the rate of increase of η is relatively slow with the inward flow.

For the III-region, the pressure loss due to the Bernoulli's law is added to what obtained by Eq. (13) for $\alpha=\alpha_p'$. The equation is

$$\left. \begin{aligned} \xi_{III} &= -\frac{p}{(\rho/2)V^2} = \left\{ \frac{3}{(3-\eta)\alpha} \right\}^2 + f_{III(R)} \\ f_{III(R)} &= \frac{24}{R} \log_e \frac{\alpha_p}{\alpha_p'} - \frac{99}{140} \left(\frac{1}{\alpha_p'^2} - \frac{1}{\alpha_p^2} \right) \end{aligned} \right\} \dots\dots\dots (14)$$

The first term of the right side of Eq. (14) is the same as Eq. (12), and the second term $f_{III(R)}$ is the value which should be added for a Poiseuille flow

Fig. 5 Pressure distributions

appearing on the way. The latter term is the function of R only and becomes zero when $R=5.42$. In practice, when R is small, α_p' becomes also small, hence the III-region is scarcely realized.

The pressure distribution with R as a parameter is shown in Fig. 5. The pressure decreases only towards the center and at the boundaries of each region discontinuities of the pressure gradient are seen on the curves for the reason mentioned in Sec. 2.2.

3. Discussion of result

The above results are obtained by solving Eq. (1) with the initial condition, $\eta=0$ at $\alpha=1$. However, other initial conditions, such as $\eta=\eta_0$ at $\alpha=1$ or $\eta=0$ at $\alpha=\alpha_0$, are conceivably possible. Therefore, calculating the integration constant with these conditions and putting it into Eq. (4)', we have

$$\left. \begin{aligned} X &= \frac{1}{b} \left[\left\{ c\alpha \frac{1+m \tan c(\alpha_0-\alpha)}{m-\tan c(\alpha_0-\alpha)} - 1 \right\}^{-1} + \frac{3}{c^2\alpha^2} \right]^{-1} \\ m &= c\alpha_0 \left[1 + \left(\frac{1}{bX_0} - \frac{3}{c^2\alpha_0^2} \right)^{-1} \right]^{-1} \end{aligned} \right\} \dots\dots\dots (15)$$

where X_0 is the value of X corresponding to η_0 and is obtained by Eq. (2); η_0 and α_0 may take arbitrary values but restricted as $0 \leq \eta_0 \leq 1$ and $\alpha_0 \leq 1$. For example, Fig. 6 shows the calculated results with $R=18.9$. In spite of the difference in conditions, η converges to zero at the center of a nozzle, and therefore, the effect of the initial conditions diminishes where α is small. When a Poiseuille flow appears, the curves of η are the same in all cases after α_p' . These phenomena are remarkably different from the case of the outward flow. The values of ξ , of course, depend not only upon η_0 and α_0 but also upon R and the radius of the outlet $\alpha_1 (=r_1/r_2)$. The deviations of ξ from ξ_0 which

Fig. 6 Effects of initial conditions: $R=18.9$

obtained in the case of $\alpha_0=1$ and $\eta_0=0$ are shown in Fig. 7. With a large flanged nozzle which has a small value of α_1 , the effect of the entrance condition is less than that with a small flanged one. This phenomenon is the same as in the outward flow, and may be predicted by the fact mentioned above. Similarly to the outward flow, when R is small, α_0 scarcely becomes large, so that the influence of the initial condition on ξ is negligibly small.

To check the effect of the approximation in Eq. (17) in the 1st Report, strict solutions are obtained in the same manner as the 1st Report and are compared with the approximate solutions for two different R . In the outward flow, fairly close consistence was gained between two solutions, but in this case, appreciable discrepancies are seen between the two values when R is large, as shown in Fig. 8. The reason for this is that, in this case, as η is limited in the range of small value, the positive and negative deviations of the approximation for Y cannot cancel out each other. However, the difference of η is less than 0.02, so that the approximate equation (17) in the 1st Report is proved to be applicable in practice.

Now the pressure losses of actual nozzles shall be calculated. To compare the characteristics of the inward flow with that of the outward flow, the pressure loss coefficient ξ and R must be expressed on the basis of the inner radius of the nozzle. Fig. 9 shows the ξ - R curves for nozzle 4/3 and nozzle 12/3. Suffix of a nozzle means that a numerator and a denominator indicate the values of the outer and the inner diameters in millimeters respectively. Therefore, nozzle 12/3 has a large flange than nozzle 4/3. With the same condition, i.e. with the same

nozzle geometries, clearance and flow rate, the pressure losses in the case of the inward flow are always greater than that of the outward flow. The reason is that the velocity head of the flow which is spent at the outlet is greater in the case of the inward flow than in that of the outward flow. In the case of the outward flow, when R is small, i.e. the effect of the viscosity of fluid is large, the pressure loss becomes larger with the large flanged nozzle than with the small flanged nozzle. But when R is large, this relation is reversed due to the diffuser effect. Meanwhile, in the case of the inward flow, as the diffuser effect does not exist, the pressure loss of the small flanged nozzle is always smaller than that of the large flanged nozzle, but its difference becomes less as R becomes large.

4. Conclusion

The pressure-flow characteristics of the inward flow of a radial-flow nozzle is studied in consideration of the inlet length in this report. The equation of

Fig. 8 Comparison of strict and approximate solutions

Fig. 7 Influence of initial condition on ξ

Fig. 9 Losses in actual nozzle

the boundary layer is solved with the assumptions mentioned in the 1st Report, and the change of the thickness of the boundary layer, the velocity and the pressure distribution, the pressure losses, etc. are determined theoretically. As a result, the following points are clarified.

(1) $R = Vh^2/(r_2\nu)$ must be employed as a parameter corresponding to the Reynolds number in this case, too. Then, the velocity distribution at any radial station and the pressure distribution of any nozzle can be determined by R only.

(2) Three regions of the flow are established by the combination of α and R , and the pressure loss must be evaluated by one of different formulas depending on the regions.

(3) The thickness of the boundary layer is not increasing only but decreases from a certain position and converges to zero at the center of a

nozzle.

(4) When R is less than 5.42, the Poiseuille flow may appear on the way.

(5) The velocity of the flow at the central core increases only and the pressure decreases only along the flow direction.

(6) Deviations of the values of ξ due to the entrance condition are less than that when fluid flows outwards, specially, its influence can be neglected with the large flanged nozzle.

(7) The pressure loss of any nozzle with the inward flow is always larger than with the outward flow.

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531.717.082.36:532.525

Studies on the Characteristics of Radial-Flow Nozzles*

(3rd Report, Experimental Results)

By Tatsuya HAGIWARA**

This paper concerns the experimental results of the outward and inward flows in radial-flow nozzles. With an air source of 500 mmAq gauge pressure, the overall pressure losses and pressure distributions are obtained for various flow rates. It follows that the results of the theoretical analysis obtained in the 1st and 2nd Reports are applicable to the practical problems with reasonable accuracy in spite of simple assumptions. However, when the flow is outward and the inner corner of a nozzle is sharp, a severe contraction will occur and experimental results will not be consistent with theoretical values. Since the clearances of the nozzles dealt with are extremely narrow, the velocity distribution in the clearance cannot be obtained by the experiment.

1. Introduction

Theoretical analyses of a radial-flow nozzle were carried out concerning the outward flow in the 1st Report⁽¹⁾ and the inward flow in the 2nd Report⁽²⁾. The theoretical equations for evaluating the velocity and the pressure distribution were deduced by simple assumptions. Main ones among them are such as assumptions of incompressibility of fluid and the velocity distribution which insists that the velocity in a boundary layer is expressed by a quadratic

equation of the distance from the nozzle wall. The former seems to materialize rationally because of the premise that the pressure of an air source used is less than 500 mmAq gauge. However, as to the latter, it is necessary to prove whether such a simple assumption can be applied in practice.

This report concerns experiments in which the pressure losses in various nozzles and the pressures along walls of these nozzles are measured when air flows outwards and inwards. The experimental data are compared with the theoretical values to investigate the adaptability of the preceding analyses. The velocity distribution in the clearance of a nozzle cannot be measured, because the nozzles in the test

* Received 21th June, 1961.

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