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A Study on the Exhaust Muffler of Internal Combustion Engine*

By Motokazu FUKUDA**

In this paper, it was studied on the exhaust muffler of an internal combustion engine.

Generally, the theoretical analysis of the exhaust muffler of an engine contains various difficult factors. For example, almost all the recent reports are based on the presupposition that the tail pipe of a muffler has an infinite length.

In this report, therefore, a theoretical formula of the attenuation was studied from the point of view that the tail pipe has a finite length. The formula was established by the application of the theory of electric transmission line, and the values calculated by the above formula have been found practically in good agreement with those obtained from the experiments.

Moreover, many experiments were executed as to the influence of varying the various factors of the cavities and the tail pipes.

1. Introduction

The exhaust noise of an internal combustion engine is potentially the greatest noise source of the engine. It seems that the theoretical analysis on an exhaust muffler is very difficult.

Almost all the recent reports⁽¹⁾ on the theory of a muffler, including those presented by Mr. D. D. Davis, are based on the presupposition that the tail pipe of a muffler has an infinite length.

In these experiments, therefore, two straight long pipes connected to the cavity of a muffler are terminated at both ends by nonreflective material, and so generally an engine is not used throughout the experiments.

From the practical point of view, however, it appears that such presuppositions as follows must be supported by the study of an exhaust muffler.

In the first place, it must be presupposed that the tail pipe has a finite length.

In the second place, we must realize that the exhaust noise is caused by the sound wave radiated from the ejecting port of exhaust gas. For the above reason, the radiation power of the exhaust noise must be considered.

In the third place, it must be noticed that the exhaust gas temperature has some gradient in an exhaust system.

From the above point of view therefore the

author has tried a theoretical investigation of an exhaust muffler in this report, and compared the calculated values with the measured values in each case.

Further, the author has studied in this report the effect of the cavity and the tail pipe of a muffler.

2. Theoretical consideration

The attenuation effect of an exhaust muffler is determined by the difference between the exhaust noise of an engine without muffler and that of an engine with muffler. And, the impedance in the cylinder is so high that it appears to be a constant velocity sound source.

In a muffler as shown in Fig. 1, therefore, the attenuation effect can be obtained by:

$$Att(db) = 20 \log_{10} \left| \frac{p_{2(n+1)}}{p_1} \right| \dots \dots \dots (1)$$

where $p_{2(n+1)}$: sound pressure of exhaust noise without muffler

p_1 : sound pressure of exhaust noise with muffler

At a distant point from noise source, the right side of Eq. (1) is approximately given by⁽²⁾:

$$20 \log_{10} \left| \frac{p_{2(n+1)}}{p_1} \right| = 20 \log_{10} \left| \frac{v_{2(n+1)}}{v_1} \right| \dots \dots \dots (2)$$

where $v_{2(n+1)}$: mass velocity of the noise radiating from the open port of the cylinder.

v_1 : mass velocity of the noise radiating from the open end of the tail pipe

On the other hand, the particle velocity of the

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** Professor, Faculty of Engineering, Yamaguchi University, Ube.

exhaust noise of an engine with muffler $v_{2(n+1)}$ and that of an engine with muffler $v_{(2n+1)i}$ will be approximately equal (Because, we suppose that the cylinder is constant velocity sound source).

then

$$\left. \begin{aligned} v_{2(n+1)} &= v_{(2n+1)i} \\ \rho_{2(n+1)} &= \rho_{2n+1} \\ S_{2(n+1)} &= S_{2n+1} \end{aligned} \right\} \dots\dots\dots (3)$$

where $v_{2(n+1)}$: particle velocity at the exhaust port of the cylinder

$v_{(2n+1)i}$: particle velocity at the inlet part of the muffler

$\rho_{2(n+1)}$: average density of gas at the exhaust port of the cylinder

ρ_{2n+1} : average density of gas at the inlet pipe of the muffler

$S_{2(n+1)}$: cross sectional area of the exhaust port of the cylinder

S_{2n+1} : cross sectional area of the inlet pipe of the muffler

Thus, the mass velocity ratio $v_{2(n+1)}/v_1$ is approximately:

$$|v_{2(n+1)}/v_1| = \frac{\rho_{2n+1} S_{2n+1}}{\rho_1 S_1} \left| \frac{v_{(2n+1)i}}{v_1} \right| \dots\dots\dots (4)$$

where ρ_1 : average density of gas inside the tail pipe

v_1 : particle velocity of gas at the open end of the tail pipe

S_1 : cross sectional area of the tail pipe

so that the attenuation effect of a muffler can be determined by the following equation.

$$Att (db) = 20 \log_{10} \frac{\rho_{2n+1} S_{2n+1} |v_{(2n+1)i}|}{\rho_1 S_1 |v_1|} \dots\dots (5)$$

In the above Eq. (5), the right side may be determined by calculation as shown in Eq. (16).

By the way, it is known in electrical engineering that the voltage E and the current I at the distance x from the receiving end of an electric-transmission line are shown as follows:

$$\left. \begin{aligned} E_x &= E_1 \cosh \gamma x + I_1 Z_0 \sinh \gamma x \\ I_x &= E_1 \frac{1}{Z_0} \sinh \gamma x + I_1 \cosh \gamma x \end{aligned} \right\} \dots\dots\dots (6)$$

where E_1 : voltage at the receiving end of an electric-transmission line

I_1 : current at the receiving end of an electric-transmission line

Z_0 : characteristic impedance in the line

γ : propagation constant in the line

Applying the above Eq. (6) to an acoustic duct, the following equations may be obtained:

$$\left. \begin{aligned} F_x &= F_1 \cos kx + v_1 j \rho c S \sin kx \\ v_x &= F_1 \frac{j}{\rho c S} \sin kx + v_1 \cos kx \end{aligned} \right\} \dots\dots\dots (7)$$

where F_x : force at a distance x from the receiving end of an acoustic duct

v_x : particle velocity at a distance x from the receiving end of an acoustic duct

F_1 : force at the receiving end of an acoustic duct

v_1 : particle velocity at the receiving end of an acoustic duct

S : cross sectional area of an acoustic duct

c : average sound velocity in an acoustic duct

The above equations are based merely on the consideration of plane waves in the duct.

In Eq. (7) naturally the sound pressure p_x at a distance x from the receiving end of the duct is given by:

$$p_x = F_x / S \dots\dots\dots (8)$$

The above relations on an acoustic duct can be applied to the calculation of the attenuation effect of a muffler.

If the length of a tail pipe is l_1 , the ratio of the inlet particle velocity v_{1i} to the outlet particle velocity v_1 may be given by:

$$\left. \begin{aligned} v_{1i}/v_1 &= \frac{jZ_1}{\rho_1 c_1 S_1} \sin k_1 l_1 + \cos k_1 l_1 \\ Z_1 &= F_1/v_1 \end{aligned} \right\} \dots\dots\dots (9)$$

where l_1 : length of tail pipe

c_1 : average sound velocity in the tail pipe

S_1 : cross sectional area of the tail pipe

ρ_1 : average density of gas inside the tail pipe

k_1 : $2\pi f/c_1$

Fig. 1 Muffler formed of n -pieces of cavities

The impedance Z_{1i} in the inlet part of the tail pipe is obtained using the above Eq. (9) as follows:

$$Z_{1i} = \frac{Z_1 \cos k_1 l_1 + j \rho_1 c_1 S_1 \sin k_1 l_1}{\frac{j Z_1 \sin k_1 l_1}{\rho_1 c_1 S_1} + \cos k_1 l_1} \quad (10)$$

If the average density of gas in the cavity is ρ_2 , the mean sound velocity c_2 and the cross-sectional area S_2 , the mass flow of gas will be continuous at the junction of the cavity and tail pipe.

$$\left. \begin{aligned} F_{1i}/F_2 &= S_1/S_2 \\ v_2/v_{1i} &= \rho_1 S_1/\rho_2 S_2 \end{aligned} \right\} \quad (11)$$

where F_{1i} : force at inlet part of tail pipe

F_1 : force at outlet part of tail pipe

On the other hand, the load impedance at the end of the cavity may be obtained from the following equation.

$$Z_2 = F_2/v_2 = (\rho_2/\rho_1)(S_2/S_1)^2 Z_{1i} \quad (12)$$

Substituting Eq. (10) in Eq. (12), Z_2 is:

$$Z_2 = (\rho_2/\rho_1)(S_2/S_1)^2 \frac{Z_1 \cos k_1 l_1 + j \rho_1 c_1 S_1 \sin k_1 l_1}{\frac{j Z_1 \sin k_1 l_1}{\rho_1 c_1 S_1} + \cos k_1 l_1} \quad (13)$$

Similarly, the load impedance Z_3 of the connecting pipe l_3 is:

$$Z_3 = (\rho_3/\rho_2)(S_3/S_2)^2 \frac{Z_2 \cos k_2 l_2 + j \rho_2 c_2 S_2 \sin k_2 l_2}{\frac{j Z_2 \sin k_2 l_2}{\rho_2 c_2 S_2} + \cos k_2 l_2} \quad (14)$$

In the same way, the ratio of the particle-velocity $v_{(2n+1)i}$ at the inlet part of the exhaust pipe to the particle-velocity v_1 at the outlet part of the tail pipe can be obtained as follows:

$$\begin{aligned} v_{(2n+1)i}/v_1 &= \left(\frac{v_{2n+1}}{v_{2ni}} \frac{v_{2n}}{v_{(2n-1)i}} \dots \frac{v_3}{v_{2i}} \frac{v_2}{v_{1i}} \right) \left(\frac{v_{(2n+1)i}}{v_{2n+1}} \frac{v_{2ni}}{v_{2n}} \dots \frac{v_{2i}}{v_2} \frac{v_{1i}}{v_1} \right) \\ &= \frac{\rho_1}{\rho_{2n+1}} \frac{S_1}{S_{2n+1}} \prod_{\nu=1}^{2n+1} \left\{ \frac{j Z_\nu}{\rho_\nu c_\nu S_\nu} \sin k_\nu l_\nu + \cos k_\nu l_\nu \right\} \quad (15) \end{aligned}$$

From the Eqs. (5) and (15), the attenuation of the exhaust muffler may be determined by the following equation:

$$Att(db) = 20 \log_{10} \left| \prod_{\nu=1}^{2n+1} \left\{ \frac{j Z_\nu}{\rho_\nu c_\nu S_\nu} \sin k_\nu l_\nu + \cos k_\nu l_\nu \right\} \right| \quad (16)$$

where

$$Z_\nu = \frac{\rho_\nu}{\rho_{\nu-1}} \left(\frac{S_\nu}{S_{\nu-1}} \right)^2 \frac{Z_{\nu-1} \cos(k_{\nu-1} l_{\nu-1}) + j \rho_{\nu-1} c_{\nu-1} S_{\nu-1} \sin(k_{\nu-1} l_{\nu-1})}{\frac{j Z_{\nu-1} \sin(k_{\nu-1} l_{\nu-1})}{\rho_{\nu-1} c_{\nu-1} S_{\nu-1}} + \cos(k_{\nu-1} l_{\nu-1})} \quad (17)$$

The above Eq. (16) is a general expression for the attenuation effect of a muffler.

The result obtained by calculation from the above expression ought to be in accord with the value measured by using acoustical instruments.

In the above study the sound velocity and density are assumed to be constant in each cavity or duct.

Suppose $Z_1 \neq 0$, then, the attenuation of a muffler having a cavity may be given by:

$$\begin{aligned} Att(db) &= 20 \log_{10} \left\{ \cos k_1 l_1 \cos k_2 l_2 \cos k_3 l_3 - \frac{c_1 S_2}{c_2 S_1} \sin k_1 l_1 \sin k_2 l_2 \cos k_3 l_3 \right. \\ &\quad \left. - \frac{c_1 S_3}{c_3 S_1} \sin k_1 l_1 \cos k_2 l_2 \sin k_3 l_3 - \frac{c_2 S_3}{c_3 S_2} \cos k_1 l_1 \sin k_2 l_2 \sin k_3 l_3 \right\} \quad (18) \end{aligned}$$

These results may be obtained otherwise from the four terminal constants as follows. Now, consider plane waves in an acoustic duct, then the four terminal constants of the duct may be:

$$\begin{bmatrix} p_I \\ v_I \end{bmatrix} = \begin{bmatrix} \cosh \theta & Z_{01} \sinh \theta \\ \frac{\sinh \theta}{Z_{01}} & \cosh \theta \end{bmatrix} \begin{bmatrix} p_{II} \\ v_{II} \end{bmatrix} \quad (19)$$

where

$$\left. \begin{aligned} \theta &= j\omega \sqrt{\frac{\rho}{k}} l \\ Z_{01} &= \frac{1}{S} \sqrt{\frac{k}{\rho}} \end{aligned} \right\} \quad (20)$$

v =mass velocity, p =sound pressure, and k =volume elasticity of gas

Next, consider a series connection of n cavity elements as shown in Fig. 1, and the four terminal constants may be:

$$\begin{aligned} \begin{bmatrix} p_{(2n+1)i} \\ v_{(2n+1)i} \end{bmatrix} &= \begin{bmatrix} \cosh \theta_{2n+1} & Z_{0(2n+1)} \sinh \theta_{2n+1} \\ \frac{\sinh \theta_{2n+1}}{Z_{0(2n+1)}} & \cosh \theta_{2n+1} \end{bmatrix} \begin{bmatrix} \cosh \theta_{2n} & Z_{02n} \sinh \theta_{2n} \\ \frac{\sinh \theta_{2n}}{Z_{02n}} & \cosh \theta_{2n} \end{bmatrix} \cdots \begin{bmatrix} \cosh \theta_1 & Z_{01} \sinh \theta_1 \\ \frac{\sinh \theta_1}{Z_{01}} & \cosh \theta_1 \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix} \\ &= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix} \dots\dots\dots (21) \end{aligned}$$

If we suppose that the impedance in the cylinder is so high that the cylinder is a constant velocity sound source, then the attenuation will be :

$$Att(\text{db}) = 20 \log_{10} \left| \frac{v_{2n+1}}{v_1} \right| = 20 \log_{10} |D| \dots\dots\dots (22)$$

In case of $n=1$,

$$Att(\text{db}) = 20 \log_{10} \left| \cosh \theta_1 \cosh \theta_2 \cosh \theta_3 + \frac{Z_{01}}{Z_{02}} \sinh \theta_1 \sinh \theta_2 \cosh \theta_3 + \frac{Z_{01}}{Z_{03}} \sinh \theta_1 \cosh \theta_2 \sinh \theta_3 + \frac{Z_{02}}{Z_{03}} \cosh \theta_1 \sinh \theta_2 \sinh \theta_3 \right| \dots\dots\dots (23)$$

where $Z_{0n}'/Z_{0n} = C_n' S_n / C_n S_n'$, $\theta_n = jk_n l_n$

The above Eq. (23) is approximately the same as the Eq. (18). Thus, the four terminal constants may be in use.

Next, in Fig. 2, the attenuation between the inlet part of a cavity and the outlet part of the tail pipe will be obtained from Eq. (18) as follows:

$$Att(\text{db}) = 20 \log_{10} \left| \cos k_1 l_1 \cos k_2 l_2 - \frac{c_1 S_2}{c_2 S_1} \sin k_1 l_1 \sin k_2 l_2 \right| \dots\dots\dots (24)$$

Suppose a muffler, which has small dimension in comparison with the wave length (i.e. $l < \lambda/2$), then the attenuation between I and II is:

$$Att(\text{db}) = 20 \log_{10} \left| 1 - \left(\frac{2\pi}{\lambda} \right)^2 \left(\frac{S_2}{S_1} \right) l_1 l_2 \right| \dots\dots\dots (25)$$

Replacing the above Eq. (20) with the common expressions of the electric circuit, we have:

$$Att(\text{db}) = 20 \log_{10} |1 - \omega^2 m C| \dots\dots\dots (26)$$

Fig. 2 Muffler which has one cavity

Fig. 3 Scheme of apparatus for the measurement

where

$$\left. \begin{aligned} m &= \rho S_1 l_1 \\ C &= V / \rho c^2 S_1^2 = S_2 l_2 / \rho c^2 S_1^2 \end{aligned} \right\} \dots\dots\dots (27)$$

3. Experimental apparatus and method

The schematic arrangement of the experimental apparatus is shown in Fig. 3.

A small 4-cycle gasoline engine has been used throughout the experiment. Its main dimensions are as follows:

Type	Mitsubishi G-3-L (1 cyl., air cooled)
Cylinder bore and stroke	62φ×52.5 mm
Normal output	3 PS/3 600 rpm
Max. torque	0.74 m·kg/3 000 rpm
Compression ratio	5.5
Diameter of exhaust pipe	21 mm

The engine was cooled by a 3 kW fan and put into a sound-proof box, the inside and outside walls of which had been covered with sound absorptive material.

Moreover the engine was connected with a generator the characteristics of which already had been known, and the electric load was kept in the same driving condition using a wattmeter.

Engine tests were run at 2 600 rpm, operating under about half-throttle load.

To measure the sound pressure of the exhaust noise, a condenser microphone (Brüel, 4131) was employed. The exhaust noise was analyzed by a third-octave band spectrometer (Brüel, 2111) and attenuation curves derived.

In the case of the experiment the object of which was comparing the theoretical values on a muffler with the experimental values, engine tests and speaker tests were performed.

In speaker tests, a beat frequency oscillator

(Brüel, 1014) was used to send single frequency sounds through a muffler, and was recorded by a high speed level recorder (Brüel, 2305).

In engine tests, the exhaust noise was recorded on a magnetic tape recorder (Sony, KP-2), and thereafter, the recorded noise was analyzed in line spectrum of 5-cps pass band by means of a heterodyne analyzer.

4. Comparison of theoretical results with experimental results

An example of the comparison of theoretical results with experimental results is shown in Fig. 6.

The exhaust noise was picked up at a distance of 20 cm rear from the outlet end of the tail pipe and at a height of 20 cm above the pipe center line.

Fig. 4 shows the 5-cps pass band spectrum in the case of the exhaust system completely removed. In this spectrum are recognized the resonance waves

of the firing frequency 21.7 c/sec.

Fig. 5 shows the spectrum of the exhaust system as shown in Fig. 6. In this spectrum are recognized the resonance waves depending on the cavity and the tail pipe. The ordinates of the plots in Fig. 6 represent the differences between Fig. 4 and Fig. 5.

The temperatures at the center points of the cavity and ducts were 300°C in the exhaust pipe l_3 , 240°C in the cavity and 220°C in the tail pipe l_1 respectively. Then, sound velocities in those parts will be given as: $c_3 = 480$ m/sec, $c_2 = 450$ m/sec and $c_1 = 445$ m/sec.

Substituting these values in Eq. (18) we have the theoretical values of the transmission loss of this muffler as shown by full lines in Fig. 6. It will be noticed that the curves are approximately in accord the experimental values at lower frequency, but not at higher frequency. This discord at high frequency may be due to the assumption of plane wave.

Fig. 4 Exhaust noise from a 3 PS-engine without muffler (5-cps passband)

Fig. 5 Exhaust noise from a 3 PS-engine with a muffler (5-cps passband)

Now, here will be seen the numerical calculation of the cut-off frequency and other resonance frequencies. At first, the cut-off frequency of this muffler has the expression:

$$f_v = \frac{c}{2\pi} \sqrt{\frac{S_1}{V l_1}} \dots\dots\dots (28)$$

Substituting $\pi(2.1)^2/4 \text{ cm}^2$ in S_1 , $(36+2 \times 0.6R)$

Frequency c/sec	Fig. 4 db	Fig. 5 db	Diff. db	Frequency c/sec	Fig. 4 db	Fig. 5 db	Diff. db
50	86	92	-6	450	80	(60)	(20)
70	82	84	-2	500	80	69	11
90	88	79	9	550	73	67	6
110	94	84	10	600	64	72	-8
130	95	81	14	650	68	68	0
150	100	81	19	700	68	(60)	(8)
200	89	74	15	800	71	63	8
320	74	66	8	900	67	66	1
360	70	66	4	1 000	(63)	61	2
400	73	60	13	1 100	64	68	-4

Fig. 6 Theoretical attenuation values and experimental values

Fig. 7 Influence of varying diameter of a cavity

cm in l_1 and 450 m/sec in c , f_v is

$$f_v = 60 \text{ c/sec}$$

Next, the resonance frequency f_{l_3} caused by the exhaust pipe may be calculated as follows:

$$f_{l_3} = \frac{c_3}{4l_3} = \frac{48\,000}{4(36+0.85R)} = 325 \text{ c/sec} \dots\dots\dots (29)$$

where $R = 2.1/2 \text{ cm}$

The resonance frequency f_{l_1} caused by the tail pipe is

$$f_{l_1} = \frac{c_1}{2l_1} = \frac{44\,500}{2(36+2 \times 0.6R)} = 600 \text{ c/sec} \dots\dots\dots (30)$$

The resonance frequency f_{l_2} caused by the length of the cavity is

$$f_{l_2} = \frac{c_2}{2l_2} = 1\,400 \text{ c/sec} \dots\dots\dots (31)$$

Those values calculated are practically in good agreement with those obtained from the experiments.

5. Influence of the diameter of cavity

As already considered in previous chapters, the attenuation of a muffler is influenced by the ratio between cross-sectional areas of pipe and cavity.

Therefore, the author has investigated the frequency characteristics of attenuation of a muffler by changing the diameter of a cavity into 41, 81, 116, 155 mmφ, keeping the length equal to 160 mm.

In each case, the exhaust pipe is 21φ × 60 mm and the pipe is 21φ × 35 mm.

In Fig. 7, full lines are calculated values of transmission loss and dots are experimented values by a speaker. It is recognized that they are approximately in good agreement, and the following points were ascertained:

- (1) To enlarge the diameter of a cavity makes the maximum attenuation value of each diagram remarkably large.
- (2) The larger the diameter of a cavity becomes, the farther the cut-off frequency of a muffler moves to low frequency.

6. Influence of the length of cavity

Fig. 8 shows the values obtained by speaker experiments and the values by calculation, in the case of keeping the inner diameter of a cavity at 81 mmφ and changing its length into 42, 68, 160, 278, 325 and 580 mm.

Both values are comparatively in good agreement, and the following points were recognized:

(1) The change of the length of a cavity does not cause the change of the values of maximum attenuation. Namely, the attenuation does not always increase even if the length increases.

(2) The longer a cavity becomes, the farther the cut-off frequency moves to low frequency.

(3) As the length of a cavity increases, the numbers of passing bands increase.

Fig. 9 shows the results of engine-tests. In view

Fig. 8 Influence of varying length of a cavity

Fig. 9 Exhaust noise

of these facts, it is recognized even in the engine-tests that the rate of the increase of attenuation effect becomes remarkably small in the case of the changing of length compared with the changing of diameter.

7. Influence of the length-diameter ratio of cavity

The experiments were made keeping the volume of the cavity constant at 1 400 cc, and changing its diameter and length as shown in Table 1. These experiments were made by means of a speaker, and the values calculated by Eq. (24) are shown in Fig. 10.

Table 1 Dimensions of cavities used in the experiments of varying the length-diameter ratio

Vol. cc	Dia. D mm	Length L mm	Ratio L/D	Sectional area of cavity Sectional area of pipe
1 400	52	660	12.7	6.1
	68	387	5.7	10.5
	80	272	3.4	14.9
	108	162	1.5	23.2
	152	76	0.5	53.3
1 000	52	470	9.0	6.1
	68	277	4.1	10.5
	80	194	2.4	14.9
	120	96	0.8	32.7
400	39	334	8.6	3.5
	52	190	3.6	6.1
	68	110	1.6	10.5
	80	80	1.0	14.5

Fig. 10 Influence of varying the length-diameter ratio of a cavity (vol. 1 400 cc)

In this case, the following points are remarkable:

(1) Generally, the larger the diameter of cavity of a constant volume, the higher the maximum value of the attenuation of a muffler becomes.

(2) If the volume of the cavity is constant, the cut-off frequency is nearly constant in spite of the change of the length-diameter ratio of the cavity.

Fig. 11 and Fig. 12 show the results of the engine test. In these figures, it will be noticed that the noise level increases when the cavity has an extremely flat shape. The author intends to make this clear in his future study.

8. Influence of interior partitions of a cavity

As above, the performance of an exhaust muffler without any interior partitions, may be approximately deduced from theoretical calculations. However, from the point of practical design, the values of attenuation obtainable from the calculations are almost insufficient. Therefore, it is necessary to set in the cavity interior partitions and the like.

The theoretical values calculated about the muffler shown in Fig. 13 in which a partition plate is set or not, are shown in Fig. 14. In this calculation, the dimensions of the tail pipe were neglected to

know the characteristics of the cavity only.

In Fig. 14, it is remarkable that the partitions deteriorate the attenuation effect in low frequency region and improve it in high frequency region.

Fig. 15 shows the results of an engine-test with regard to the effect of the number of interior partitions set in a cavity of constant volume.

Fig. 16 shows the influence of the shape of the partition-plate. In this figure, (1) corresponds to a plate having a connecting tube of $21\text{ mm}\phi \times 20\text{ mm}$ and (2) a multiply-drilled plate having 29 holes of $4\text{ mm}\phi$ (Both plates have the same hole area 3.5 cm^2 in total).

From Fig. 16, it will be considered in the case of a multiply-drilled plate that the divided parts couple closely one another.

9. Influence of the length of tail pipe

9.1 Experiments by an internal combustion engine

In this experiments, a 3-PS gasoline engine was run at 2 600 rpm, and a cavity of $105\phi \times 160\text{ mm}$ and a tail pipe of $21\text{ mm}\phi$ were used as shown in Fig. 17, where the length of the pipe was varied in 20 cases ranging between 0 and 143 cm.

Fig. 11 Noise level of exhaust noise with muffler

Fig. 13 Mufflers for the experiments of interior partitions 1 400 cc cavity)

Fig. 14 Calculated attenuation characteristics

(a) $l_1 = 66$ mm (b) $l_2 = 500$ mmFig. 12 Analysis of exhaust noise with muffler of various ratios ($V=1400$ cc)

(a)

(b)

(c)

- (1) Muffler not divided, (2) Muffler divided into half parts,
(3) Muffler divided into three parts

Fig. 15 Attenuation of exhaust noise with a divided muffler

Fig. 17 A muffler for the tests of tail pipe

- (1) A plate having $21\phi \times 21$ mm pipe, (2) A plate having 29 holes of 4 mm ϕ

Fig. 16 Exhaust noise with a partition plate

Fig. 18 Sound pressure level of exhaust noise depending on the length of a tail pipe

Fig. 18 shows the results of frequency analysis, and Table 2 shows the numerical values of the measured resonance frequency and thereafter calculated sound velocity c_1 .

The procedure as above may be a way to con-

firm the sound velocity in a tail pipe.

Next, substitution of $c_3 = 480$ m/sec, $c_2 = 450$ m/sec, $c_1 = 445$ m/sec and the corrected values of length of the tail pipe in the general expression (18), gives the theoretical attenuation effect as shown by full

Fig. 19 Attenuation effect depending on the length of a tail pipe (engine test)

Fig. 20 Exhaust noise depending on the length of a tail pipe

lines in Fig. 19.

These theoretical values well correspond to the measured values determined from Fig. 18. Besides, the values of noise level are shown in Fig. 20.

9.2 Experiments by a constant sound pressure source

Fig. 21 shows the results of measurement which was made by application of the constant sound pressure source in regard to the mufflers as above.

From this figure it will be known that the passing band moves and the attenuation changes continuously as the tail length changes.

In Fig. 21, f_{13} will be calculated as:

$$f_{13} = \frac{c_3}{2l_3} = \frac{34\,000}{2(36+2 \times 0.6R)} = 460 \text{ c/sec} \dots (32)$$

Hence, the passing band at 460 c/sec visible in the figure must be due to the internal resonance of exhaust pipe.

Likewise it will be noticed that the passing bands at 1750 c/sec for $l_1=8.5$ cm, 1450 c/sec for $l_1=10.5$ cm, 870 c/sec for $l_1=18.5$ cm, 660 c/sec for $l_1=24.5$ cm, 170 c/sec for $l_1=100$ cm, and 120 c/sec for $l_1=143$ cm

are caused by the internal resonance of tail pipe l_1 .

It should be noted that the calculated values of the frequency of these passing bands are in good agreement with the measured values.

The cut-off frequency of a low-pass filter, appears between 200 c/sec ($l_1=0$) and 50 c/sec ($l_1=143$ cm), and moves down to the lower frequency zone as the length of a tail pipe increases.

10. Influence of the diameter of tail pipe

It will be clearly known, from the equations referred to in the first chapter in this report, that the attenuation effect of a muffler decreases when a tail pipe has a larger diameter.

The author confirmed this from a series of tail pipes of 270 mm in length, and 39 mm, 27 mm and 16 mm in diameter.

Fig. 22 shows that the exhaust noise lessens considerably with the use of tail pipes of smaller diameter, and the cut-off frequency also moves down to the lower frequency zone.

Table 2 Calculated sound velocity in various tail pipes

Practical length of l_1 cm	3.6	6.6	14.6	36.0	72.0	100.0	143.0
Corrected length of l_1 cm	4.85	7.85	15.85	37.25	73.25	101.25	144.25
Measured resonance f_{11} c/sec	(4 600)	2 900	1 400	600	305	220	150
Calculated sound velocity m/sec	(446)	455	445	447	446	445	435

Fig. 21 Sound pressure level through the muffler shown in Fig. 17 (Tested by use of B & K, 1 014, 4 211 and 2 305)

11. Influence of the number of tail pipe

11.1 Tail pipes of an equal diameter

Fig. 23 shows the frequency spectrum of the exhaust noise of the engine with a muffler having a cavity of $105\phi \times 160$ mm and having tail pipes of $21\phi \times 270$ mm, number of which is 1, 2, and 3.

In this figure, it is shown that the attenuation effect of the muffler can lessen as the number of tail pipes increases.

11.2 Tail pipes of an equal total cross-sectional area

The comparison of the attenuation effect of the

number of tail pipes which have total cross sectional area of 6 cm^2 is made in Fig. 24. It will be known from this figure that the two curves have a corresponding nature in all zones except in the vicinity of 800 c/sec. This 800 c/sec may be calculated as follows:

$$f_{l_1} = \frac{44\,500}{2(27 + 2 \times 0.6R)} = 800 \text{ c/sec} \quad \dots\dots\dots (33)$$

Namely, this peak is due to the resonance of tail pipe and it will be deduced from the figure that the resonance will be stronger in larger pipes.

Fig. 22 Exhaust noise depending on the diameter of a tail pipe

Fig. 23 Exhaust noise depending on the number of tailpipes

Fig. 24 Effect of the number of tail pipes of an equal area in total (engine test)

Fig. 25 Influence of the number of ejecting holes of an equal diameter (engine test)

Fig. 26 Exhaust noise of an engine with a muffler having a total exhaust area 6 cm^2

12. Influence of the number of outlet-ports of cavity

Some commercial mufflers have many exhaust outlet ports, which are designed to eject the exhaust gas directly from the rear plate of the cavity without the use of tail pipes.

The attenuation effects, therefore, were tested as to the influence of the number of the outlet-ports of an equal diameter, and as to the influence of the number of the ports of an equal area in total. The results are shown in Fig. 25 and Fig. 26 respectively.

It is clearly noticed from the above experiments that the tail pipe gives a very distinct effect on the attenuation of a muffler.

13. Conclusion

In this report, the attenuation effect of the

exhaust muffler of an internal combustion engine was studied.

A theoretical formula of the attenuation was established by the application of the theory of an electric transmission line and it was ascertained by the experiments.

Moreover, many experiments were executed as to the influence of the various factors of the cavity and the tail pipe and like.

Reference

- (1) D.D. Davis, Jr., G.M. Stokes, D. Moore, and G.L. Stevens, Jr.: *NACA Rep.* 1192 (1954); Igarashi and Toyama: *Aeronautical Research Institute Univ. of Tokyo*, Rep. 339 (1958).
- (2) M. Fukuda: *Trans. Japan Soc. Mech. Engrs.*, Vol.26, No.168 (1960-8), 1168; Okuda: unpublished.