Geschwindigkeiten und die Beschleunigungen lösen, ohne die Glieder sich einander zu abwechseln.

Wenn die Konstruktion ungemein groß würde und die Konstruktionslinien von dem Papier aus gingen im Falle, daß das Glied *b* Antriebsglied ist, muß man das Glied *d* als Antriebsglied aufnehmen, und nach der bereits bei Abb. 12 erörterten Methode lösen.

5.3 Die Formen C und D

Im Falle, daß das Glied *a* Festglied ist, hält es hart, daß man das Glied *b* oder *f* als Antriebsglied aufnimmt und die Beschleunigungen löst.

Angenommen, daß eins von ternären Gliedern Festglied sei, darf man dann von dem dieses Glied enthaltenen Viergelenkgetriebe die Konstruktion anfangen. Eins von den an das ternäre Glied gegrenzten zwei Gliedern muß als Antriebsglied gewählt werden. Wenn man das Glied, das zu lang ist, als Antriebsglied aufnimmt, wird die Konstruktion leicht übermäßig werden. Man muß aufmerksam auf es sein.

6. Schlüsse

Die Verfasser haben das neue Verfahren für die Beschleunigungen des Viergelenkgetriebes im Falle, daß das Antriebsglied gleichförmig rotiert, vorgeschlagen. Die Verfasser haben das vorerwähnte Verfahren bis zum Verfahren für die Beschleunigungen im eine Bewegungsfreiheit besitzenden ebenen Gelenkgetriebe für den Fall, daß das Antriebsglied oder andres, beliebiges Glied die angegebene Geschwindigkeit und Beschleunigung hat, entwickelt.

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Research on Externally Pressurized Journal Gas Bearing*

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A theoretical research on externally pressurized journal gas-bearing has been reported by T. Sasaki and H. Mori. In their paper, the experimental data employed in comparison with the theoretical results were for the bearing with a single gas supply hole without restriction. It is common to use externally pressurized gas-bearings with multiple gas supply holes arranged circumferentially and axially. In this paper, a gas-bearing with multiple gas supply holes is designed theoretically and the pressure distribution, flow rate and optimum restriction are examined and compared with the complex potential theory.

The experimental results show good agreement with the theoretical results.

1. Introduction

A theoretical research on externally pressurized journal gas bearings has been reported by T. Sasaki and H. Mori. In their paper, the experimental data

employed in comparison with the theoretical results were for the bearing with a single gas supply hole without restriction. Today it is common to use externally pressurized gas-bearings with multiple gas supply holes arranged circumferentially and axially. In this paper, a gas-bearing with multiple gas supply holes is designed theoretically, and the pressure distribution, flow rate and optimum restriction are examined and compared with the complex potential

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theory advanced by T. Sasaki and H. Mori before.

Nomenclature

- B : distance of supply holes in complex field
 c_r : radial clearance
 $c_{r\text{opt}}$: optimum radial clearance
 c_f : discharge coefficient
 D : diameter of shaft
 h : bearing clearance
 K : constant to be determined by bearing shape and size
 L : length of bearing
 M : mass flow rate
 n : polytropic index
 p : pressure
 p_0 : pressure in gas supply hole
 p_a : ambient pressure
 p_s' : pressure just after flow meter
 $p_{sg}(p_s)$: supply pressure (gauge)
 Q' : volume flow rate measured by rota-meter
 r_0 : radius of gas supply hole
 R_0 : radius of gas supply hole in complex field
 x, y : co-ordinates
 e : eccentricity ratio
 ξ, η : co-ordinates in complex field
 θ : angular co-ordinate
 κ : adiabatic index
 ρ : density at ordinary temperature
 ρ_a : density of air at ambient pressure
 ρ_s : density of air at supply pressure

2. Experimental equipment

The journal gas bearing employed in the experiment is 28.057 mm in inner diameter and 74 mm in length with 12 gas supply holes arranged circumferentially and axially as shown in Fig. 1. Each hole has its own nozzle-type restriction. As the chief object of this research lies in full and minute examination of the theory, the shafts used in the experiment are assumed to be immobile. The shafts have two holes bored from both ends as shown

Fig. 1

in Fig. 1, which serve to measure continuously the circumferential and axial pressure distribution respectively by turning the shafts slowly and by moving them axially. Fig. 2 illustrates schematically the experimental equipment for this research, where the gas supply pressure just after restriction can also be measured from one side of the bearing.

The bearing size and hole position shown in Fig. 1 were not decided theoretically but considered to be suitable from experience. Theoretically, the optimum clearance ($c_{r\text{opt}}$) can be given approximately by the following formula regarding the gas as an incompressible fluid.

$$c_{r\text{opt}} = \sqrt[6]{\frac{9.66c_f^2\pi^2r_0^4}{4K^2p_s\rho}} \quad \dots\dots\dots (1)$$

where,

- c_f : discharge coefficient at nozzle
 r_0 : nozzle radius
 p_s : gas supply pressure (gage)
 ρ : gas density at ordinary temperature
 K : constant to be determined by bearing shape and size, diameter of gas supply hole after restriction and gas viscosity

From the above formula, a non-eccentric clearance which is expected to provide maximum stiffness can be obtained for this experiment by deciding the shaft diameter properly. Now, when c_f is given as 0.6, $2r_0$ as 0.5 mm, p_s as 3 kg/cm², ρ as 1.21×10^{-9} kg sec²/cm⁴ (air) and the diameter of the hole after restriction as 2 mm, K is calculated as 6.28×10^8 cm²/kg, from which $2c_{r\text{opt}}$ is obtained as 75 μ . So as to make the clearance closest to the theoretically optimum one, two shafts were prepared by precise and careful grinding; one with a diameter of 27.978 mm and the other with 27.999 mm. But, when these two shafts were placed in position, $2c_r$ became 79 μ for the former shaft and 58 μ for the latter. In the latter case, the clearance was considered too small. Hereafter these two clearances will be called $c_r = 40 \mu$ and $c_r = 30 \mu$.

The load on the bearing is fixed at 2.5kg, which corresponds to the weights of bearing and housing.

Fig. 2

3. Fundamental theory

The gas pressure distribution in an externally pressurized journal gas bearing, which evolves on such complex plane as is shown in Fig. 3, is already known from the complex potential function of flow supposing that sources are placed instead of gas supply holes. Namely, taking up the part enclosed by the dotted line in Fig. 3 in correspondence to non-eccentricity of the shaft, the gas pressure here can be given by the following formula when gas flows isothermally,

$$p^2 = \sum_{k=1}^2 K_k \ln \{ e^{(\xi - \xi_k)} + e^{-(\xi - \xi_k)} - 2 \cos(\eta - \eta_k) \} + K_3 \dots \dots \dots (2)$$

where ;

$$\xi = 2\pi x/B$$

$$\eta = 2\pi y/B$$

When the position of sources, hole radius and boundary conditions are given as below ;

$$\left. \begin{aligned} \xi_1 = -\xi_2 = \pi L'/B, \quad \eta_1 = \eta_2 = 0 \\ \xi_0 = 2\pi R_0/B \end{aligned} \right\}$$

$$\left. \begin{aligned} \xi = \xi_1 + \xi_0, \quad \eta = 0 : p = p_0 \\ \xi = \xi_a = \pi L/B, \quad \eta = 0 : p = p_a \end{aligned} \right\}$$

K_1 , K_2 and K_3 in Eq. (2) are obtained as below ;

$$K_1 = K_2 = \frac{p_0^2 - p_a^2}{\ln \left[\frac{\{e^{(2\xi_1 + \xi_0)} + e^{-(2\xi_1 + \xi_0)} - 2\} \{e^{\xi_0} + e^{-\xi_0} - 2\}}{\{e^{(\xi_a + \xi_1)} + e^{-(\xi_a + \xi_1)} - 2\} \{e^{(\xi_a - \xi_1)} + e^{-(\xi_a - \xi_1)} - 2\}} \right]} \dots \dots \dots (3)$$

$$K_3 = \frac{K_1}{p_0^2 - p_a^2} \left[\ln \frac{[\{e^{(2\xi_1 + \xi_0)} + e^{-(2\xi_1 + \xi_0)} - 2\} \{e^{\xi_0} + e^{-\xi_0} - 2\}] p_a^2}{[\{e^{(\xi_a + \xi_1)} + e^{-(\xi_a + \xi_1)} - 2\} \{e^{(\xi_a - \xi_1)} + e^{-(\xi_a - \xi_1)} - 2\}] p_0^2} \right] \dots \dots \dots (4)$$

Based on the pressure distribution obtained as before for a non-eccentric shaft, the pressure distribution can also be known approximately in the case of a slightly eccentric shaft as shown in Fig. 4, by only observing the change of gas supply pressure p_0 to be caused by the eccentricity. Namely, Fig. 4 shows a pressure distribution curve drawn exemplarily on the supposition that the clearance at a gas supply hole is kept parallel up to 60° on both circumferential sides of each hole.

The mass rate of flow which corresponds to the pressure distribution obtained by Eq. (2) is expressed as follows :

$$M = \frac{1}{2} K h^3 (p_0^2 - p_a^2) p_a^{-1} \rho_a \dots \dots \dots (5)$$

where, if h is given as the clearance at the gas supply hole, the flow in the part enclosed by the dotted line in Fig. 3 is obtained. This is equal to the rate of flow supplied adiabatically through the nozzles of two supply holes.

On the other hand, assuming that the nozzle is an orifice restriction ;

$$M = 2c_f \pi r_0^2 \left[\frac{2\kappa}{\kappa - 1} p_s \rho_s \left\{ \left(\frac{p_0}{p_s} \right)^{2/\kappa} - \left(\frac{p_0}{p_s} \right)^{(\kappa+1)/\kappa} \right\} \right]^{1/2} \dots \dots \dots (6)$$

where,

κ : ratio of specific heats of the gas

p_s : gas pressure before restriction

ρ_s : gas density before restriction

Fig. 3

Fig. 4

and the following is assumed :

$$p_s \rho_s^{-\kappa} = \rho_0^{-\kappa} p_0$$

$$p_0 \rho_0^{-1} = \rho_a^{-1} p_a$$

Now, considering that both M in Eqs. (5) and (6) are equal,

$$h^6 = \frac{4 \frac{A^2}{K^2} \frac{\kappa}{\kappa-1} p_s^2 p_a \left\{ \left(\frac{p_0}{p_s} \right)^{(\kappa+1)/\kappa} - \left(\frac{p_0}{p_s} \right)^{2/\kappa} \right\}}{(p_0^2 - p_a^2)^2} \dots\dots\dots (7)$$

where

$$A = 2c_f \pi r_0^2 \sqrt{2/\rho_a}$$

By the above Eq. (7), the relation between bearing clearance h and gas supply pressure p_0 can be obtained. When the discharge coefficient is defined

Fig. 5

Fig. 6

as in Eq. (6), it generally comes to be more than 0.6 in the case of compressible flow at high pressure. Fig. 5 shows the discharge coefficients c_f for the nozzles employed in this experiment obtained from the result of flow measurement using Eq. (6). As p_0/p_s was 0.7 to 0.9 in the actual bearing operation, 0.80 can properly be taken as c_f for calculation. Therefore, the relation represented by Eq. (7) is drawn as shown in Fig. 6, from which the pressure in each gas supply hole can be obtained by giving clearance c_r and eccentricity ϵ . This graph also serves to prove that 30μ is too small and 40μ is appropriate as clearance.

4. Examination on experimental results

4.1 Pressure in gas supply hole p_0

The air pressure after restriction measured from a bearing side differs from the value measured through the holes in the shaft. This difference corresponds to the dynamic pressure which is not measured in the former case, because the latter stands for the pressure at the stagnation point. More strictly, in consideration of the variation of the dynamic pressure just after restriction, at the stagnation point and at the annular part where gas flows out into the clearance, the pressure should be determined separately. But, as far as this experiment is concerned, neither experimental result nor the theoretical calculation has shown much difference in pressure among respective points above mentioned. Consequently, the value measured from a bearing side is used as the pressure in the gas supply hole and plotted in Figs. 7 and 8. Several broken lines represent the change of the pressure in the gas supply hole obtained from Eq. (7) by giving various eccentricities ϵ on the assumption of $h = c_r(1 - \epsilon \cos \theta)$.

Although the eccentricity should be found directly

Fig. 7

by calibrating the clearance, no precise measuring method has been available so far. Therefore, various eccentricities were assumed and the one which showed the best accordance with the experimental results was selected out of them for theoretical calculation on the flow rate and pressure distribution. Judging from Figs. 7 and 8, Eq. (7) can be considered to have good conformity with the actual results of experiment.

4.2 Flow rate

When the gas pressure in each gas supply hole situated at every hexasectional point on the circumference has been given by assuming the eccentricity as above, the total flow can be obtained from Eq. (5) as follows:

$$M = \sum_{k=1}^6 \frac{1}{2} K h_k^3 (p_{0k}^2 - p_a^2) p_a^{-1} \rho_a \dots \dots \dots (8)$$

where k is 1 to 6 corresponding to each hole position.

On the other hand, the flow rate can be measured with the rota-meter shown in Fig. 2. When the value determined by the rota-meter is given as Q' l/min, and the air pressure through the meter as p_s' , and the air is assumed to show a polytrope change (index: n) in flowing from the compressor to the meter, the flow rate can be expressed in mass as

follows:

$$M = Q' p_s'^{(2-n)/2n} \rho_a \dots \dots \dots (9)$$

The above M can be converted into volume under the atmospheric pressure. The flow rate obtained from the experimental results by Eq. (9), converted into volume rate under the atmospheric pressure and at 15°C, is compared with the theory in Table 2. The values of p_s , p_s' and ϵ employed in this calculation are put in Table 1. The actual flow rate shows good agreement with the theoretical one when n is given as 1.1 from Table 2.

4.3 Pressure distribution

The pressure distribution was investigated varying the supply pressure as 3.60, 3.20 and 2.75 kg/cm² for $c_r=30\mu$ and as 3.35, 3.00 and 2.60 for $c_r=40\mu$. Though the experiment was so arranged that the pressure might become 4.0, 3.5 and 3.0 kg/cm² just after the flow meter for both 40 μ and 30 μ clearances as shown in Table 1, a little difference in p_s value was occasioned by the resistance existing in the passage from the meter to the supply hole in the bearing. An example of the experimental result obtained at the pressure of 3.00 kg/cm² for $c_r=40\mu$ and of 2.75 kg/cm² for $c_r=30\mu$ is shown in Figs. 9 and 10 respectively. Solid lines in the graphs are drawn theoretically using the supply pressures at 0°, 60° (300°), 120° (240°) and 180° calculated from the eccentricity shown in Table 1.

For such points as 30° (330°) and 90° (270°) intermediate between the two supply holes arranged circumferentially, two theoretical lines are drawn because the theoretical pressure distribution curve is given discontinuously at these points as shown in

Fig. 8

Table 1

Radial clearance	Supply pressure kg/cm ²	Pressure after rota meter kg/cm ²	Eccentricity
$c_r=30\mu$	3.60	4.0	0.15
	3.20	3.5	0.20
	2.75	3.0	0.35
$c_r=40\mu$	3.35	4.0	0.10
	3.00	3.5	0.12
	2.60	3.0	0.15

Table 2

Radial clearance	Supply pressure kg/cm ²	Volume flow rate (Theoretical) l/min	Volume flow rate (Experimental) l/min		
			$n=1.0$	$n=1.1$	$n=1.2$
$c_r=30\mu$	3.60	34.2	41.0	35.3	31.7
	3.20	27.3	33.7	30.0	27.3
	2.75	22.6	26.3	23.8	21.9
$c_r=40\mu$	3.35	51.9	56.6	50.0	44.4
	3.00	43.6	45.8	40.9	37.2
	2.60	33.8	37.2	33.7	31.3

Fig. 4.

The general tendency seen in pressure distribution is that the experimental results accord very well with the theory in the part where gas flows outwards (towards bearing ends) from the gas supply holes, but not so well between two holes arranged axially. Namely, the actual results appear lower

than the theoretical curve at 0° and 120° (240°) while higher at 120° (240°) and 180° . This difference is considered to be derived from the fact that, while the theory is based on the supposition of dividing the clearance into respectively parallel six parts and joining them together, the actual clearance is not parallel but diverge gradually towards 90° and 240°

Fig. 9

from the loading side ($\theta=0^\circ$) and the reverse is the case on no-load side ($\theta=180^\circ$) owing to the eccentricity of the shaft. This explanation is supported by the fact that the experimental results appear between two theoretical curves at $90^\circ(270^\circ)$ where the clearance is considered to be parallel.

In order to bridge the gap between the theory and the experimental result, it is necessary to alter the arrangement of strength of gas sources on the ordinates (y coordinates) in Fig. 3 according to the gas pressure p_0 in respective supply holes or use another potential function in accordance with the

Fig. 10

difference in clearance in addition, instead of such approximation as in Fig. 4. But in this case, the calculation gets more complicated because it requires more constants to be determined by the boundary conditions contained in Eq. (2). So the simpler method is taken in this research for convenience. Nevertheless, this method could prove satisfactorily that the pressure distribution in an externally pressurized journal gas bearing can be given by the complex potential functions to be obtained from the arrangement of sources, and is considered to be of much use in determining the pressure p_0 in the supply hole, or in other words, in designing the restriction.

4.4 Load carrying capacity

As regards the load carrying capacity, comparison was made between the average pressure on the shaft p' under the fixed load, under which the experiment was carried out, and the gage pressure p_{sg} of gas supply pressure p_s . The change of p'/p_{sg} is shown in Fig. 11 with an eccentricity given on the horizontal axis. Though the load capacity is not the chief object of this research, just a few data obtained have shown well that $c_r=30\mu$ is inappropriate in connection with the design of restriction since the load carrying capacity has appeared much lower for $c_r=30\mu$ than for $c_r=40\mu$.

5. Conclusion

It was the chief object of this research to experimentally prove the theory reported before. All the above experimental results may well be considered to have established that the flow rate and pressure distribution in an externally pressurized

Fig. 11

journal gas bearing with multiple gas supply holes and restrictions can be given with due precision. This experiment has also made it clear that the bearing with the best load carrying capacity can be designed by determining the nozzle size and clearance in accordance with the theory. Further research will be continued hereafter on the case of a larger clearance in which the pressure distribution in the part where gas flows out from the supply hole into the clearance is still left unknown.

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