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Statical Deflection of a Rhomboidal Plate with Clamped Edges Subjected to Uniformly Distributed Pressure*

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The problem of the statical deflection of a rhomboidal plate with clamped edges subjected to uniformly distributed pressure, is studied in this paper on the basis of the energy method, which was once used by one of the authors in the eigenvalue problems⁽¹⁾. As numerical examples, the deflection and the bending moments at the center of the plate, and the maximum bending moments along the edges are calculated for three oblique angles — 75°, 60° and 45°.

1. Introduction

The problem of the statical deflection of a rhomboidal plate with clamped edges is important in practice, since such plate is used widely as a fundamental element of many kinds of structures. In recent years, R. Heinen⁽²⁾ treated this problem by means of the conformal mapping method, but numerical calculations were carried out only for one oblique angle 60°, but more data are required for structural designing.

In this report, the statical deflection of a rhomboidal plate with clamped edges subjected to uniformly distributed pressure is studied on the basis of the energy method, which was once used by one of the authors in the eigenvalue problems⁽¹⁾. Numerical calculations are carried out for three oblique angles — 75°, 60° and 45°, and the graphs are obtained which give the deflection, the bending moments at the center of the plate and the maximum bending moments along the edges for any oblique angle.

2. Solution

Let the length of the equal edges and the oblique angle of the plate be $2a$ and α . The skew coordinates $O-XY$ and the orthogonal coordinates $O-X'Y'$ are taken as shown in Fig. 1. Also non-dimensional variables $x=X/a$ and $y=Y/a$ are used. The displacement of a rhomboidal plate is assumed as follows :

$$w = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_{mn} \cos m\pi x \cos n\pi y \\ + \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} b_{mn} \cos m\pi x \sin n\pi y \\ + \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} c_{mn} \sin m\pi x \cos n\pi y \\ + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} d_{mn} \sin m\pi x \sin n\pi y \dots\dots\dots (1)$$

Fig. 1

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Since the boundary conditions are

$$\left. \begin{aligned} x = \pm 1 : w = 0, \quad \partial w / \partial x = 0 \\ x = \pm 1 : w = 0, \quad \partial w / \partial y = 0 \end{aligned} \right\} \dots\dots\dots (2)$$

the following relations should be satisfied :

$$\sum_{m=0}^{\infty} (-1)^{m+n} \cdot a_{mn} = 0 \quad (n=0, 1, 2, \dots) \dots (3 \cdot a)$$

$$\sum_{n=0}^{\infty} (-1)^{m+n} \cdot a_{mn} = 0 \quad (m=0, 1, 2, \dots) \dots (3 \cdot b)$$

$$\sum_{m=0}^{\infty} (-1)^{m+n} \cdot b_{mn} = 0 \quad (n=1, 2, 3, \dots) \dots\dots (3 \cdot c)$$

$$\sum_{n=1}^{\infty} (-1)^{m+n} \cdot n b_{mn} = 0 \quad (m=0, 1, 2, \dots) \dots (3 \cdot d)$$

$$\sum_{m=1}^{\infty} (-1)^{m+n} \cdot m c_{mn} = 0 \quad (n=0, 1, 2, \dots) \dots (3 \cdot e)$$

$$\sum_{n=0}^{\infty} (-1)^{m+n} \cdot c_{mn} = 0 \quad (m=1, 2, 3, \dots) \dots\dots (3 \cdot f)$$

$$\sum_{m=1}^{\infty} (-1)^{m+n} \cdot m d_{mn} = 0 \quad (n=1, 2, 3, \dots) \dots (3 \cdot g)$$

$$\sum_{n=1}^{\infty} (-1)^{m+n} \cdot n d_{mn} = 0 \quad (m=1, 2, 3, \dots) \dots (3 \cdot h)$$

The elastic energy is given by

$$V = \frac{D}{2} \int \int \left(\frac{\partial^2 w}{\partial X'^2} + \frac{\partial^2 w}{\partial Y'^2} \right)^2 dX' dY' \dots\dots (4)$$

where D is the flexural rigidity of the plate. Transforming Eq. (4) by the relations $X' = X + Y \cos \alpha$, $Y' = Y \sin \alpha$; $X = ax$, $Y = ay$, we obtain

$$V = \frac{D}{2a^2 \sin^3 \alpha} \int_{-1}^1 \int_{-1}^1 \left(\frac{\partial^2 w}{\partial x^2} - 2 \cos \alpha \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2} \right)^2 dx dy \dots\dots\dots (5)$$

Substitution of Eq. (1) into Eq. (5) yields

$$\begin{aligned} V = & \frac{2\pi^4 D}{a^2 \sin^2 \alpha} \left[\frac{1}{2} \sum_{m=1}^{\infty} m^4 (a_{m0}^2 + c_{m0}^2) + \frac{1}{2} \sum_{n=1}^{\infty} n^4 (a_{0n}^2 + b_{0n}^2) \right. \\ & + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\frac{1}{4} \{ (m^2 + n^2)^2 + 4m^2 n^2 \cos^2 \alpha \} (a_{mn}^2 + b_{mn}^2 + c_{mn}^2 + d_{mn}^2) \right. \\ & \left. \left. + 2 \cos \alpha \cdot mn (m^2 + n^2) (a_{mn} d_{mn} - b_{mn} c_{mn}) \right] \right] \dots\dots\dots (6) \end{aligned}$$

Here the theorem of the stationary total energy is applied, thereby Eqs. (3·a) to (3·h) are considered the constraining conditions. Eq. (6) and Eqs. (3·a) to (3·h) show that the two groups of the coefficients (a_{mn} , d_{mn}) and (b_{mn} , c_{mn}) are separated. Apparently the group (a_{mn} , d_{mn}) belongs to the present problem. Moreover, according to its symmetricity

$$a_{mn} = a_{nm}, \quad d_{mn} = d_{nm} \dots\dots\dots (7)$$

Therefore Eqs. (1) and (6) become as follows :

$$\begin{aligned} w = & \sum_{m=0}^{\infty} a_{mm} \cos m\pi x \cos m\pi y + \sum_{m=1}^{\infty} \sum_{n=0}^{m-1} a_{mn} (\cos m\pi x \cos n\pi y + \cos n\pi x \cos m\pi y) \\ & + \sum_{m=1}^{\infty} d_{mm} \sin m\pi x \sin m\pi y + \sum_{m=2}^{\infty} \sum_{n=1}^{m-1} d_{mn} (\sin m\pi x \sin n\pi y + \sin n\pi x \sin m\pi y) \dots\dots\dots (8) \end{aligned}$$

$$\begin{aligned} V = & \frac{2\pi^4 D}{a^2 \sin^3 \alpha} \left[\sum_{m=1}^{\infty} m^4 a_{m0}^2 + \sum_{m=1}^{\infty} \{ m^4 (1 + \cos^2 \alpha) (a_{mm}^2 + d_{mm}^2) + 4m^4 \cos \alpha \cdot a_{mm} d_{mm} \} \right. \\ & \left. + \sum_{m=2}^{\infty} \sum_{n=1}^{m-1} \left[\frac{1}{2} \{ (m^2 + n^2)^2 + 4m^2 n^2 \cos^2 \alpha \} (a_{mn}^2 + d_{mn}^2) + 4mn \cos \alpha (m^2 + n^2) a_{mn} d_{mn} \right] \right] \dots\dots\dots (9) \end{aligned}$$

Also from Eqs. (7), Eqs. (3·b) and (3·h) are same as Eqs. (3·a) and (3·g), respectively.

Now the method of undetermined coefficients in the calculus of variations (Lagrangian multiplier method) is applied, thereby $u_n^{(a)}$ and $u_n^{(d)}$ are defined as the undetermined coefficients for Eqs. (3·a) and (3·g), respectively, and $Q_{mn}^{(a)}$, $Q_{mn}^{(d)}$ as the generalized components of external forces. Then the theorem of the stationary total energy gives

$$\begin{aligned} \delta \left\{ V - \sum_{m=0}^{\infty} Q_{mm}^{(a)} a_{mm} - \sum_{m=1}^{\infty} \sum_{n=0}^{m-1} Q_{mn}^{(a)} a_{mn} - \sum_{m=1}^{\infty} Q_{mm}^{(d)} d_{mm} - \sum_{m=2}^{\infty} \sum_{n=1}^{m-1} Q_{mn}^{(d)} d_{mn} \right. \\ \left. - \sum_{n=0}^{\infty} u_n^{(a)} \sum_{m=0}^{\infty} (-1)^{m+n} a_{mn} - \sum_{n=1}^{\infty} u_n^{(d)} \sum_{m=1}^{\infty} (-1)^{m+n} m d_{mn} \right\} = 0 \dots\dots\dots (10) \end{aligned}$$

Since V is the elastic energy and the terms of $Q_{mn}^{(a)}$, $Q_{mn}^{(d)}$ mean the works done by external forces, the last two terms in Eq. (10) must be those of the works done by the supporting forces and the bending moments along the edges. Now let the bending moment along the edge $y=1$ be given by

$$(IM_{Y'})_{y=1} = \sum_{m=1}^{\infty} K_m \sin m\pi x \dots\dots\dots (11)$$

In this report the bending moment which compresses the upper side of the plate is taken as positive. The inclination along the edge $y=1$ is

$$\left(\frac{\partial w}{\partial Y'}\right)_{y=1} = \frac{1}{a \sin \alpha} \left(\frac{\partial w}{\partial y}\right)_{y=1}$$

which yields by Eq. (1)

$$\left(\frac{\partial w}{\partial Y'}\right)_{y=1} = \frac{\pi}{a \sin \alpha} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^n n d_{mn} \sin m\pi x$$

Therefore the work done by the edge bending moment is

$$\begin{aligned} - \int_{-a}^a (M_{Y'})_{y=1} \left(\frac{\partial w}{\partial Y'}\right)_{y=1} dX &= \frac{-\pi}{\sin \alpha} \int_{-1}^1 \left(\sum_{m=1}^{\infty} K_m \sin m\pi x\right) \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^n n d_{mn} \sin m\pi x dx \\ &= \frac{\pi}{\sin \alpha} \sum_{m=1}^{\infty} K_m \sum_{n=1}^{\infty} (-1)^{n+1} n d_{mn} \end{aligned}$$

Since the same work is done on each edge, four times the work above is equal to the seventh term in Eq. (10).

$$\frac{4\pi}{\sin \alpha} \sum_{m=1}^{\infty} K_m \sum_{n=1}^{\infty} (-1)^{n+1} n d_{mn} = \sum_{m=1}^{\infty} u_m^{(d)} \sum_{n=1}^{\infty} (-1)^{m+n} n d_{mn}$$

Hence

$$K_n = \frac{(-1)^{n+1} \sin \alpha}{4\pi} u_n^{(d)} \quad \dots\dots\dots (12)$$

The edge bending moment given by Eq. (11) is of asymmetric distribution. On the other side the symmetric distribution is obtained from Eq. (24). The sum of both distributions gives the edge bending moment.

Now returning to Eq. (10), and using a_{mn}' , d_{mn}' instead of a_{mn} , d_{mn}

$$a_{mn}'/a_{mn} = d_{mn}'/d_{mn} = 2\pi^4 D/a^2 \sin^3 \alpha \quad \dots\dots\dots (13)$$

we obtain

$$m=n=0; \quad 0 = Q_{00}^{(a)} + u_0^{(a)} \quad \dots\dots\dots (14)$$

$$m \neq n=0; \quad 2m^4 a_{m0}' = Q_{m0}^{(a)} + (-1)^m u_0^{(a)} + (-1)^m u_m^{(a)} \quad \dots\dots\dots (15)$$

$$m=n \neq 0; \quad 2m^4 (1 + \cos^2 \alpha) a_{mn}' + 4m^4 \cos \alpha \cdot d_{mn}' = Q_{mn}^{(a)} + u_m^{(a)} \quad \dots\dots\dots (16)$$

$$2m^4 (1 + \cos^2 \alpha) d_{mn}' + 4m^4 \cos \alpha \cdot a_{mn}' = Q_{mn}^{(d)} + m u_m^{(d)} \quad \dots\dots\dots (17)$$

$$\begin{aligned} m \neq n, m \neq 0, n \neq 0; \\ \{ (m^2 + n^2)^2 + 4m^2 n^2 \cos^2 \alpha \} a_{mn} + 4mn \cos \alpha (m^2 + n^2) d_{mn} \\ = Q_{mn}^{(a)} + (-1)^{m+n} \cdot u_m^{(a)} + (-1)^{m+n} \cdot u_n^{(a)} \quad \dots\dots\dots (18) \end{aligned}$$

$$\begin{aligned} \{ (m^2 + n^2)^2 + 4m^2 n^2 \cos^2 \alpha \} d_{mn} + 4mn \cos \alpha (m^2 + n^2) a_{mn} \\ = Q_{mn}^{(d)} + (-1)^{m+n} \cdot m u_m^{(d)} + (-1)^{m+n} \cdot n u_n^{(d)} \quad \dots\dots\dots (19) \end{aligned}$$

Since the external force is the uniformly distributed pressure in this report, $Q_{mn}^{(a)}$ and $Q_{mn}^{(d)}$ are given by

$$\left. \begin{aligned} Q_{00}^{(a)} &= A p_0 \\ Q_{mn}^{(a)} &= 0 \quad (\text{except } m=n=0) \\ Q_{mn}^{(d)} &= 0 \end{aligned} \right\} \quad \dots\dots\dots (20)$$

where A is the total area of the plate.

In order to solve the problem, by expressing a_{mn}' and d_{mn}' with $u_n^{(a)}$, $u_n^{(d)}$ and p_0 by Eqs. (14) to (20), and by substituting these into Eqs. (3·a) and (3·g), we obtain the simultaneous linear equations about $u_n^{(a)}$ and $u_n^{(d)}$. By solving these equations, a_{mn}' and d_{mn}' can be evaluated, and the problem is solved.

The maximum deflection which occurs at the center of the plate, is given from Eq. (8) as

$$w_{\max} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_{mn} \quad \dots\dots\dots (21)$$

Next the orthogonal coordinates $O-X_0$, Y_0 are taken as shown in Fig. 2. The bending moments at the center point corresponding to those new coordinates are

$$\left. \begin{aligned} M_{X_0} &= -D \left(\frac{\partial^2 w}{\partial X_0^2} + \nu \frac{\partial^2 w}{\partial Y_0^2} \right)_{\substack{X_0=0 \\ Y_0=0}} \\ M_{Y_0} &= -D \left(\frac{\partial^2 w}{\partial Y_0^2} + \nu \frac{\partial^2 w}{\partial X_0^2} \right)_{\substack{X_0=0 \\ Y_0=0}} \end{aligned} \right\} \quad \dots\dots\dots (22)$$

where ν is the Poisson's ratio of the material. Using the transforming

Fig. 2

relations

$$X_0 = (X+Y) \cos \frac{\alpha}{2}, \quad Y_0 = (Y-X) \sin \frac{\alpha}{2}; \quad X = ax, \quad Y = ay$$

we obtain

$$\left. \begin{aligned} \frac{\partial^2 w}{\partial X_0^2} &= \frac{1}{4a^2 \cos^2 \frac{\alpha}{2}} \left(\frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{\partial^2 w}{\partial Y_0^2} &= \frac{1}{4a^2 \sin^2 \frac{\alpha}{2}} \left(\frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2} \right) \end{aligned} \right\}$$

From Eq. (1)

$$\left. \begin{aligned} \left(\frac{\partial^2 w}{\partial x^2} \right)_{x=y=0} &= -\pi^2 \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} m^2 a_{mn} \\ \left(\frac{\partial^2 w}{\partial y^2} \right)_{x=y=0} &= -\pi^2 \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} n^2 a_{mn} \\ \left(\frac{\partial^2 w}{\partial x \partial y} \right)_{x=y=0} &= \pi^2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m n d_{mn} \end{aligned} \right\}$$

Substitution of these relations into Eq. (22) yields

$$\left. \begin{aligned} M_{X_0} &= \frac{\pi^2 D}{4a^2} \left\{ \left(\frac{1}{\cos^2 \frac{\alpha}{2}} + \frac{\nu}{\sin^2 \frac{\alpha}{2}} \right) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (m^2 + n^2) a_{mn} - 2 \left(\frac{1}{\cos^2 \frac{\alpha}{2}} - \frac{\nu}{\sin^2 \frac{\alpha}{2}} \right) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m n d_{mn} \right\} \\ M_{Y_0} &= \frac{\pi^2 D}{4a^2} \left\{ \left(\frac{\nu}{\cos^2 \frac{\alpha}{2}} + \frac{1}{\sin^2 \frac{\alpha}{2}} \right) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (m^2 + n^2) a_{mn} - 2 \left(\frac{\nu}{\cos^2 \frac{\alpha}{2}} - \frac{1}{\sin^2 \frac{\alpha}{2}} \right) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m n d_{mn} \right\} \end{aligned} \right\} \dots (23)$$

Next, the symmetric distribution of the edge bending moment is considered. Along the edge $y=1$, it is given by

$$(\text{II } M_{Y'})_{y=1} = -D \left\{ \left(\frac{\partial^2 w}{\partial Y'^2} \right)_{y=1} + \nu \left(\frac{\partial^2 w}{\partial X'^2} \right)_{y=1} \right\} = -D \left(\frac{\partial^2 w}{\partial Y'^2} \right)_{y=1}$$

By the transformation of the coordinates

$$\frac{\partial^2 w}{\partial Y'^2} = \frac{1}{a^2 \tan^2 \alpha} \frac{\partial^2 w}{\partial x^2} - \frac{2}{a^2 \sin \alpha \tan \alpha} \frac{\partial^2 w}{\partial x \partial y} + \frac{1}{a^2 \sin^2 \alpha} \frac{\partial^2 w}{\partial y^2}$$

From Eq. (1)

$$\left. \begin{aligned} \left(\frac{\partial^2 w}{\partial x^2} \right)_{y=1} &= -\pi^2 \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} (-1)^n m^2 a_{mn} \cos m\pi x \\ \left(\frac{\partial^2 w}{\partial y^2} \right)_{y=1} &= -\pi^2 \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (-1)^n n^2 a_{mn} \cos m\pi x \\ \left(\frac{\partial^2 w}{\partial x \partial y} \right)_{y=1} &= \pi^2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^n m n d_{mn} \cos m\pi x \end{aligned} \right\}$$

Therefore

$$\begin{aligned} (\text{II } M_{Y'})_{y=1} &= \frac{\pi^2 D}{a^2} \left\{ \frac{1}{\tan^2 \alpha} \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} (-1)^n m^2 a_{mn} \cos m\pi x + \frac{2}{\sin \alpha \tan \alpha} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (-1)^n m n d_{mn} \cos m\pi x \right. \\ &\quad \left. + \frac{1}{\sin^2 \alpha} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (-1)^n n^2 a_{mn} \cos m\pi x \right\} \dots (24) \end{aligned}$$

The total distribution of the edge moment is given by

$$(M_{Y'})_{y=1} = (\text{I } M_{Y'})_{y=1} + (\text{II } M_{Y'})_{y=1}$$

3. Numerical calculations

Numerical calculations are carried out for three oblique angles — 75° , 60° and 45° . Their outline is given in the following for $\alpha=60^\circ$ as an example. For the first approximation, only $u_0^{(a)}$, $u_1^{(a)}$ and $u_1^{(d)}$ are considered, and $u_2^{(a)}$, $u_2^{(d)}$... are all taken as zero. From Eqs. (14) and (20)

$$u_0^{(a)} + A p_0 = 0 \dots (25)$$

Expressing a_{m0}' with $u_0^{(a)}$ and $u_1^{(a)}$ by Eq. (15) and substituting these into Eq. (3·a), we obtain the result as follows:

$$\begin{aligned} +a_{00}' &= +a_{00}' \\ -a_{10}' &= +0.500\,000\,0\,u_0^{(a)} + 0.500\,000\,0\,u_1^{(a)} \\ +a_{20}' &= +0.031\,250\,0\,u_0^{(a)} \\ -a_{30}' &= +0.006\,172\,8\,u_0^{(a)} \\ &\vdots \\ -a_{19,0}' &= +0.000\,003\,837\,u_0^{(a)} \\ +a_{20,0}' &= +0.000\,003\,125\,u_0^{(a)} \\ &\vdots \\ a_{00}' + 0.541\,17\,u_0^{(a)} + 0.500\,00\,u_1^{(a)} &= 0 \end{aligned} \quad S = +0.000\,037\,5\,u_0^{(a)} \quad (26)$$

S is the estimated sum of $\sum_{m=19}^{\infty} a_{m,0}'$, and it is obtained by assuming the form of convergence of the series as $S = \alpha + \beta/(\xi + \gamma)$, where α , β and γ are constants, and S , ξ are the sum and the number of the terms, respectively, therefore

$$S = \lim_{\xi \rightarrow \infty} \eta = a_{19,0}' \left(\frac{2}{(a_{19,0}'/a_{20,0}') - 1} + 1 \right)$$

In this report the series which converges monotonously are all calculated in this method.

In the same way as above, we form the equations for $\sum_{m=0}^{\infty} (-1)^{m+1} a_{m1} = 0$ in Eq. (3·a) and for $\sum_{m=1}^{\infty} (-1)^{m+1} m d_{m1} = 0$ in Eq. (3·g), thus

$$0.500\,00 u_0^{(a)} + 1.697\,81 u_1^{(a)} - 1.015\,61 u_1^{(d)} = 0 \quad (27)$$

$$1.782\,56 u_1^{(d)} - 1.015\,60 u_1^{(a)} = 0 \quad (28)$$

By solving Eqs. (25) to (28), we obtain

$$\begin{aligned} u_0^{(a)} &= -A p_0, \quad u_1^{(a)} = 0.446\,76 A p_0, \\ u_1^{(d)} &= 0.254\,54 A p_0, \quad a_{00}' = 0.317\,79 A p_0 \end{aligned}$$

Therefore all a_{mn}' are evaluated, and Eq. (21) gives the maximum deflection as

$$w_{\max I} = 0.012\,43 a^4 p_0 / D$$

For the second approximation $u_0^{(a)}$, $u_1^{(a)}$, $u_1^{(d)}$, $u_2^{(a)}$, $u_2^{(d)}$ are considered. By solving the simultaneous equations with those five known values, we obtain

$$\begin{aligned} u_0 &= -A p_0, \quad u_1^{(a)} = 0.443\,88 A p_0, \\ u_1^{(d)} &= 0.259\,42 A p_0, \quad u_2^{(a)} = 0.169\,01 A p_0, \\ u_2^{(d)} &= 0.028\,231 A p_0, \quad a_{00}' = 0.313\,94 A p_0 \end{aligned}$$

The maximum deflection is

Table 1 Maximum deflections

α		75°	60°	45°
$\frac{10^2 w_{\max} D}{a^4 p_0}$	1st approx.	1.798	1.243	0.625
	2nd approx.	1.795	1.232	0.606
	3rd approx.	1.795	1.231	0.604

Table 2 Bending moments at the center point

α	75°	60°	45°
$10^2 M_{Y0}/a^2 p_0$	9.02 (9.04)	7.91 (7.91)	5.77 (6.32)
$10^2 M_{X0}/a^2 p_0$	7.96 (7.99)	6.18 (6.19)	3.91 (4.01)

(3rd approx. outside and 2nd approx. inside the parentheses)

$$w_{\max II} = 0.012\,32 a^4 p_0 / D$$

For the third approximation $u_3^{(a)}$ and $u_3^{(d)}$ are added to the five coefficients above, and the results are:

$$\begin{aligned} u_0^{(a)} &= -A p_0, \quad u_1^{(a)} = 0.443\,74 A p_0, \\ u_1^{(d)} &= 0.260\,14 A p_0, \\ u_2^{(a)} &= 0.165\,88 A p_0, \quad u_2^{(d)} = 0.030\,360 A p_0, \\ u_3^{(a)} &= 0.103\,68 A p_0, \\ u_3^{(d)} &= 0.005\,785\,5 A p_0, \quad a_{00}' = 0.313\,49 A p_0 \\ w_{\max III} &= 0.012\,31 a^4 p_0 / D \end{aligned}$$

Same calculations are carried out for $\alpha = 75^\circ$ and $\alpha = 45^\circ$. These results are shown in Table 1.

Next the bending moments at the center of the plate are calculated by Eq. (23). Since d_{mn}' is necessary in addition to a_{mn}' , we calculate it, and obtain the sums of $\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (m^2 + n^2) a_{mn}'$ and $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m n d_{mn}'$. In these calculations, we must sum up the series which converges alternatively. Twenty terms are evaluated, and the sum of the series after the nineteenth term is estimated with the mean value of the last two terms.

Calculations are carried out for the second and the third approximation, thereby the Poisson's ratio ν is taken as 0.3. The results are shown in Table 2.

Next the bending moments along the edge are calculated. Double series in Eq. (24) are summed up. Thereby the series which converges periodically must be treated, and its sum is estimated by evaluating twenty terms and by using the mean value during the last one period.

Table 3 Distribution of the edge bending moments $(M_Y)_{y=1/a^2 p_0}$

$x \backslash \alpha$	75°	60°	45°
3/4	-0.020(-0.018)	-0.007(-0.004)	+0.003(-0.004)
1/2	-0.078(-0.080)	-0.039(-0.038)	-0.019(-0.014)
1/4	-0.140(-0.141)	-0.080(-0.087)	-0.037(-0.041)
0	-0.184(-0.181)	-0.129(-0.129)	-0.069(-0.064)
-1/4	-0.192(-0.195)	-0.156(-0.161)	-0.098(-0.095)
-1/2	-0.165(-0.164)	-0.160(-0.162)	-0.114(-0.120)
-3/4	-0.084(-0.084)	-0.111(-0.105)	-0.102(-0.099)

(3rd approx. outside and 2nd approx. inside the parentheses)

We calculate the bending moments for the points $y=1$; $x=0, \pm 1/4, \pm 1/2, \pm 3/4$. Their symmetric components are obtained by Eq. (24), and the asymmetric ones by Eqs. (11) and (12). The results are shown in Fig. 3 and Table 3. The maximum edge

Fig. 3

Table 4 Maximum bending moments along the edge

α	75°	60°	45°
$10(M_Y')_{y=1}/a^2 p_0$ max	1.93	1.61	1.15

Fig. 4

Fig. 5

Table 5 Maximum deflection, bending moments at the center point, maximum bending moments along the edge

α	0	45°	60°	75°	90°	105°	120°	135°	180°
$10^4 w_{max} D / l^4 p_0$	26.0	11.00	7.69	5.11	3.16	1.77	0.85	0.32	0
$10^2 M_{X0} / l^2 p_0$	1.25	1.67	1.55	1.34	1.15	0.90	0.66	0.42	0
$10^2 M_{Y0} / l^2 p_0$	4.17	2.46	1.98	1.52	1.15	0.79	0.52	0.29	0
$10^2 (M_Y')_{y=1} / l^2 p_0$ max	8.33	4.80	4.03	3.25	2.57	1.92	1.34	0.82	0

moment obtained from Fig. 3 is shown in Table 4.

4. Considerations

In order to relate the results of numerical calculations above to the data of a square plate ($\alpha=90^\circ$) and those of an infinite strip ($\alpha=0^\circ$), we keep the length l of the diagonal line to be constant, and change the oblique angle α from 0° to 180° , as shown in Fig. 4. Using the relation $l=4a \sin(\alpha/2)$ and rewriting the data in Tables 1, 2 and 4, the results shown in Table 5 and Figs. 5 to 7 are obtained. By reading the data for $\alpha=30^\circ$ and $\alpha=150^\circ$ in Figs. 5 to 7, and by using the data in Tables 1, 2 and 4, the graphs in Figs. 8 to 10 are obtained.

By comparing the numerical results of each approximation, it is found that the accuracy of the third approximation is almost enough for practical purposes, and that the accuracy becomes worse as the oblique angle α is farther apart from 90° .

R. Heinen⁽²⁾ gives the maximum deflection for $\alpha=60^\circ$ as

$$w_{max} = 0.012 4a^4 p_0 / D$$

Fig. 6

Fig. 7

Fig. 8

This agrees nearly with the data in Table 1.

5. Conclusions

In the problem of the statical deflection of the rhomboidal plate with the clamped edges subjected to uniformly distributed pressure, the maximum deflection, the bending moments at the center of the plate and the maximum bending moment along the edge are evaluated for three oblique angles—75°, 60° and 45°, and the graphs showing these data for any oblique angle are obtained. Figs. 8 to 10 are the conclusions of this report.

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Fig. 9

Fig. 10

earnest help in numerical calculations.

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