only one grain size thick) damaged layer appears after an infinite number of stress repetitions. This view will have some close relation with the view of Watanabe(5) or Wadsworth(6) that fatigue limit is a stress under which fatigue crack does not propagate at all.

Conclusion

After repeated polishing and annealing procedure we reached the following conclusions about fatigue damage of 6~4 brass specimens under rotary bending stressing.

a) Fatigue damage due to stress above fatigue limit concentrates near the surface of the specimen. The damage can not be removed by stress relief annealing. It proceeds inward into the specimen when stress repetitions continue. The thickness of the damaged layer is a function of applied stress and its number of repetitions.

b) Fatigue damage seems to proceed grain by grain from the surface to the interior of the specimen.

c) Most part of the fatigue damage in the interior part of the specimen can be removed by stress relief annealing but not perfectly.

d) Fatigue limit will be the stress under which the thickness of the damaged layer remains within some limit (perhaps 0 or one grain size thick) even if the number of stress repetitions is infinitely large.

Acknowledgments

The author would like to thank assistant T. Matake for carrying out fatigue tests and postgraduate students K. Setoguchi, M. Shibata for their aids in electro-polishing of the specimens.

References

(3) T. Endo: Private communication.
(8) N.J. Wadsworth: Phil. Mag., Vol. 6, p. 387.

On the Strength of Hoisting Drums for Windlasses*

By Motoharu Taneda**

Theoretical and experimental studies were conducted in order to establish a method of computing the strength of hoisting drums for windlasses. It was assumed that external pressure is the only external force applied to a hoisting drum. Further, the strength of the hoisting drum was discussed in terms of the strength of the cylindrical parts and the strength of the stiffening rings. Each of the 16 model drums failed due to the yielding of either the cylindrical parts or the stiffening rings. As a result of the research, it was made clear that the initial deflection of the hoisting drum has a considerable effect on the strength, and formulas were obtained for computing the strength of the drum. Finally, applicability of these results to actual design of hoisting drums was discussed.

1. Introduction

Methods of computing the strength of hoisting drums for windlasses have been proposed heretofore by several research workers(1~3), and these methods have been in general used in designing the hoisting drums. None of these methods, however, is satisfactory for computing the strength of hoisting drums. Accordingly, it is urgently required in designing windlasses to establish a method of computing the strength of safe and economical drums.
Consequently, the author previously clarified the increase in the external pressure to which the hoisting drum was subjected as the number of rope layers was increased, when a rope was wound in multi-layers about a hoisting drum under tension. Subsequently, he studied the effect of the initial deflection on the strength of hoisting drums without a stiffener.

In this paper, the strength of hoisting drums with stiffening rings has been explored, and, as a result, a new computing method has been obtained.

2. Theory

Fig. 1 is a sectional view of a hoisting drum. It is assumed that, as shown in the figure, the stiffening rings are placed at equal intervals; the part between two stiffening rings is called "the cylindrical part". The notations shown in Fig. 1 are used for the dimensions of each part of the hoisting drum. It is assumed that the hoisting drum is a thin-walled cylinder, and that the material used for hoisting drums is mild steel. It is assumed that the external pressure is the only external force applied to the drum, and also that this external pressure is applied uniformly to the lateral side of the drum.

When a rope is wound in multi-layers about a hoisting drum under tension, the external pressure \( p \) to which the drum is subjected can be expressed approximately as follows:

\[
 p = \frac{C_s T}{d_I R}
\]

where

- \( T \): the initial rope tension
- \( d_I \): the center distance between the adjacent sections of rope in same layer (Fig. 1)
- \( C_s \): the ratio of the external pressure acting on the drum when the rope has been wound

in \( n \) layers to the external pressure caused by first one layer (called "coefficient of rope layers")

Let us now divide the strength of the hoisting drum into the strength of the cylindrical parts and the strength of the stiffening rings.

2-1 Cylindrical parts Buckling strength and yield strength are conceivably as the strength of the cylindrical parts. Within the range of the actual dimensions, it was verified by computations that the buckling strength was sufficiently high in comparison with the yield strength. Consequently, the yield strength is dealt with here.

In the case when a thin-walled cylinder without initial deflection, whose ends were fixed, was subjected to uniform external pressure on its lateral sides, the yielding pressure of the cylinder was obtained theoretically by Hodge. His analysis is a limit analysis. The result of the numerical computation of the analysis is shown as the curve in Fig. 2.

On the other hand, the theoretical formula of failure pressure of an unrounded thin-walled cylinder under uniform external pressure applied to the lateral side was obtained by Sturm, assuming that the cylinder had such an initial deflection as a wave profile of completely round cylinder in elastic buckling under uniform external pressure. It was assumed further that the yielding pressure was equal to the external pressure at which the maximum circumferential stress became yield point \( \sigma_{y_2} \). Sturm's formula is as follows:

\[
p_{st} = \frac{2\sigma_{y_2} D}{1 + \frac{4A_0}{t} \left[ \frac{N^2 - \left( 1 + \nu \right) \frac{E}{t} R}{E} \right] \left( \frac{t}{D} \right)^3}
\]

where

- \( A_0 \) is theoretical value (for completely round cylinders)
- \( \nu \) is Poisson’s ratio
- \( E \) is modulus of elasticity
- \( R \) is radius of cylinder
- \( t \) is thickness of cylinder
- \( D = \frac{2R}{t} \)
- \( \sigma_{y_2} \) is yield point of cylindrical parts

Experimental values \( \times \) of drums R-6, R-6, RA-7 which have failed due to yielding of stiffening rings are shown for reference, too.

Fig. 1 Sectional view of hoisting drum

Fig. 2 Yielding pressures of cylindrical parts
where

\[ p_s = KE_s(tD)^3 \]  \hspace{1cm} \text{(3)}

\( p_s \): yielding pressure of unround cylinder

\( A_0 \): amplitude of the initial profile of cylinder, assuming that the profile is the same as the wave-profile of completely round cylinder in elastic buckling (to be called by the name "magnitude of out-of-roundness")

\( E_s \): modulus of longitudinal elasticity of cylinder material

\( \nu_s \): Poisson's ratio of cylinder material

\( N \): number of lobes in elastic buckling of completely round cylinder

\( p_r \): elastic buckling pressure of completely round cylinder

\( K \): buckling coefficient depending on \( t/D, l/R, \)

\( \nu_s, N \) and supporting condition at both ends of cylinder\(^(5)\).

As the value of \( K \), the smallest of all the \( K \) values obtained assuming \( N \) to be 2, 3, 4, ... is used, and the corresponding value of \( N \) is used here.

It is a fact that the actual cylindrical parts have not a regular wave-profile such as that assumed in Eq. (2) usually. Consequently, the method for determining the magnitude of out-of-roundness \( A_0 \) should be discussed, when Eq. (2) is used for an actual hoisting drum. The author studied on this point previously, and proposed the most appropriate method for determining \( A_0 \)\(^(5)\).

2-2 Stiffening rings

In order to compute the strength of stiffening rings, one must know the loads applied to the stiffening rings as the reaction forces from the cylindrical parts. Timoshenko dealt with this point\(^(6)\). However, this analysis involves certain assumptions which would not appear to correspond with the actual conditions in cases when the thickness \( b \) of the stiffening rings and their height \( h' \) are not especially small, as is true in the case of a hoisting drum. For this reason, these assumptions will be given up, and the calculation will be done anew.

2-2-1 Loads applied to the stiffening rings

If the hoisting drum has no stiffening ring, the decrement \( \Delta w \) in the radius of the cylindrical parts caused by the application of external pressure \( p \) will be:

\[ \Delta w = \frac{pR^2}{E_t} \]  \hspace{1cm} \text{(4)}

However, as there are stiffening rings actually,

\[ \Delta w = \frac{2\beta R^2}{E_t} \left\{ \begin{array}{c}
Q_1 \cosh 2\alpha - \cosh 2\alpha \sinh 2\alpha \\
2 \sinh 2\alpha + \sin 2\alpha
\end{array} \right\} + \beta M_1 \frac{\sinh 2\alpha - \sin 2\alpha}{\sinh 2\alpha + \sin 2\alpha} \]  \hspace{1cm} \text{(5)}

reaction forces will occur between the stiffening rings and the cylindrical parts, and the uniform deformation as expressed by Eq. (4) will not appear, as shown in Fig. 3.

The hatched parts shown in Fig. 3 are considered as the stiffening rings, and the reaction forces from the cylindrical parts of the length \( l \) are considered to be applied to the stiffening rings as shearing forces \( Q/l \) and moments \( M_1 \) (per unit length on the circumference). In the hoisting drums, the effect of the moment \( M_1 \) on the deflection and stress of the stiffening rings can be considered as slight. Consequently, let us neglect the effect of \( M_1 \) on the deflection of the stiffening rings. Further, it is assumed that the stiffening rings are subjected to pressures \( Q \) at their upper surfaces, instead of shearing forces \( Q/l \) at both sides. Besides being subjected to these pressures \( Q \), the stiffening rings are also subjected to external pressures \( p \) at their upper surfaces directly. As a result, the stiffening rings are subjected to pressure \( (Q_1 + bp) \) per unit length on the circumferences.

Therefore, the decrement \( w_1 \) in the radius of the stiffening rings will be:

\[ w_1 = \frac{R^2}{E_t h'} \left( \frac{Q_1}{b} + p \right) \]  \hspace{1cm} \text{(5)}

where

\( E_r \): modulus of longitudinal elasticity of the ring material.

Consequently, the deformation of the cylindrical part may be considered as follows: (i) at first, the radius of the cylinder (length \( l \)) having free ends decreases by \( \Delta w \) due to the application of external pressure \( p \); (ii) further, the radii at both ends increase by \( \Delta w \) and the slopes at the ends become 0 due to the applications of outward shearing forces \( Q/l \) and moments \( M_1 \) at the ends.

Therefore, we obtain the following equations, according to the theory of cylindrical shells\(^(8)\):

\[ (w) - w_1 = \frac{2\beta R^2}{E_t} \left\{ \begin{array}{c}
Q_1 \cosh 2\alpha - \cosh 2\alpha \\
2 \sinh 2\alpha + \sin 2\alpha
\end{array} \right\} + \beta M_1 \frac{\sinh 2\alpha - \sin 2\alpha}{\sinh 2\alpha + \sin 2\alpha} \]  \hspace{1cm} \text{(6)}

\[ 0 = \frac{2\beta R^2}{E_t} \left\{ \begin{array}{c}
Q_1 \cosh 2\alpha - \cosh 2\alpha \\
2 \sinh 2\alpha + \sin 2\alpha
\end{array} \right\} + \frac{2\beta M_1}{\sinh 2\alpha + \sin 2\alpha} \]  \hspace{1cm} \text{(7)}

where
\[ \beta = \frac{3(1 - \nu^2)}{R^2} \]  
(8)

\[ \alpha = \beta l^2 \]  
(9)

By using Eqs. (4) and (5) in Eqs. (6) and (7), we obtain

\[ Q_i = p \frac{R^2}{R_i^2} \frac{t}{h'} E_r \frac{1}{E_r} \]  
(10)

where

\[ \mathcal{J}(2\alpha) = \frac{\sinh 2\alpha + \sin 2\alpha}{2(\cosh 2\alpha - \cos 2\alpha)} \]  
(11)

If the external pressure per unit area applied to the stiffening rings is expressed by \( p \), \( p_1 \) will be as follows:

\[ p_1 = Q_i \frac{b}{h'} \frac{t}{R_i^2} E_r \frac{1}{E_r} \]  
(12)

2.2.2 Yield strength According to Lamé, the circumferential stress \( \sigma_{r1} \) in the stiffening rings due to the external pressure \( p_1 \) is as follows (the stress due to the moment \( M_i \) is ignored):

\[ \sigma_{r1} = -z_{11/2} h_i / h' \]  
(13)

where

\[ z_{11} = \frac{2}{R_i} \left[ \frac{1 + h'}{2R_i} \right] \left[ \frac{1 - h'}{2R_i} \right] + 1 \]  
(14)

\( z_{11} \): correction factor for considering that \( h'/R_i \) is very small

\( r \): radius of the point where the stress is sought

Since \( z_{11} \) at the outer surfaces of the stiffening rings are quite close to 1 in hoisting drums, it is assumed to be 1. Then, if the mean circumferential stress \( \langle \sigma_{11} \rangle_{\text{mean}} \) in the stiffening rings is assumed to be approximately equal to the mean of the stress at the inner and outer surfaces, we obtain

\[ \langle \sigma_{11} \rangle_{\text{mean}} = -\frac{2}{p} \frac{h_i}{h'} \]  
(15)

If the radial stress is ignored, the pressure when the stiffening rings will yield can be found by substituting the yielding stress in compression \( \sigma_{r1} \) of the stiffening rings for \( -\langle \sigma_{11} \rangle_{\text{mean}} \) in Eq. (15).

As strength of the stiffening rings, strength for lateral buckling and strength for buckling inside the plane of the rings are also conceivable, besides the above-mentioned yield strength. However, it was

<table>
<thead>
<tr>
<th>Table 1 Specifications of model drums</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model number</td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td>R-2</td>
</tr>
<tr>
<td>R-1</td>
</tr>
<tr>
<td>R-3</td>
</tr>
<tr>
<td>R-4</td>
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<tr>
<td>R-5</td>
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<td>R-6</td>
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<td>R-7</td>
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<td>R-8</td>
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<td>R-9</td>
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<td>R-10</td>
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<td>R-11</td>
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<tr>
<td>R-12</td>
</tr>
<tr>
<td>R-13</td>
</tr>
<tr>
<td>R-14</td>
</tr>
<tr>
<td>R-15</td>
</tr>
<tr>
<td>R-16</td>
</tr>
</tbody>
</table>

\( D = 310 - 324 \text{ mm}, \) (c):cylindrical part, (r):stiffening ring
verified by the experiments that the strengths for those bucklings are sufficiently high in comparison with the yield strength. Consequently, explanations of those bucklings are omitted here.

3. Experimental methods

3.1 Model drums Two types of model drums were used: (a) 7 machined drums (with comparatively small magnitude of out-of-roundness), (b) 9 plate-worked drums (with comparatively large magnitude of out-of-roundness). The outlines of the model drums are shown in Fig. 4, and their specifications are listed in Table 1; \( \Delta R \) is the maximum value of deviation in \( R \). The magnitude of out-of-roundness \( A_0 \) was found by the method proposed by the author for the cylindrical parts whose deflections were measured during the experiment.

3.2 Experimental methods Oil pressure was used as the external pressure applied to the model drums. As the apparatus for experiments, the apparatus for model drums without stiffeners was used. The circumferential strain of the inner and lateral surfaces of the stiffening rings as well as the circumferential strain of the inner surfaces of the cylindrical parts were measured by an electric resistance wire strain meter. These strain measurements were taken at 24 equidistant points on the same circumference.

4. Experimental results and their discussions

As the external pressure was gradually increased, there appeared a strain corresponding to the yielding stress either on the entire circumference or on a section of the cylindrical parts or the stiffening rings. Also a permanent set (20 \( \times \) 10\(^{-6}\) to 60 \( \times \) 10\(^{-6}\)) appeared. However, there were no great deformations for some time, and the applied pressure was still kept constant. When this pressure was further increased, the deflection of the cylindrical parts in the radial direction began to increase suddenly, and the cylindrical parts—or the cylindrical parts and the stiffening rings together—were deformed largely.

In the present experiment, the occurrence of a yielding either in the cylindrical parts or in the stiffening rings was considered to be the failure of the hoisting drum, and the pressure at which the permanent set appeared was taken as the experimental value \( p_{ex} \) of failure pressure of the drum.

4.1 Strength of the stiffening rings Tables 2 and 3 are comparisons of experimental values with computed values of pressures on the model drums. As mentioned later, the experimental values of failure pressure in the machined drum R-6 and in the plate-worked drums RA-6 and RA-7 are considerably lower than the computed values of failure pressure of their cylindrical parts. Furthermore, as evident from Tables 2 and 3, in these drums the pressures \( \bar{p} \) applied to the stiffening rings at failure are nearly equal to the computed values \( p_T \) of the yielding pressure of the stiffening rings. Consequently, drums R-6, RA-6 and RA-7 are presumed to have failed due to the yielding of the stiffening rings. In the

<table>
<thead>
<tr>
<th>Model number</th>
<th>( \sqrt{2R/R} )</th>
<th>( p_{ex} ) kg/cm(^2)</th>
<th>( p_T ) kg/cm(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-2</td>
<td>3.696</td>
<td>62.0</td>
<td>1.067</td>
</tr>
<tr>
<td>R-1</td>
<td>3.680</td>
<td>61.8</td>
<td>1.038</td>
</tr>
<tr>
<td>R-3</td>
<td>2.144</td>
<td>70.8</td>
<td>1.382</td>
</tr>
<tr>
<td>R-4</td>
<td>1.597</td>
<td>70.5</td>
<td>1.470</td>
</tr>
<tr>
<td>R-5</td>
<td>1.286</td>
<td>87.9</td>
<td>1.531</td>
</tr>
<tr>
<td>R-7</td>
<td>0.975</td>
<td>116.0</td>
<td>1.497</td>
</tr>
<tr>
<td>R-6</td>
<td>1.286</td>
<td>72.0</td>
<td>1.694</td>
</tr>
</tbody>
</table>

\( p_{ex} \): experimental value of failure pressure of drum
\( \bar{p} \): pressure in radial direction on stiffening rings at failure of drum (the value \( p_T \) in Eq. (12) when \( p = p_{ex} \))
\( p_T \): computed value of yielding pressure of stiffening rings (the value \( p_T \) in Eq. (15) when \( (a_{11})_{mean}^- = \sigma_T \))

In computations, Poisson's ratio of the cylindrical parts was assumed to be 0.3.

* The pressure at this time was 1 to 4% larger than the pressure at which a permanent set appeared in the machined drums and 7 to 26% larger in the plate-worked drums.
plate-worked drums RA-6 and RA-7, the computed values \( p_s \) are somewhat higher than the experimental values \( \bar{p} \). The reason for this is considered as follows. Excessive stress appears locally in the stiffening rings because of the initial deflection of the drums, and accordingly the strength of the stiffening rings has decreased.

### 4-2 Strength of the cylindrical parts

Studies will be made of the other hoisting drums besides R-6, RA-6 and RA-7, which failed due to the yielding of the stiffening rings. As is evident from Tables 2 and 3, in these drums the pressures \( \bar{p} \) applied to the stiffening rings at failure, are lower than the computed values \( p_s \) of the yielding pressure of the stiffening rings. Therefore, it is considered that, in all other drums except R-6, RA-6 and RA-7, the failure occurred first in the cylindrical parts.

Fig. 2 shows comparison of the experimental values of the failure pressure with the theoretical values (according to Hodge) of the yielding pressure of completely round cylinders. As shown in this figure, the experimental values for the machined drums having comparatively small magnitudes of initial out-of-roundness coincide well with their theoretical values. On the other hand, in the plate-worked drums having comparatively large magnitudes of initial out-of-roundness, their experimental values gradually become smaller than the theoretical values, as the value of dimensionless length \( l/\sqrt{Rl} \) of the cylindrical parts increases. In the cylindrical parts of these plate-worked drums, a permanent set began to appear first from the vicinity where the magnitude}

of out-of-roundness \( A_0 \) had existed. Consequently, it is considered that the failure pressure of plate-worked drums was effected by the magnitude of out-of-roundness.

Table 3 shows comparison of the computed values of Sturm’s failure pressure \( p_{st} \), which take into consideration the effect of out-of-roundness, with the experimental values \( p_{ex} \) of the plate-worked drums. An examination of this table shows that there is little correspondence between the values \( p_{st} \) and \( p_{ex} \). The difference increases as the value \( l/\sqrt{Rl} \) decreases. The cause of this is believed to be as follows.

In Sturm’s theory, analysis is made considering that failure occurs when the sum of the local bending stress due to the wave-profile of the initial deflection and the stress in a completely round cylinder becomes a yielding stress \( \sigma_y \). Consequently, in the case when the magnitude of out-of-roundness \( A_0 \) is 0, the failure pressure computed by Sturm’s formula would correspond to the external pressure when the stress of a completely round cylinder became \( \sigma_y \). However, as presumed from Eq. (2), the pressure at this time will be:

\[
p_{st} = 2\sigma_y D.
\]

This pressure is equal to the failure pressure \( p_o \) of an infinitely long cylinder. Since the failure pressure of a comparatively short cylinder will be larger than \( p_o \), this formula is not suitable.

In order to compensate the formula, using \( 2\sigma_y D, 2\sigma_y D \) in Eq. (2) multiplied by Hodge’s coefficient \( P \) as shown in curve in Fig. 2, instead of \( 2\sigma_y D, \) Eq. (2) will be rewritten as follows:

\[
(p_{st}) = \frac{2P \sigma_y D}{1 + 4A_0 \left( N^2 - 1 + \nu_s \frac{\pi^2 R^2}{P} \right) \left( \frac{t}{D} \right)^2}
\]

where

\[
(p_{st}) \text{ : compensated value of } p_{st}.
\]

Values computed by the above formula are shown in Table 3. The computed values \( p_{st} \) are close to the experimental values \( p_{ex} \), without regard to the value of \( l/\sqrt{Rl} \).

Further in these drums, the experimental values \( p_{ex} \) are somewhat higher than the computed values \( p_{st} \). The following is thought to be the chief reason for this. It is believed that a yielding occurred first locally in that section of the cylindrical parts where the initial deflection was largest. However, the strain gauge happened not to have been pasted to exactly this section, so that it was impossible to detect the yielding at this section. Consequently, it can be presumed that the detection of the failure.

<table>
<thead>
<tr>
<th>Model number</th>
<th>( l/\sqrt{Rl} )</th>
<th>( p_{ex} )</th>
<th>( p_{st} )</th>
<th>( \frac{p_{ex}}{p_{st}} )</th>
<th>( \frac{\pi^2 R^2}{P} )</th>
<th>( \frac{t}{D} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA-5</td>
<td>4.715</td>
<td>16.2</td>
<td>0.774</td>
<td>0.774</td>
<td>2.480</td>
<td></td>
</tr>
<tr>
<td>RA-4</td>
<td>3.185</td>
<td>51.0</td>
<td>0.753</td>
<td>0.787</td>
<td>1.515</td>
<td></td>
</tr>
<tr>
<td>RA-3</td>
<td>2.364</td>
<td>87.9</td>
<td>0.834</td>
<td>0.994</td>
<td>1.331</td>
<td></td>
</tr>
<tr>
<td>RA-9</td>
<td>1.398</td>
<td>62.5</td>
<td>0.685</td>
<td>0.810</td>
<td>1.551</td>
<td></td>
</tr>
<tr>
<td>RA-1</td>
<td>1.631</td>
<td>110.0</td>
<td>0.597</td>
<td>0.848</td>
<td>1.066</td>
<td></td>
</tr>
<tr>
<td>RA-8</td>
<td>1.310</td>
<td>62.5</td>
<td>0.440</td>
<td>0.785</td>
<td>1.641</td>
<td></td>
</tr>
<tr>
<td>RA-2</td>
<td>1.193</td>
<td>86.9</td>
<td>0.390</td>
<td>0.803</td>
<td>1.449</td>
<td></td>
</tr>
<tr>
<td>RA-7</td>
<td>1.671</td>
<td>82.0</td>
<td>0.711</td>
<td>1.130</td>
<td>1.119</td>
<td></td>
</tr>
<tr>
<td>RA-6</td>
<td>1.128</td>
<td>90.7</td>
<td>0.681</td>
<td>1.500</td>
<td>1.095</td>
<td></td>
</tr>
</tbody>
</table>

\( p_{ex} \) : computed value of yielding pressure of unround cylinders (computed from Eq.(2), assuming that both ends of cylinder are simply supported.)

\( (p_{st}) \) : compensated value of \( p_{st} \) (computed from Eq.(16) and the curve in Fig. 2, assuming that both ends of cylinder are simply supported.)

The others are the same as the notes in Table 2.
was delayed.

5. Application of the research results to design of actual hoisting drums

In designing a hoisting drum, both the strength of the cylindrical parts and that of the stiffening rings must be sufficient for the given specification load. As observed in the preceding chapters, it is possible to compute from Eq. (1) the external pressure applied to the actual hoisting drum, and also to compute the strength of the cylindrical parts and that of the stiffening rings from Eq. (16) and Eq. (15), respectively.

5.1 Design computations

Since Eq. (16) and Eq. (15) are quite complex, design computation diagrams etc. were prepared for greater convenience.

5.1.1 Cylindrical parts $\gamma$ is the yielding pressure $(\sigma_{y})$ of the cylindrical parts, in which the effect of out-of-roundness has been taken into consideration, divided by the yielding pressure $P_{0} = \frac{P \cdot \sigma_{y}}{d_{2} \cdot R}$ of a completely round cylinder ($P$ is Hodge's theoretical value). This $\gamma$ is called by the term “failure pressure decrease rate due to magnitude of out-of-roundness.” $\gamma$ is less than 1 and is expressed from Eqs. (16) and (3) as follows:

$$\gamma = \frac{\sigma_{y}}{P_{0}} = \frac{1}{2P} \left[ \frac{P + \frac{K}{8} \cdot \frac{E_{s}}{\sigma_{y}} \left( \frac{t}{R} \right)^{2} + \frac{A_{0}}{2} \cdot \frac{E_{s}}{\sigma_{y}} \left( \frac{N^{2} - 1 + \nu + \frac{\pi^{2} R^{2}}{j^{2}}}{1 - \nu} \right)}{\left( \frac{t}{R} \right)^{2}} \right]$$

$$\pm \sqrt{\left[ \frac{P + \frac{K}{8} \cdot \frac{E_{s}}{\sigma_{y}} \left( \frac{t}{R} \right)^{2} + \frac{1}{2} \cdot \frac{A_{0}}{2} \cdot \frac{E_{s}}{\sigma_{y}} \left( \frac{N^{2} - 1 + \nu + \frac{\pi^{2} R^{2}}{j^{2}}}{1 - \nu} \right)}{\left( \frac{t}{R} \right)^{2}} \right] - \frac{K}{2} \cdot \frac{E_{s}}{\sigma_{y}} \cdot P \left( \frac{t}{R} \right)^{2}}$$

$P$ is a function solely of $l / \sqrt{R t} = [(l/R) / \sqrt{l/R}]$ and is expressed by the curve in Fig. 2.

In Eq. (17), $\nu$, $E_{s}$, and $\sigma_{y}$ depend on the material of the cylindrical parts, and $A_{0} / t$ is to be the tolerance standard in manufacturing. When these values are determined as fixed ratings, both $K$ and $N$ will become functions of $l / R$ and $t / R$, as mentioned above. Therefore, $\gamma$ also will become a function of $l / R$ and $t / R$. Consequently, it is possible to express graphically the relation between $\gamma$ and $l / R$ with parameter $t / R$, as in Fig. 5.

If $\gamma$ is found from this figure, the failure pressure $(\sigma_{y})$ of cylindrical parts can be computed from the following equation:

$$(\sigma_{y}) = \gamma P_{0}$$

In the design computations, the value $(\sigma_{y})$ in which the value $\sigma_{y}$ in the above equation is converted into the allowable stress $\sigma_{as}$, in compression of the cylindrical parts must not be smaller than the design pressure (the value computed by Eq. (1)). In other words:

$$\frac{t}{R} P \geq \frac{1}{\gamma} \cdot \frac{C_{t} \cdot T}{d_{1} \cdot \sigma_{as}}$$

This is the formula for the design computation of cylindrical parts. In the above equation, the coefficient of rope layers $C_{t}$, the rope tension $T$, and the center distance between the adjacent sections of rope $d_{1}$ in the right side of the equation can be determined depending upon the specifications. Also the drum radius $R$ and the allowable stress $\sigma_{as}$ can be determined conventionally, while the failure pressure decrease rate $\gamma$ can be determined from Fig. 5. Consequently, the relation between $t / R$ and $l / R$ giving a sufficient strength of the cylindrical parts is obtained from Eq. (19). Since it is bothersome to compute with this equation directly, it will be convenient to show graphically the relation between $t / R$ and $l / R$ with parameter $1 / \gamma \cdot (C_{t} T / d_{1} \cdot R s_{as})$ previously as shown in Fig. 6, based on Eq. (19). In Fig. 6, the domain where the sign of inequality in Eq. (19) applies is the section on the right-hand side of each curve.
The actual designing of the cylindrical parts may be done according to the following sequence.

(i) Determine the value \( l/R \), taking into consideration the economic requirement of the hoisting drum as described in section 5-2.

(ii) Choose the value \( l/R \) arbitrarily.

(iii) Seek from Fig. 5 the value \( \zeta \) corresponding to the values \( l/R \) and \( t/R \) determined in (i) and (ii).

(iv) Utilizing this \( \zeta \), compute \( 1/\zeta \cdot (C_n T/d_4 R_{\sigma_e}) \).

(v) Seek in Fig. 6 the point corresponding to the values \( l/R \) and \( t/R \) determined in (i) and (ii).

(vi) If this point is to the left of the curve for the value \( 1/\zeta \cdot (C_n T/d_4 R_{\sigma_e}) \) computed in (iv), choose a larger value \( t \) and repeat the processes after (ii). If it is to the right, choose a smaller value \( t \) and repeat the processes after (ii). As a result, find the value \( t \) giving the point closest to the curve in the domain to the right of the curve, and take this as the thickness \( t \).

**5-1-2 Stiffening rings** The absolute value of the stress \((\sigma_{\text{Stiffening}})_{\text{mean}}\) computed by substituting the design pressure (the value expressed by Eq. (1)) for the value \( p \) in Eq. (15) must not exceed the allowable stress \( \sigma_{\text{stiffening}} \) in compression of the stiffening rings. That is:

\[
\sigma_{\text{stiffening}} \geq \frac{\left(1 + \frac{h'}{2R_1}\right)^2 + 1}{2} \cdot \frac{C_n T}{d_4 R_{\sigma_e}} \cdot \frac{R_1}{b \cdot \beta' \left(2 \gamma + 1\right)} \cdot \frac{R_{\sigma_e}}{h'} \cdot \frac{E_s}{E_r}.
\]

In the design computations, it is convenient if it is possible to express the height \( h'/R \) of the stiffening rings as an explicit function of \( l/R, t/R, b/R \) and the external pressure \( C_n T/d_4 R_{\sigma_e} \) on the drum. However, this is difficult with Eq. (20). Therefore, for the sake of convenience, it is assumed that

\[
R_1 = R \left[\left(1 + \frac{h'}{2R_1}\right)^2 + 1\right]^{1/2} = 1 \quad \text{the correction factor}
\]

for considering that \( h'/R_1 \) is very small.

Instead of this, a different correction factor \( \zeta \) is used, and Eq. (20) will be expressed as follows:

\[
\sigma_{\text{stiffening}} \geq \frac{C_n T}{d_4 R_{\sigma_e}} \cdot \frac{R_1}{b \cdot \beta' \left(2 \gamma + 1\right)} \cdot \frac{R_{\sigma_e}}{h'} \cdot \frac{E_s}{E_r}.
\]

\[
\zeta \text{ can be computed by equaling both right-hand sides of Eq. (20) and Eq. (21). Generally } \zeta \text{ is greater than 1. If we compute } \zeta \text{ with respect to various values in the actual range of } t/R, b/R, l/R \text{ and } h'/R, \text{ and determine one of these values on the safe side in design (a value as large as possible) as the fixed rating of } \zeta, \text{ then the height } h'/R \text{ of the}
\]

stiffening rings can be expressed as an explicit function of \( l/R, t/R, b/R \) and \( C_n T/d_4 R_{\sigma_e} \) from Eq. (21), using Eqs. (8) and (9).

**5-2 Consideration on hoisting drums of minimum weight** If both the cylindrical parts and the stiffening rings are designed to have an equal strength, the drum will be economical. Such a hoisting drum will be one in which the signs of equality in Eq. (19) and Eq. (21) both apply. In cases when these signs of equality apply together, if the values \( 1/\zeta \cdot (C_n T/d_4 R_{\sigma_e}) \) and \( \zeta (C_n T/d_4 R_{\sigma_e}) \) are given, and if it is possible to obtain a suitable value for \( b/R \) also, then these two equations will prescribe the relations between three values \( t/R, l/R \) and \( h'/R \). Consequently, under the condition that the cylindrical parts and the stiffening rings have the identical strengths, it will further be possible to choose at will any one of the values \( t/R, l/R \) and \( h'/R \). Therefore, if we add the condition that the weight of the drum is minimum, it will be possible to determine these three values uniquely. Consequently, a drum determined in this manner will be the most economical drum. Computations were made with regard to various specification loads within the actual range, in the case when the total lengths \( L \) were within \((0.7 \sim 1.3)R\). As a result, it was learned that the weight of drum becomes minimum when the number of stiffening rings is usually 1 to 3 (assuming that \( b/R = 0.02, \nu_r = 0.3, E_s = E_r \)).

**6. Conclusion**

The results of present research are summarized as follows:

(1) The experimental values of the pressure at which a yielding occurred in the stiffening rings of the model drums coincided nearly with their computed values.

(2) It was learned that the cylindrical parts failed due to yielding, and that the magnitude of out-of-roundness has an important effect on the failure. In the machined drums having comparatively small magnitudes of initial out-of-roundness, the experimental values of the failure pressure of the cylindrical parts coincided well with Hodge's theoretical values. In the plate-worked drums having comparatively large magnitudes of initial out-of-roundness, the formula (16) compensated by the insertion of Hodge's coefficient in Sturm's formula gave values nearly coinciding with the experimental values.

(3) The design computations of the hoisting drum may be made by the method described in

* Since the actual range of the value \( b/R \) is generally narrower than those of the values \( l/R, t/R \) and \( h'/R \), we here think of \( b/R \) as being fixed at a definite value.
section 5-1.

(4) In order to reduce the weight of hoisting drum to a minimum, the number of stiffening rings should be 1 to 3 usually.

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References


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Strength and Deformation of a Cylindrical Shell
Fixed at One End and Supported at the
Other under the Weight of Lining*

By Koki Mizoguchi** and Keinosuke Inoue***

In this paper, the strength and deformation of a cylindrical shell fixed at one end and supported at the other under the weight of lining is discussed, utilizing the fundamental differential equation introduced by Dr. K. Mizoguchi.

The method of solving and the results here obtained can be applied to the design of a rotary kiln.

1. Solution of problem

Referring to Fig.1, for the sake of simplicity, the following assumptions are made, i.e., one end of a thin-walled pipe is fixed and the other end of it is supported and fitted with a thin plate, the extensional and the flexural rigidities of which are sufficiently large and small respectively; furthermore the lining, the flexural rigidities and resistances of which are neglected for safety's sake, is completely fixed on the inner surface of the pipe.

The fundamental differential equation in a single displacement under radial and circumferential loads introduced by Dr. K. Mizoguchi is as follows:

\[
\frac{\partial^2 w}{\partial x^2} + \frac{4B}{a^2} \frac{\partial^3 w}{\partial x^2 \partial \theta^2} + \frac{6c}{a^4} \frac{\partial^4 w}{\partial x^4 \partial \theta^4}
+ \frac{8 - 2\nu^2}{a^4} \frac{\partial^2 w}{\partial x^2 \partial \theta^2} + \frac{6L}{a^4} \frac{\partial^4 w}{\partial \theta^4} + \frac{4}{a^4} \left[ \frac{\partial^2 w}{\partial x^2 \partial \theta^2} + \frac{2}{a^2} \frac{\partial^4 w}{\partial x^4 \partial \theta^4} + \frac{\partial^4 w}{\partial \theta^4} \right]
+ \frac{1}{a^4} \left[ \frac{\partial^2 w}{\partial \theta^4} + \frac{2}{a^2} \frac{\partial^4 w}{\partial x^2 \partial \theta^2} + \frac{\partial^4 w}{\partial \theta^4} \right]
= \frac{1}{D} \left[ \frac{\partial^2 p}{\partial x^2} - \frac{2}{a^2} \frac{\partial^4 p}{\partial x^4 \partial \theta^4} + \frac{B}{a^4} \frac{\partial^4 p}{\partial \theta^4} \right]
- \frac{1}{D} \left[ \frac{2 - \nu}{a^2} \frac{\partial^2 q}{\partial x^2 \partial \theta} - \frac{(2 + \nu)}{a^4} \frac{\partial^4 q}{\partial x^4 \partial \theta^4}
+ \frac{4 - 3\nu}{a^4} \frac{\partial^2 q}{\partial x^2 \partial \theta} \frac{h^2}{a^2} \frac{\partial^2 q}{\partial x^2 \partial \theta} + \frac{1}{12(1 - \nu)a^4} \frac{\partial^4 q}{\partial x^4 \partial \theta^4} \right]
- \frac{1}{a^4} \frac{\partial^2 \theta^2}{\partial \theta^2} \right]
\]

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Fig. 1