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Fundamental Frequencies of Simply Supported but Partially Clamped Square Plates*

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The present paper deals with the fundamental vibrations of simply supported but partially clamped square plates. The energy method is applied, which was proposed in the previous paper⁽¹⁾ for solving the general problems of statical deflections, stationary vibrations, bucklings, etc. of rectangular plates. Numerical calculations are carried out for fourteen cases, and five diagrams are obtained which give the fundamental frequencies of the square plates under typical boundary conditions.

1. Introduction

In the previous paper⁽¹⁾ the energy method for solving many problems of rectangular plates, the edges of which are simply supported but partially clamped, was investigated. That method is applied in the present paper to the problem of the free vibrations of square plates, and the diagrams are obtained which indicate the effects of the edge constraints upon the fundamental frequencies for the plates under typical boundary conditions.

2. Solution

The general method in the previous paper⁽¹⁾ is related briefly in the form suitable for the free vibration problem of the square plate. The orthogonal coordinates $O-X, Y$ are taken as shown in Fig. 1, and non-dimensional variables $x=X/a$ and $y=Y/a$ are used, where a is the length of the edge. The transverse displacement of the free vibration

is assumed by

$$W = w \cos \omega t \dots\dots\dots (1)$$

and

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin m\pi x \sin n\pi y \dots\dots\dots (2)$$

where ω is the circular frequency, and C_{mn} is the undetermined constant. Eq. (2) satisfies the conditions for the simply supported edge. In order to satisfy the conditions for the partially clamped edge, the distributions of the edge bending moments are assumed as follows:

Fig. 1

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$$\left. \begin{aligned} m_{y=0}(x) \cos \omega t &= \left(\sum_{m=1}^{m'} K_m \sin m\pi x \right) \cos \omega t \\ m_{y=1}(x) \cos \omega t &= \left(\sum_{m=1}^{\bar{m}'} K_{\bar{m}} \sin m\pi x \right) \cos \omega t \\ m_{x=0}(y) \cos \omega t &= \left(\sum_{n=1}^{n'} K_n \sin n\pi y \right) \cos \omega t \\ m_{x=1}(y) \cos \omega t &= \left(\sum_{n=1}^{\bar{n}'} K_{\bar{n}} \sin n\pi y \right) \cos \omega t \end{aligned} \right\} \dots (3)$$

Replacing these moment distributions by the equivalent oscillating loads distributed in the narrow strip regions along the edges, considering the forced vibration problem of the plate under the oscillating loads mentioned above, and applying the principle of the stationary total energy, the following relation is introduced⁽¹⁾:

$$\frac{1}{2} \{ (m^2 + n^2)^2 - \Omega^2 \} c_{mn} = \frac{\pi}{2} \{ nK_m + (-1)^{n+1} nK_{\bar{m}} + mK_n + (-1)^{m+1} mK_{\bar{n}} \} \dots (4)$$

where

$$\Omega^2 = a^4 \rho h \omega^2 / \pi^4 D, \quad c_{mn} = (\pi^4 D / 2a^2) C_{mn} \dots (5)$$

and ρ , h , D are the density, the thickness and the flexural rigidity of the plate, respectively.

Now the conditions of the edges are considered. If the supported parts on the edge $y=0$ are $x_j < x < x_j'$ ($j=1, 2, \dots, i$), then the condition that the edge bending moments should vanish on the supported parts is given by the following relation⁽¹⁾:

$$\left. \begin{aligned} \sum_{j=1}^i \int_{x_j}^{x_j'} \sum_{\mu=1}^{\mu'} K_{\mu} \sin \mu\pi x \sin m\pi x dx &= 0 \\ \sum_{j=1}^i \int_{x_j}^{x_j'} \sum_{\mu=1}^{\bar{\mu}'} K_{\bar{\mu}} \sin \mu\pi x \sin m\pi x dx &= 0 \\ \sum_{j=1}^i \int_{y_j}^{y_j'} \sum_{\nu=1}^{\nu'} K_{\nu} \sin \nu\pi y \sin n\pi y dy &= 0 \\ \sum_{j=1}^i \int_{y_j}^{y_j'} \sum_{\nu=1}^{\bar{\nu}'} K_{\bar{\nu}} \sin \nu\pi y \sin n\pi y dy &= 0 \end{aligned} \right\} \dots (6)$$

x_j , x_j' ; y_j , y_j' and i in each equation above must take the values of each edge. Next, the conditions that the inclination of the plate should vanish along the clamped parts on each edge are given by the following relations⁽¹⁾:

$$\left. \begin{aligned} \sum_{j=1}^i \int_{x_j}^{x_j'} \sum_{\mu=1}^{\mu'} L_{\mu} \sin \mu\pi x \sin m\pi x dx &= 0 \\ \sum_{j=1}^i \int_{x_j}^{x_j'} \sum_{\mu=1}^{\bar{\mu}'} L_{\bar{\mu}} \sin \mu\pi x \sin m\pi x dx &= 0 \\ \sum_{j=1}^i \int_{y_j}^{y_j'} \sum_{\nu=1}^{\nu'} L_{\nu} \sin \nu\pi y \sin n\pi y dy &= 0 \\ \sum_{j=1}^i \int_{y_j}^{y_j'} \sum_{\nu=1}^{\bar{\nu}'} L_{\bar{\nu}} \sin \nu\pi y \sin n\pi y dy &= 0 \end{aligned} \right\} \dots (7)$$

where

$$\left. \begin{aligned} L_m &= \sum_{n=1}^{\infty} n\pi C_{mn} \\ L_{\bar{m}} &= \sum_{n=1}^{\infty} (-1)^n n\pi C_{mn} \\ L_n &= \sum_{m=1}^{\infty} m\pi C_{mn} \\ L_{\bar{n}} &= \sum_{m=1}^{\infty} (-1)^m m\pi C_{mn} \end{aligned} \right\} \dots (8)$$

The free vibration problem can be solved from Eqs. (4) to (8) by the following processes. Firstly, the numbers of terms in the series in Eqs. (3), m' , $\bar{m}'$, n' , $\bar{n}'$ are determined appropriately, then the value of Ω^2 in Eq. (4) is assumed and c_{mn} 's are expressed with $K_m \sim K_{\bar{n}}$. Those c_{mn} 's are put into Eqs. (8) and $L_m \sim L_{\bar{n}}$ are expressed with $K_m \sim K_{\bar{n}}$. Substituting these $L_m \sim L_{\bar{n}}$ into Eqs. (7), the homogeneous linear simultaneous equations about $K_m \sim K_{\bar{n}}$ are obtained together with Eqs. (6). The lowest value of Ω among those which make the coefficient determinant of those simultaneous equations vanish, gives the fundamental frequency. This can be found by trial and error. In the numerical examples, the fundamental frequencies of some square plates under typical boundary conditions are calculated.

3. Numerical examples

(I) The case of the square plate which is simply supported but partially clamped along the length αa of the middle parts of two opposite edges (Fig. 2).

$K_n = K_{\bar{n}} = 0$, since the edges $x=0$ and $x=1$ are simply supported. Also by the symmetry of the problem, $K_m = K_{\bar{m}}$ and $L_m = -L_{\bar{m}}$, and all of m and n are odd numbers. Therefore Eqs. (6) and (7) are described as

$$\int_0^{(1-\alpha)/2} \sum_{\mu=1}^{\mu'} K_{\mu} \sin \mu\pi x \sin m\pi x dx = 0 \dots (9)$$

($m=1, 3, 5, \dots$)

$$\int_{(1-\alpha)/2}^{1/2} \sum_{\mu=1}^{\mu'} L_{\mu} \sin \mu\pi x \sin m\pi x dx = 0 \dots (10)$$

($m=1, 3, 5, \dots$)

Fig. 2

From Eq. (4)

$$\frac{1}{2} \{ (m^2 + n^2)^2 - Q^2 \} c_{mn} = n\pi K_m$$

Substituting this relation into Eqs. (8),

$$L_m = \frac{4a^2}{\pi^2 D} K_m \sum_{n=1,3,\dots}^{\infty} \frac{n^2}{(m^2 + n^2)^2 - Q^2} \dots (11)$$

The following mathematical formulae are used:

$$\left. \begin{aligned} \cot Z &= \frac{1}{Z} + 2Z \sum_{n=1}^{\infty} \frac{1}{Z^2 - n^2 \pi^2} \\ \frac{1}{\sin Z} &= \frac{1}{Z} + 2Z \sum_{n=1}^{\infty} \frac{(-1)^n}{Z^2 - n^2 \pi^2} \end{aligned} \right\} \dots (12)$$

Then Eq. (11) is transformed into

$$\left. \begin{aligned} Q > m^2: L_m &= \frac{a^2}{2\pi D} \frac{K_m}{Q} \{ \sqrt{Q + m^2} (\coth \pi \sqrt{Q + m^2} - \operatorname{cosech} \pi \sqrt{Q + m^2}) \\ &\quad - \sqrt{Q - m^2} (\cot \pi \sqrt{Q - m^2} - \operatorname{cosec} \pi \sqrt{Q - m^2}) \} \\ Q < m^2: L_m &= \frac{a^2}{2\pi D} \frac{K_m}{Q} \{ \sqrt{m^2 + Q} (\coth \pi \sqrt{m^2 + Q} - \operatorname{cosech} \pi \sqrt{m^2 + Q}) \\ &\quad - \sqrt{m^2 - Q} (\coth \pi \sqrt{m^2 - Q} - \operatorname{cosech} \pi \sqrt{m^2 - Q}) \} \end{aligned} \right\} \dots (13)$$

(i) $\alpha = 2/3$

As the first approximation, K_1, K_3, K_5 and K_7 are considered. Eq. (9) for $m=1$ and $m=3$ is written as

$$0.181\,172\,1K_1 + 0.433\,012\,7K_3 + 0.433\,012\,7K_5 + 0.216\,506\,4K_7 = 0$$

$$0.433\,012\,7K_1 + 1.047\,197\,6K_3 + 1.082\,531\,8K_5 + 0.606\,217\,8K_7 = 0$$

Next, putting $Q^2 = 8.6$ as a trial, expressing L_1, L_3, L_5, L_7 with K_1, K_3, K_5, K_7 by Eqs. (13) and substituting those into Eq. (10) for $m=1$ and $m=3$, then we obtain

$$-0.000\,637\,171\,0K_1 - 0.057\,587\,45K_3 - 0.034\,067\,76K_5 - 0.012\,151\,42K_7 = 0$$

$$0.000\,093\,197\,28K_1 + 0.278\,538\,9K_3 - 0.085\,169\,41K_5 - 0.034\,023\,99K_7 = 0$$

The value of coefficient determinant of the above homogeneous simultaneous equations with four unknowns is calculated, and we obtain ${}_I \Delta_{8.6} = 1.790\,9 \times 10^{-4}$. Next we assume $Q^2 = 8.5$ and obtain ${}_I \Delta_{8.5} = 0.288\,8 \times 10^{-4}$. Further we obtain ${}_I \Delta_{8.3} = -2.925\,6 \times 10^{-4}$ for $Q^2 = 8.3$. Therefore the value of Q^2 which makes ${}_I \Delta = 0$ is concluded to be $8.48 (\equiv Q_{I^2})$. This is the first approximation of the fundamental frequency of the square plate under consideration.

In order to obtain the second approximation, $K_1, K_3, K_5, K_7, K_9, K_{11}$ should be considered, and the coefficient determinants of six orders must be calculated. The results of calculations are ${}_II \Delta_{8.6} = -25.271 \times 10^{-8}$, ${}_II \Delta_{8.5} = -0.638 \times 10^{-8}$, ${}_II \Delta_{8.3} = 52.074 \times 10^{-8}$, therefore we obtain $Q_{II^2} = 8.50$ as the second approximation.

(ii) $\alpha = 1/2$

The processes of calculation are same as in the case of $\alpha = 2/3$. The results are

$${}_I \Delta_{8.3} = 1.374\,0 \times 10^{-3}, \quad {}_I \Delta_{8.0} = -2.076\,8 \times 10^{-3},$$

$${}_I \Delta_{7.8} = -4.672\,9 \times 10^{-3}$$

therefore

$$Q_{I^2} = 8.18$$

Also

$${}_II \Delta_{8.3} = -8.360\,5 \times 10^{-6}, \quad {}_II \Delta_{8.2} = 1.206\,6 \times 10^{-6},$$

$${}_II \Delta_{8.0} = 21.740 \times 10^{-6}$$

therefore

$$Q_{II^2} = 8.21$$

(iii) $\alpha = 1/3$

$${}_I \Delta_{7.8} = 1.458\,5 \times 10^{-3}, \quad {}_I \Delta_{7.5} = -0.352\,8 \times 10^{-3},$$

$${}_I \Delta_{7.3} = -1.741\,0 \times 10^{-3}$$

therefore

$$Q_{I^2} = 7.56$$

$${}_II \Delta_{7.8} = -4.061\,0 \times 10^{-6}, \quad {}_II \Delta_{7.5} = 0.895\,6 \times 10^{-6},$$

$${}_II \Delta_{7.3} = 4.704\,8 \times 10^{-6}$$

therefore

$$Q_{II^2} = 7.56$$

(II) The case of the square plate which is simply supported but partially clamped along the length αa from the two opposite ends of two opposite edges (Fig. 3).

Fig. 3

Since the edges $x=0$ and $x=1$ are simply supported, $K_n=K_{\bar{n}}=0$. Also by the symmetry of the problem, $K_m=K_{\bar{m}}$ and $L_m=-L_{\bar{m}}$, and n is odd but m is an integer. Therefore Eqs. (6) and (7) are written as

$$\int_0^1 \sum_{\mu=1}^{\mu'} K_{\mu} \sin \mu \pi x \sin m \pi x dx = 0 \quad \dots\dots\dots (14)$$

$$(m=1, 2, 3, \dots)$$

$$\int_0^1 \sum_{\mu=1}^{\mu'} L_{\mu} \sin \mu \pi x \sin m \pi x dx = 0 \quad \dots\dots\dots (15)$$

$$(m=1, 2, 3, \dots)$$

Eqs. (11) and (13) are satisfied in this case, too.

(i) $\alpha=2/3$

As the first approximation, K_1, K_2, K_3 and K_4 are considered. The results of calculations are

$$I A_{8,2} = 4.8214 \times 10^{-2}, \quad I A_{8,0} = 1.9558 \times 10^{-2},$$

$$I A_{7,7} = -2.8802 \times 10^{-2}$$

therefore

$$Q_I^2 = 7.88$$

As the second approximation, $K_1 \sim K_8$ are considered.

The results are

$$I A_{8,2} = 7.3982 \times 10^{-4}, \quad I A_{8,0} = 1.8618 \times 10^{-4},$$

$$I A_{7,7} = -7.4781 \times 10^{-4}$$

therefore

$$Q_{II}^2 = 7.94$$

(ii) $\alpha=1/2$

$$I A_{7,0} = 2.9336 \times 10^{-1}, \quad I A_{6,5} = -0.5416 \times 10^{-1},$$

$$I A_{6,2} = -3.0025 \times 10^{-1}$$

therefore

$$Q_I^2 = 6.58$$

$$I A_{7,0} = 1.2662 \times 10^{-2}, \quad I A_{6,5} = -0.7762 \times 10^{-2},$$

$$I A_{6,2} = -2.4311 \times 10^{-2}$$

therefore

$$Q_{II}^2 = 6.66$$

(iii) $\alpha=1/3$

$$I A_{5,5} = 7.8734 \times 10^{-2}, \quad I A_{5,0} = -1.4498 \times 10^{-2},$$

$$I A_{4,8} = -8.8375 \times 10^{-2}$$

therefore

$$Q_I^2 = 5.06$$

$$I A_{5,5} = -3.0745 \times 10^{-2}, \quad I A_{5,0} = 3.1467 \times 10^{-2},$$

$$I A_{4,8} = 8.0607 \times 10^{-2}$$

therefore

$$Q_{II}^2 = 5.18$$

Fig. 4

(III) The case of the square plate which is simply supported but partially clamped along the length αa of the middle parts of one edge (Fig. 4).

Since the edges $x=0$, $x=1$ and $y=1$ are simply supported, $K_n=K_{\bar{n}}=K_{\bar{m}}=0$. Also by the symmetry of the problem, m is odd, but n is an integer. Eqs. (9) and (10) in the case (I) are used in this case, too, but Eq. (11) must be replaced by

$$L_m = \frac{2a^2}{\pi^2 D} K_m \sum_{n=1,2,\dots}^{\infty} \frac{n^2}{(m^2+n^2)^2 - Q^2} \quad \dots\dots\dots (16)$$

Applying Eqs. (12), Eq. (16) becomes

$$\left. \begin{aligned} Q > m^2: \\ L_m &= \frac{a^2}{2\pi D} \frac{K_m}{Q} \left\{ \sqrt{Q+m^2} \coth \pi \sqrt{Q+m^2} \right. \\ &\quad \left. - \sqrt{Q-m^2} \cot \pi \sqrt{Q-m^2} \right\} \\ Q < m^2: \\ L_m &= \frac{a^2}{2\pi D} \frac{K_m}{Q} \left\{ \sqrt{m^2+Q} \coth \pi \sqrt{m^2+Q} \right. \\ &\quad \left. - \sqrt{m^2-Q} \coth \pi \sqrt{m^2-Q} \right\} \end{aligned} \right\} \quad \dots\dots\dots (17)$$

(i) $\alpha=2/3$

As the first approximation, K_1, K_3, K_5 and K_7 are considered. The results of numerical calculations are

$$I A_{5,5} = -9.1845 \times 10^{-2}, \quad I A_{5,7} = -0.0186 \times 10^{-2},$$

$$I A_{5,9} = 6.1485 \times 10^{-2}$$

therefore

$$Q_I^2 = 5.70$$

As the second approximation, K_1, K_3, K_5, K_7, K_9 and K_{11} are considered.

$$I A_{5,5} = 3.4535 \times 10^{-5}, \quad I A_{5,7} = 0.1447 \times 10^{-5},$$

$$I A_{5,9} = -2.4913 \times 10^{-5}$$

therefore

$$Q_{II}^2 = 5.71$$

(ii) $\alpha=1/2$

$$I A_{5,5} = -3.0940 \times 10^{-2}, \quad I A_{5,6} = -0.0998 \times 10^{-2},$$

$$I A_{5,7} = 2.1298 \times 10^{-2}$$

therefore

$$Q_I^2 = 5.60$$

$$I A_{5,5} = 7.4828 \times 10^{-4}, \quad I A_{5,6} = 1.4983 \times 10^{-4},$$

$$I A_{5,7} = -4.4061 \times 10^{-4}$$

therefore

$$Q_{II}^2 = 5.62$$

(iii) $\alpha=1/3$

$$I A_{5,3} = -2.0052 \times 10^{-2}, \quad I A_{5,4} = -1.1224 \times 10^{-2},$$

$$I A_{5,5} = 1.3611 \times 10^{-2}$$

therefore

$$Q_I^2 = 5.41$$

$$I A_{5,3} = 6.4362 \times 10^{-4}, \quad I A_{5,4} = 0.2298 \times 10^{-4},$$

$$I A_{5,5} = -0.6422 \times 10^{-4}$$

therefore

$$Q_{II}^2 = 5.42$$

(IV) The case of the square plate which is simply supported but partially clamped along the length αa from the one end of one edge (Fig. 5).

$K_n = K_{\bar{n}} = K_{\bar{m}} = 0$, since the edges $x=0$, $x=1$ and $y=1$ are simply supported; m and n are all integers. Eqs. (17) are used in this case, too.

(i) $\alpha = 2/3$

In the first approximation, K_1, K_2, K_3 and K_4 are considered.

$$I A_{5,3} = -0.9155, \quad I A_{5,5} = -0.1003,$$

$$I A_{5,7} = 0.4861$$

therefore

$$Q_I^2 = 5.52$$

In the second approximation, K_1, K_2, K_3, K_4, K_5 and K_6 are considered.

$$II A_{5,5} = -0.7739 \times 10^{-2}, \quad II A_{5,6} = 0.7541 \times 10^{-2},$$

$$II A_{5,7} = 2.1033 \times 10^{-2}$$

therefore

$$Q_{II}^2 = 5.55$$

(ii) $\alpha = 1/2$

$$I A_{5,0} = -1.0338, \quad I A_{5,1} = -0.1213,$$

$$I A_{5,2} = 0.9748$$

therefore

$$Q_I^2 = 5.11$$

$$II A_{5,0} = -3.0893 \times 10^{-1}, \quad II A_{5,1} = -0.8462 \times 10^{-1},$$

$$II A_{5,2} = -0.1419 \times 10^{-1}$$

therefore

$$Q_{II}^2 = 5.14$$

(iii) $\alpha = 1/3$

$$I A_{4,4} = -4.7813 \times 10^{-1}, \quad I A_{4,5} = -0.1641 \times 10^{-1},$$

$$I A_{4,6} = 2.9213 \times 10^{-1}$$

therefore

$$Q_I^2 = 4.51$$

$$II A_{4,4} = 1.1635, \quad II A_{4,5} = 0.3809, \quad II A_{4,6} = -0.1419$$

therefore

$$Q_{II}^2 = 4.57$$

(V) The case of the square plate which is simply supported but partially clamped along the length αa of the middle part of four edges (Fig. 6).

The calculations for the cases $\alpha = 1/2$ and $\alpha = 1/3$ were reported in the previous paper⁽¹⁾. Those results were as follows:

Fig. 5

Fig. 6

(i) $\alpha = 1/2$

$$Q_I^2 = 12.92, \quad Q_{II}^2 = 12.96$$

(ii) $\alpha = 1/3$

$$Q_I^2 = 11.65, \quad Q_{II}^2 = 11.76$$

The results of numerical calculations mentioned above are shown in Table 1 and in Fig. 7 to Fig. 11, where the second approximation results are plotted. Also the well known data for $\alpha = 0$ and $\alpha = 1$ are shown in the Table and the diagrams.

Table 1 The values of Q

α	0	1/3	1/2	2/3	1
I	2	2.75(2.75)	2.86(2.87)	2.91(2.92)	2.933
II	2	2.25(2.28)	2.57(2.58)	2.81(2.82)	2.933
III	2	2.33(2.33)	2.37(2.37)	2.39(2.39)	2.396
IV	2	2.12(2.14)	2.26(2.27)	2.35(2.36)	2.396
V	2	3.41(3.43)	3.59(3.60)	—	3.646

Note. First or second approximations are inside or outside the parentheses, respectively.

Fig. 7 For the case of Fig. 2

Fig. 8 For the case of Fig. 3

4. Experiments

In experiments mild steel plates with the edge length $a=165$ mm and the thickness $h=1.6$ mm

Fig. 9 For the case of Fig. 4

Fig. 10 For the case of Fig. 5

Fig. 11 For the case of Fig. 6

or $h=2.3$ mm were used (Fig. 13). They were forced to vibrate by an electric magnet and a low frequency oscillator as shown in Figs. 12 and 14, and the resonance frequencies were found out. They were almost in accordance with the numerical results as shown in Fig. 7 to Fig. 11, and the experiments proved the propriety of the method adopted in this paper.

5. Considerations

It is considered that the second approximation results in numerical calculations have sufficient accuracy for the practical purposes, because they are in accordance with the first approximation results within three effective figures. Also they are almost but not perfectly in agreement with the experimental data, since the realization of the ideal boundary

Fig. 12

Fig. 13

Fig. 14

Fig. 15

conditions of being fixed or simply supported is very difficult. Now it is checked whether the values of Ω in Fig. 2, Fig. 4 and Fig. 6 tend to the other value than $\Omega=2$ as the value of α tends to zero. The case of Fig. 2 is considered for example. As shown in Fig. 15, the forced concentrated oscillating load $P_0 \cos \omega t$ is assumed to act at two points $(x=1/2, y=\delta)$ and $(x=1/2, y=1-\delta)$. If the deflections at these two points remain to be zero and δ tends to zero, then the frequency equation for the case when α tends to zero will be obtained. Applying the general method in the previous paper⁽¹⁾ to the forced vibration problem mentioned above, thereby taking the forced term as

$$Q_{mn} = P_0 \sin \frac{m\pi}{2} \{ \sin n\pi\delta + \sin n\pi(1-\delta) \}$$

the undetermined coefficient in the equation of the displacement is given by

$$c_{mn} = \frac{Q_{mn}}{\{(m^2+n^2)^2 - \Omega^2\}/2} = \frac{(-1)^{(m-1)/2} 4n\pi\delta}{(m^2+n^2)^2 - \Omega^2}$$

Hence the displacements at the two points $(x=1/2, y=\delta)$ and $(x=1/2, y=1-\delta)$ are

$$\begin{aligned} w(1/2, \delta) &= w(1/2, 1-\delta) \\ &= \sum_{m=1,3,\dots}^{\infty} \sum_{n=1,3,\dots}^{\infty} C_{mn} \sin \frac{m\pi}{2} \sin n\pi\delta = 0 \end{aligned}$$

Therefore by letting δ tend to zero, we obtain as

the frequency equation

$$\sum_{m=1,3,\dots}^{\infty} \sum_{n=1,3,\dots}^{\infty} \frac{n^2}{(m^2+n^2)^2 - \Omega^2} = 0$$

But this double series does not converge for any finite value of Ω . In introducing the above equation, it was assumed that $(m^2+n^2)^2 \neq \Omega^2$. Therefore there can not be the frequencies of the free vibrations except $(m^2+n^2)^2 = \Omega^2$, which are nothing other than those of the simply supported square plate. Hence it is concluded that the value of the fundamental frequency of the plate shown in Fig. 2 tends to 2 as α tends to zero. For the plates shown in Figs. 4 and 6, the same results as above can be obtained.

6. Conclusions

The general method for the simply supported but partially clamped rectangular plates, proposed in the previous paper, was applied to the fundamental vibration problem of the square plate, and five diagrams were obtained which give the fundamental frequencies for the square plates under five boundary conditions. Also the experiments showed the propriety of the above diagrams.

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