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The Theoretical Research on the Blanking of a Sheet Material*

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In this paper, the author applied the theory of limit analysis to a theoretical investigation of the blanking process. In Part I, the statically admissible stress field in a sheet material was analysed on the assumptions: (1) The sheet material is a non-hardening plastic-rigid body. (2) The material near tool edges is approximately in the state of plane strain. (3) The stress field consists of several regions of constant stress separated by straight lines of discontinuity.

In Part II, the deformation process of the sheet material was divided into four stages, and the kinematically admissible velocity field was analysed in each stage. Furthermore, comparing the solution of the stress field with that of the velocity field, an exact solution of the blanking force was obtained for blanking with smooth punch and die.

Introduction

It is well known that blanking, together with bending and deep drawing, are the most important techniques in the press working. Among these techniques, the plastic deformation of the sheet material in the process of bending and deep drawing has been studied by many researchers. However, in the blanking, the material in the narrow region between both edges of punch and die flows unsteadily, but theoretical researches of this phenomenon have been very few⁽¹⁾.

Especially in such unsteady plastic flow as the blanking, it is difficult to obtain exact solutions of stress and velocity, which fulfill all the fundamental equations of plasticity and the boundary condition of the sheet material.

Therefore, the author applied the theory of limit analysis in the following treatment. In Part I, the statically admissible stress distribution in the sheet material during the blanking process was analysed, and, in Part II, the kinematically admissible velocity distribution was analysed.

Part I. The statically admissible stress field

1.1 Basis of analysis

(a) Assumptions Fig. 1 illustrates the shape of the sheet material in the process of circular blanking. The stress field, which occurs in the material,

consists of three regions: I is a plastic region, II and III are rigid regions where no plastic deformation occurs.

To simplify the analytical treatment of the problem, the following assumptions are adopted.

(1) The sheet material is a non-hardening plastic-rigid body.

(2) The ratio of the tool diameter to the sheet thickness is large enough to assume that the material near tool edges is approximately in the state of plane strain.

(3) All stress components are zero in both rigid regions II and III.

(4) The surface traction acting on the contact surface with a punch is equal to that with a die. From the above assumption (2), the theory of plane strain is used in the following treatment.

(b) Boundary conditions In Fig. 1, Carte-

Fig. 1 Configuration of sheet material during blanking

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sian coordinates are taken so that the z axis coincides with the direction parallel to that of the punch penetration. For simplicity, let us take the sheet thickness as unit of length, and moreover take the downward velocity of the punch as unit of velocity.

Then, the boundary conditions on velocity are given as follows;

$$\left. \begin{array}{ll} \text{at the front of a punch} & v_z = -1 \\ \text{at the front of a die} & v_z = 0 \\ \text{at the side of a punch} & v_x \leq 0 \\ \text{at the side of a die} & v_x \geq 0 \end{array} \right\} \dots\dots\dots (1)$$

All boundaries of the sheet material are free surfaces except those which contact with the punch or the die.

(c) First extremum principle⁽²⁾⁽³⁾ In the theory of limit analysis, the term of the statically admissible stress implies such stress that fulfills the equilibrium equations, the yield condition and the stress boundary conditions.

When the front and the side of tool edges are perfectly smooth or perfectly rough, a first extremum principle is represented as follows: The exact solutions of blanking force can not be smaller than the blanking forces which are calculated by equilibrium consideration in the statically admissible stress field.

1.2 Analysis of the statically admissible stress field⁽²⁾

The shape of the sheet material varies with the punch penetration. But, it is difficult to find the valid shape of the deformed material theoretically. We shall therefore neglect the obtuse plane l_1 through the blanking process, and assume the deformed shape of the sheet material as shown in Fig. 2.

In Fig. 2, the plastic region I consists of the fields ①, ② and ③ of constant stress separated by two straight lines of discontinuity OA and O'A. The broken line expresses the maximum shearing direction that is obtained from the algebraically maximum normal stress direction by a clockwise rotation through 45° . This line is called the first

shear line. And the line perpendicular to the first shear line is called the second shear line.

Then the jump condition across the line of discontinuity is given as follows;

$$\left. \begin{array}{l} \text{across OA} \\ \theta_2 = 2\delta - \theta_1, \\ 2k\omega_2 = 2k\omega_1 + 2k \sin 2(\theta_1 - \delta) \end{array} \right\} \dots\dots\dots (2)$$

$$\left. \begin{array}{l} \text{across O'A} \\ \theta_3 = 2\gamma - \theta_2, \\ 2k\omega_3 = 2k\omega_2 + 2k \sin 2(\theta_2 - \gamma) \end{array} \right\} \dots\dots\dots (3)$$

where k is the yield shearing stress, and $2k\omega$ is the mean normal stress, and θ means the angle of inclination of the first shear line to the z axis as shown in Fig. 2. Suffixes 1, 2, 3 refer to the fields ①, ②, ③, respectively.

Let us take the surface tractions as positive when they have the directions shown in Fig. 2. Then, the normal stress and the shearing stress on the front of punch and die are given by

$$N_f = 2k\omega_1 - k \sin 2\theta_1, \quad T_f = k \cos 2\theta_1 \dots\dots\dots (4)$$

and the apparent coefficient of friction is

$$\mu_f = T_f / N_f \dots\dots\dots (5)$$

The stress components acting on the side of punch and die are given by

$$N_s = 2k\omega_3 + k \sin 2\theta_3, \quad T_s = -k \cos 2\theta_3 \dots\dots\dots (6)$$

and the apparent coefficient of friction is

$$\mu_s = T_s / N_s \dots\dots\dots (7)$$

The stress components acting on the boundaries between the plastic region and the rigid region are represented by

$$\left. \begin{array}{l} \text{along DE or AB} \\ N_1 = 2k\omega_1 + k \sin 2(\theta_1 - \phi_1) \\ T_1 = -k \cos 2(\theta_1 - \phi_1) \end{array} \right\} \dots\dots\dots (8)$$

$$\left. \begin{array}{l} \text{along CD or AF} \\ N_3 = 2k\omega_3 + k \sin 2(\theta_3 - \phi_3) \\ T_3 = -k \cos 2(\theta_3 - \phi_3) \end{array} \right\} \dots\dots\dots (9)$$

The punch penetration is

$$t = \frac{(\tan \delta + c) \sin (\phi_3 - \gamma)}{(\tan \phi_3 - \tan \delta) \sin \gamma \cos \phi_3} \dots\dots\dots (10)$$

where c is the tool clearance (See Fig. 1). The length of the contact surface of the material with the front of punch and die is given by

$$a = \frac{(\tan \delta - \tan \phi_1)(\tan \phi_3 + c)}{(\tan \phi_3 - \tan \delta)} \dots\dots\dots (11)$$

The blanking force acting on a unit length of the circumference of the punch is

$$P = N_f \cdot a + T_s \cdot t \dots\dots\dots (12)$$

and the side force acting on the edge of the die is

$$Q = N_s \cdot t - T_f \cdot a \dots\dots\dots (13)$$

The first term on the right hand side of Eq. (13) shows the pressure acting on the side of the die, and the second term shows the frictional force

Fig. 2 A statically admissible stress field

on the front of the die.

The normal stress and the shearing stress acting on the line OO' connecting the edges of punch and die are as follows:

$$\left. \begin{aligned} N_2 &= 2k\omega_2 + k \sin 2(\theta_2 + \alpha), \\ T_2 &= -k \cos 2(\theta_2 + \alpha), \\ \alpha &= \tan^{-1} \frac{c}{1-t} \end{aligned} \right\} \dots\dots\dots (14)$$

where α represents the inclination of the line OO' to the z axis.

Since the stress components in the rigid regions II and III are zero from the assumption (3) of 1.1,

$$N_1 = N_3 = T_1 = T_3 \dots\dots\dots (15)$$

Substituting Eq. (15) into Eqs. (8) and (9), we obtain

$$\left. \begin{aligned} \omega_1 &= \omega_3 = -\frac{1}{2}, \\ \theta_1 &= \phi_1 + \frac{\pi}{4}, \quad \theta_3 = \phi_3 + \frac{\pi}{4} \end{aligned} \right\} \dots\dots\dots (16)$$

If we substitute Eq. (16) into Eqs. (2) and (3), we have

$$\theta_2 = \frac{\phi_1 + \phi_3}{2} - \frac{\pi}{4}, \quad \delta = \frac{3\phi_1 + \phi_3}{4}, \quad \gamma = \frac{\phi_1 + 3\phi_3}{4} \dots\dots\dots (17)$$

The second and the third equations of Ep. (17) show that the relation of $\angle BAO' = \angle FAO = (1/2)\angle OAO'$ is satisfied

After all, the quantities that represent the blanking characteristic can be solved from the following equations using three variables μ_f , μ_s and c .

$$\left. \begin{aligned} \mu_f &= \tan \phi_1, \quad \mu_s = -\cot \phi_3, \\ t &= \frac{\left(\tan \frac{3\phi_1 + \phi_3}{4} + c \right) \sin \frac{\phi_3 - \phi_1}{4}}{\sin \frac{\phi_1 + 3\phi_3}{4} \cos \phi_3 \left(\tan \phi_3 - \tan \frac{3\phi_1 + \phi_3}{4} \right)}, \\ a &= \frac{\left(\tan \frac{3\phi_1 + \phi_3}{4} - \tan \phi_1 \right) (\tan \phi_3 + c)}{\tan \phi_3 - \tan \frac{3\phi_1 + \phi_3}{4}}, \\ P &= -k(1 + \cos 2\phi_1)a + k \sin 2\phi_3 \cdot t, \\ Q &= -k(1 - \cos 2\phi_3)t + k \sin 2\phi_1 \cdot a, \\ N_2 &= 2k\omega_2 - k \cos (\phi_1 + \phi_3 + 2\alpha) \\ &= -k + 2k \cos \frac{\phi_1 - \phi_3}{2} - k \cos (\phi_1 + \phi_3 + 2\alpha), \\ T_2 &= -k \sin (\phi_1 + \phi_3 + 2\alpha) \end{aligned} \right\} \dots\dots\dots (18)$$

Since the surface tractions on the front and the side of the tool exist in the range of $N_f < 0$, $k \geq T_f \geq -k$, $N_s < 0$ and $0 \geq T_s \geq -k$, the apparent coefficients of friction μ_f and μ_s must satisfy the following condition:

$$1 \geq \mu_f \geq -1, \quad 1 \geq \mu_s \geq 0 \dots\dots\dots (19)$$

This restricts the range of ϕ_1 and ϕ_3 as follows.

$$\frac{\pi}{4} \geq \phi_1 \geq -\frac{\pi}{4}, \quad \frac{3}{4}\pi \geq \phi_3 \geq \frac{\pi}{2} \dots\dots\dots (20)$$

Especially, in the case of $\mu_f = -1.0$ and $\mu_s = 1.0$, that is, $T_f = k$ and $T_s = -k$, the statically admissible stress field of Fig. 2 may be simplified as shown in Fig. 3. In the plastic region I, only a compressive stress $-2k$ acts parallel to the boundary BF and CE. In the rigid regions II and III, all stress components are zero. We have then

$$\left. \begin{aligned} P &= -k(1 - c + t), \\ Q &= P \end{aligned} \right\} \dots\dots\dots (21)$$

Eq. (21) is irrational in advanced states of punch penetration, because the solution of blanking force increases monotonously with increasing punch penetration. Therefore, Eq. (18) is preferred to Eq. (21) when both of these equations have different solutions at the same penetration.

1.3 Results

(a) Blanking force The calculation of Eq. (18) was carried out under the frictional conditions:

Fig. 3 A statically admissible stress field for $\mu_f = -1.0$ and $\mu_s = 1.0$

Fig. 4 Blanking force curves
 c : tool clearance
 μ_s and μ_f : apparent coefficient of friction at the side and the front of a tool

$$\left. \begin{aligned} \mu_s = 1.0 & \quad (T_s = -k), \\ 1.0 \geq \mu_f \geq -1.0 & \quad (k \geq T_f \geq -k) \end{aligned} \right\} \dots\dots\dots(22)$$

$$\mu_s = 0 \quad (T_s = 0), \quad \mu_f = 0 \quad (T_f = 0) \dots\dots\dots(23)$$

When the frictional stress T_f on the front of a tool is neither equal to zero or k , the extremum principle mentioned in 1-1 (c) is not valid. But it may be safely concluded that the blanking force calculated under the condition (22) represents approximately the lower limit of the exact solution. Because the relative sliding velocity of the material to the front of tool is negligible.

Fig. 5 Typical stress fields ($\mu_s=1.0$)

Fig. 6 The direction of crack development

Fig. 4 shows the relation between blanking force P and punch penetration t . In the case of $\mu_s=1.0$ and $1.0 \geq \mu_f \geq -1.0$ the points A, B and C represent the solutions for $\mu_f=-1.0, 0$ and 1.0 , respectively. When the apparent coefficient of friction μ_f is negative, the frictional stress T_f is positive, and T_f is negative when μ_f is positive. The variation of μ_f in Fig. 4, therefore, suggests that the relative sliding velocity of the material to the front of the tool changes the direction with punch penetration. The maximum value of blanking force occurs in the range of $\mu_f < 0$, and decreases with increasing clearance. When the front and the side of tool edges are perfectly smooth ($\mu_f = \mu_s = 0$), the blanking force is smaller than that when a rough tool is used.

(b) Typical stress field The stress field on the point that are expressed by signs A, B and C are illustrated in Fig. 5. In the figure, the chain line represents the line across which the maximum principal stress is transmitted, and the broken line represents the first shear line.

We now denote by ζ the angle of inclination of the chain line to the z axis. Then, the angle ζ is plotted against punch penetration t as shown in Fig. 6 comparing with the angle α .

(c) Direction of crack development In the blanking of sheet metals cracks develop from the tool edges, and may cause ultimate rupture.

We shall now explain the direction of crack development on the assumption that cracks develop along the principal direction represented with the chain lines in Fig. 5 and Fig. 6

The mean normal stress σ_m acting in the material near tool edges can be calculated from the following equations.

In the region ② of Fig. 2

$$\sigma_m = 2k\omega_2 = -k + 2k \cos \frac{\phi_1 - \phi_3}{2} \dots\dots\dots(24)$$

In the region I of Fig. 3

$$\sigma_m = -k \dots\dots\dots(25)$$

Fig. 7 gives the variation of σ_m as the function of punch penetration t . The mean normal stress σ_m is high pressure equal to the yield shearing stress k at the early stages of blanking, and then with increasing punch penetration the absolute value of σ_m decreases.

We shall therefore assume that cracks initiate when the mean normal stress becomes zero. Then, the point of crack initiation may be plotted as circles on the ζ curves and the α curves in Fig. 6.

Cracks develop in the direction of $\zeta > \alpha$ for small clearance, and then in the direction of $\zeta < \alpha$ for large clearance. When clearance $c \approx 0.15$, we obtain $\zeta = \alpha$, that is, the direction of crack developed from the edge of punch coincides with that developed from the edge of the die.

The above mentioned theoretical results in the direction of crack development are in good agreement with the experimental results with sheet metals.

Part II. The kinematically admissible velocity field

2.1 Basis of analysis

Analysis of the kinematically admissible velocity field was performed under the assumptions and the boundary conditions similar to those adopted in Part I.

In the theory of limit analysis, the kinematically admissible velocity implies such velocity that fulfills the condition of incompressibility and the velocity boundary conditions.

When the tool edges are perfectly smooth or perfectly rough, a second extremum principle is represented as follows: The exact solutions of blanking force can not be larger than the blanking force calculated using the relation that the total amount of the rate of energy dissipation in the kinematically admissible velocity field is equal to the rate of energy supply for the punch.

2.2 Deformation processes of a sheet material in the blanking

From an experimental investigation on the blanking of a lead plate, it is found that the blanking deformation process of nearly ideally ductile metal may be divided into four stages as follows:

The first stage: Simple shearing deformation occurs in the material in the clearance between punch and die. So a part of the free surface of the material near tool edges deforms and becomes an obtuse plane. The second stage: As the tool

penetrates into the material, the material which contacts with the side of tool is polished and becomes a bright plane. The third stage: The part of the material near the side of tool is extended in the direction of punch penetration. Therefore, it is contracted in the perpendicular direction and a depression plane is formed. The fourth stage: Separation occurs in the material.

To analyse these blanking deformation processes, we shall now assume the four types of kinematically admissible velocity field corresponding to each stage as shown in Fig. 8.

(i) The first stage The velocity field in the first stage is assumed as Fig. 8 (a). Simple shearing deformation occurs in the plastic region I, and the rigid region II is stationary, and III moves

Fig. 7 Mean normal stress acting in the material near tool edges

(a): the first stage
(b): the second stage
(c): the third stage
(d): the fourth stage

Fig. 8

with a unit velocity, so a part of the free surface of the plastic region I deforms and becomes an obtuse plane.

Let us take Cartesian coordinates as shown in Fig. 8. Velocity components v_x and v_z will be considered positive if they have the direction of the x axis and the z axis, respectively. Then the velocity distribution in the first stage is expressed by

$$\left. \begin{array}{ll} \text{I} & v_x = -x/c, \quad v_z = 0, \\ \text{II} & v_x = 0, \quad v_z = 0, \\ \text{III} & v_x = -1, \quad v_z = 0 \end{array} \right\} \dots\dots\dots (26)$$

where c is the tool clearance.

(ii) The second stage The velocity field in the second stage is assumed as Fig. 8 (b). The plastic region consists of the regions of constant stress state (right angled triangle) and the regions of centered fan.

The velocity v_1 along the first shear line is taken positive if v_1 has the direction of $\vec{OA}$, $\vec{OCO'}$, $\vec{DO'}$, etc. The velocity v_2 along the second shear line is taken positive if v_2 has the direction of $\vec{EDCAB}$. We denote furthermore by θ the angle of inclination of the first shear line; θ is taken positive in the counterclockwise rotation.

Assuming that the rigid region II is stationary and III moves with the downward velocity V . Then, the velocity distribution can be represented by

$$\left. \begin{array}{ll} \text{II} & v_x = 0, \quad v_z = 0, \\ \text{III} & v_x = 0, \quad v_z = -V, \\ \text{OAB} & v_1 = V \cos \beta, \quad v_2 = -V \sin \alpha - V \sin \beta, \\ \text{OAC} & v_1 = V \cos \theta, \quad v_2 = -V \sin \alpha - V \sin \theta, \\ \text{O'CD} & v_1 = 0, \quad v_2 = V \sin \alpha, \\ \text{O'DE} & v_1 = 0, \quad v_2 = V \sin \alpha \end{array} \right\} \dots\dots\dots (27)$$

where α and β denote the positive angles of inclination of $\vec{OCO'}$ and $\vec{OA}$ to the z axis.

Since the velocity component v_z must be zero in the triangle O'DE contacted with the front of a die, $v_z = -V \sin \alpha \sin \beta$ must be added to the whole velocity distribution of (27). Besides, $v_z = -1$ in the triangle OAB contacted with the front of punch. Thus, we obtain

$$\left. \begin{array}{ll} \text{II} & v_x = v_z = 0, \\ \text{III} & v_x = 0, \quad v_z = -1, \\ \text{OAB, OBC, OCO'C'} & v_1 = U, \quad v_2 = 0, \\ \text{O'A'B'} & v_1 = -1 + U + \frac{1}{\sqrt{2}}, \quad v_2 = \frac{1}{\sqrt{2}}, \\ \text{O'B'C'} & v_1 = -1 + U + \sin\left(\theta - \frac{\pi}{2}\right), \quad v_2 = \cos\left(\theta - \frac{\pi}{2}\right) \end{array} \right\} \dots\dots\dots (32)$$

where U may assume an arbitrary value within the range from zero to one.

Across the plastic-rigid boundaries, the jumps of tangential velocity are

$$\left. \begin{array}{ll} \text{ABCO'} & \Delta v = U \\ \text{A'B'C'O} & \Delta v = 1 - U \end{array} \right\} \dots\dots\dots (33)$$

$$V = 1/(1 + 2 \sin \alpha \sin \beta) \dots\dots\dots (28)$$

After all, the velocity distribution in the second stage is given as follows:

$$\left. \begin{array}{ll} \text{II} & v_x = 0, \quad v_z = -V \sin \alpha \sin \beta, \\ \text{III} & v_x = 0, \quad v_z = -V(1 + \sin \alpha \sin \beta), \\ \text{OAB, OAC} & v_2 = V \sin \alpha, \\ & v_z = -V(1 + \sin \alpha \sin \beta), \\ \text{O'DE, O'CD} & v_2 = V \sin \alpha, \quad v_z = -V \sin \alpha \sin \beta \end{array} \right\} \dots\dots\dots (29)$$

The tangential components of velocity are discontinuous across OCO' , EDCAB and also the tool surfaces. The jump is given by

$$\left. \begin{array}{ll} \text{OCO'} & \Delta v = V \cos \alpha, \\ \text{EDCAB} & \Delta v = V \sin \alpha, \\ \text{The front of a tool} & \Delta v = V \sin \alpha \cos \beta, \\ \text{The side of a tool} & \Delta v = V(1 + \sin \alpha \sin \beta) \end{array} \right\} \dots\dots\dots (30)$$

The triangles OAB and O'DE move in the direction of $\vec{BA}$ and $\vec{ED}$ with the relative velocity $\Delta v = V \sin \alpha$ to the rigid regions II and III. Thus, the indentations OB and O'E occur. Since the tool edges hold the rigidity, the length of indentation, i.e., a has to be constant in the second stages. From the condition of $a = \text{constant}$, we obtain the following relation:

$$a = c/2 \sin \alpha \sin \beta = \text{constant} \dots\dots\dots (31)$$

On the other hand, the material contacted with the side of a tool is polished by the tool edge, and a bright plane is formed.

(iii) The third stage The velocity field in the third stage is assumed as Fig. 8 (c). The plastic region consists of the region of constant state, i. e., two isosceles triangles and a rectangle, and the region of centered fan. The velocity v_1 along the first shear line is taken positive when v_1 has the direction of $\vec{ABCO'}$ etc. And the velocity v_2 along the second shear line is taken positive when v_2 has the direction of $\vec{BO}$, $\vec{CO}$, $\vec{O'C'}$, $\vec{O'B'}$, etc.

Then the velocity distribution in the third stage can be solved in a similar manner to the second stage and is represented by

Since the triangles OAB and O'A'B' move in the x direction at the velocities $v_x = -U/\sqrt{2}$ and $v_x = (1-U)/\sqrt{2}$, respectively, the length of apparent clearance OC, namely g decreases with the speed of $1/\sqrt{2}$, and a depression plane is formed. g is given by

$$g = c - (t - t_{23})/\sqrt{2} \quad (34)$$

where t_{23} is the punch penetration when the blanking process enters the third stage.

(iv) The fourth stage When the punch penetration t reaches $t_{34} (= \sqrt{2}c + t_{23})$, the apparent clearance g becomes zero. When t exceeds t_{34} , the velocity field in the material is expressed as Fig. 8(d). It is the fourth stage. As the shearing strain rate along the line OO' is infinite, ultimate rupture may take place as soon as the fourth stage is reached.

2.3 Analysis of blanking force

The calculation of blanking force acting on a unit length of the circumference of the punch for each punch penetration. For simplicity it is assumed that the tool edges are perfectly smooth.

(i) The first stage For any clearance, the blanking force is given by

$$P = -k \quad (35)$$

(ii) The second stage In the region of centered fan OAC and O'CD, the maximum shearing strain rate on the point which has a distance r from the center is represented by

$$\dot{\gamma} = V \sin \alpha / r \quad (36)$$

Then the rate of energy dissipation in two fans is

$$\begin{aligned} \int_0^R k_{OAC} \dot{\gamma} (\alpha + \beta) r \cdot dr + \int_0^R k_{O'CD} \dot{\gamma} (\alpha + \beta) r dr \\ = \frac{k_{OAC} + k_{O'CD}}{2} Vc(\alpha + \beta) \quad (37) \end{aligned}$$

where R is the radius of the fan, and k_{OAC} means the yield shearing stress in the region OAC (that is represented by the suffix).

Furthermore the rates of energy dissipation along the lines of velocity discontinuity are given as follows:

$$\left. \begin{aligned} \text{OCO'} & \quad k_{OCO'} Vc \cot \alpha, \\ \text{AB and DE} & \quad \frac{k_{AB} + k_{DE}}{2} Vc, \\ \text{AC and CD} & \quad \frac{k_{AC} + k_{CD}}{2} Vc(\alpha + \beta) \end{aligned} \right\} \quad (38)$$

The rate of energy supply for the punch, i.e., $P \cdot 1$, is equal to the total amount of these rates of energy dissipation in the sheet material. Therefore, the blanking force is represented by

$$\begin{aligned} P = -\frac{Vc}{2} \{ (k_{OAC} + k_{O'CD} + k_{AC} + k_{CD})(\alpha + \beta) \\ + (k_{AB} + k_{DE}) + 2k_{OCO'} \cot \alpha \} \quad (39) \end{aligned}$$

In Eq. (38), β can be determined through Eq. (31), and α is expressed with a function of punch penetration.

$$\cot \alpha = (1-t)/c \quad (40)$$

(iii) The third stage The blanking force in the third stage can be solved in a similar way to that in the second stage as follows:

$$\begin{aligned} P = -g \{ k_{AB} \cdot U + k_{A'B'}(1-U) \\ + \frac{\pi}{4} [(k_{BC} + k_{OBC})U + (k_{B'C'} + k_{O'B'C'})(1-U)] \} \\ - k_{CO'}(1-t) \quad (41) \end{aligned}$$

In Eq. (41), g shows the apparent clearance defined by Eq. (34).

The blanking force may be calculated in another way from the equation of equilibrium (Hencky's relation). In this case, the stress components in the triangles OAB and O'A'B' are expressed by

$$\sigma_x = 0, \quad \sigma_z = 2k, \quad \tau_{xz} = 0 \quad (42)$$

And the blanking force is

$$P = -k \left[\left(1 + \frac{\pi}{2} \right) g + (1-t) \right] \quad (43)$$

Eq. (43) coincides with Eq. (41) when the yield shearing stress is uniform in the sheet material.

(iv) The fourth stage Substituting the relation of $g=0$ into Eq. (43), the blanking force is given by

$$P = -k_{CO'}(1-t) \quad (44)$$

2.4 Results

(a) Blanking force The relations between blanking force and punch penetration calculated from Eqs. (35), (39), (43) and (44) are shown in Fig. 9. In the calculation, it is assumed that the yield shearing stress is uniform in the material, and moreover

Fig. 9 Blanking force curves

the length of tool indentation a is equal to 0.5 (one-half of the sheet thickness).

In the case of null clearance, the whole deformation process in the blanking is represented with the velocity field of the second stage. Except for the case of null clearance, the blanking force curves can be divided into four stages. In the first stage, the blanking force is constant. When the second stage starts, the blanking force begins to decrease, and then decreases rapidly in the third stage. The blanking force curves in the fourth stage are equal to that for null clearance.

(b) Sheared surface of the blank The sheared surface of the blank consists of "obtuse plane", "bright plane", "depression plane" and "shear fractured plane", as is shown in Fig. 10. The length of these planes varies with the tool clearance as shown in Fig. 11.

(1) The obtuse plane l_1 is given by

$$l_1 = t_{12} \quad (45)$$

where t_{12} denotes the punch penetration on the transition point from the first stage to the second stage; l_1 is formed completely in the first stage, and

Fig. 10 Sheared surface of the blank

Fig. 11 Effect of tool clearance on sheared surface

increases with increasing clearance.

(2) The bright plane l_2 is given by

$$l_2 = t_{23} - t_{12} - \sqrt{2}c \quad (46)$$

where $(t_{23} - t_{12})$ is the length of the material polished by the side of the tool in the second stage, and $\sqrt{2}c$ is a part of the polished plane that is depressed in the third stage. The sheared surface with null clearance is composed of only the bright plane.

(3) The depression plane l_3 develops in the third stage, and is given by

$$l_3 = t_{34} - t_{23} + \sqrt{2}c \quad (47)$$

l_3 increases with clearance. The depth of depression is equal to $c/2$ when $U=0.5$.

(4) The shear fractured plane l_4 decreases with clearance, however, becomes zero in the case of null clearance. The shear fractured plane and the tool indentation are given by

$$l_4 + l_5 = 1 - t_{34} \quad (48)$$

2.5 Comparison between the statically admissible stress field and the kinematically admissible velocity field

(a) Supplement of the obtuse plane in the stress field To compare the solutions of the stress field with that of the velocity field, it is necessary to supplement the obtuse plane in the stress field as shown in Fig. 12. We shall now assume that the regions I, II, III are in the same state as described in Fig. 2 and Fig. 3, and moreover that the stress components in the regions IV and V are equal to zero.

Then, the quantities that represent the blanking characteristics may be transformed as follows.

$$\left. \begin{aligned} t &= t'(1-l_1) + l_1, & c &= c'(1-l_1), \\ P &= P'(1-l_1), & Q &= Q'(1-l_1) \end{aligned} \right\} \quad (49)$$

The relation between c and l_1 is obtained from Fig.

Fig. 12 Supplement of obtuse plane

11. Therefore, the relation between c' and l_1 can be calculated by solving the second equation of (49) graphically.

(b) Comparison of blanking force The supplement of the obtuse plane is performed on the solution of stress field which corresponds to the black spot in Fig. 4, and the result is shown in Fig. 13 comparing with the solution of velocity field.

In the case of null clearance of the velocity field, the line OO' connecting both edges of punch and die coincides with the first shear line, and the plastic strain rate is non-zero only on the line OO' . Hence, the solution of stress field that the line OO' coincides with the first shear line gives an exact solution of blanking force. When $c=0$ and $t=0.1716$, the solution of stress field fulfills the above condition. When $c=0$ and $t>0.1716$, exact solutions can be obtained as Fig. 14. The stress field of Fig. 14 consists of the plastic region (the solution for $c=0$ and $t=0.1716$ represented in Fig. 13) and the attached rigid region (shaded in Fig. 14) where all stress components are zero.

Therefore, an exact solution of blanking force is given by

$$P = -k(1-t) \quad \text{when } c=0, t \geq 0.1716 \dots (50)$$

In the case of $c=0$ and $t < 0.1716$, and another case except for null clearance, an exact solution of blanking force could not be obtained, because the solution of stress field associated with that of velocity field could not be found.

Conclusions

The author applied the theory of limit analysis to a theoretical investigation of the blanking process of a sheet material, and obtained the following results.

Part I. (1) The sheet material was separated into several regions of constant stress by using the straight lines of discontinuity, and the statically admissible stress field was analysed. (2) The lower limit of blanking force against each punch penetration was calculated. (3) The direction of crack development of sheet metals in the blanking process was well explained.

Part II. (4) The plastic deformation of a sheet material in the blanking process was divided into four stages, and the kinematically admissible velocity field in the material was analysed for each stage. (5) The upper limit of blanking force against punch penetration was calculated. (6) The sheared surface of the blank with usual clearance was confirmed to be composed of an obtuse plane, a bright

Fig. 13 Comparison between the stress field and the velocity field (for smooth tool)

Fig. 14 An exact solution of the stress field

plane, a depression plane and a shear fractured plane.

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