A Study on the Hydrodynamic Instability in Boiling Channels*
(2nd Report, The Instability in Two Parallel Boiling Channels)

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An investigation is carried out on the hydrodynamic instability in a system having two parallel boiling channels. The basic equations for the circulation flow rates and the driving heads are derived from the laws of conservation of mass, energy and momentum.

The stability of equilibrium in a hydrodynamic system and the modes of vibration are discussed. As the result, the following facts are found. (a) A hydrodynamic system is always stable when the equilibrium of the system lies in the range of positive inclination of the characteristic curve for each channel and (b) always unstable when the equilibrium is in the range of negative inclination of the characteristic curve for each channel; (c) when the equilibrium lies in the range of positive inclination for one channel and in one of negative inclination for the other channel, the system is either stable or unstable; (d) there are two normal modes of vibration in the system having two parallel boiling channels. The first is a mode in which each channel vibrates in the same phase and the second is one in which each channel vibrates in the opposite phase; (e) in the case of (b) the second mode is always unstable and a throttle or a pump in the downcomer is not effective to cause positive damping to the second mode, and (f) in the system with parallel channels there may be many equilibriums, so that non-uniform distributions of flow rates may occur.

1. Introduction

In the first report(1) an analysis was carried out on the hydrodynamic instability in a single boiling channel and it was shown that the hydrodynamic instability is a self-excited oscillation due to the dynamical instability of the equilibrium in the hydrodynamic system. In practical systems, however, most of them consist of parallel boiling channels, so that it is important to study the stability of a system with parallel boiling channels.

Several papers(2)(3)(4) are presented on the hydrodynamic instability in parallel boiling channels, concerning the stability of the flow circulation in boilers, in which the stability of the system is discussed from the statical point of view and dynamic behaviors are not considered. Wallis and Heasley(5) treated the hydrodynamic instability from the dynamical point of view and discussed the stability by Nyquist’s criterion, but they did not refer to what behaviors were expected to occur.

In Japan Akagawa(6) reported the interesting experimental results obtained from the system having four boiling channels, but he has not presented a theoretical analysis.

In this paper, as the simplest case of parallel boiling channels, the system having two boiling channels as shown in Fig. 1 is treated and the stability of the system and its dynamic behaviors are discussed theoretically, by applying the theory obtained in the previous paper to that system.

2. Basic equations

Basic equations are derived from the laws of conservation of mass, energy and momentum. Assumptions for introducing the basic equations are same as in the previous paper.

Let $V_1$ and $V_2$ be the flow rates entering the channels I and II, respectively, and $V_d$ be the flow rate in the downcomer. Form the laws of conservation of mass and energy, the driving heads $H_1$ and $H_2$ are written for each channel(1)
\[ A \frac{dH_j}{dt} = \frac{Q_j}{\gamma_j h_v} \left( 1 - \frac{l_{ij}}{L} \right) - f_{s_i} \left\{ \frac{Q_j}{\gamma_j h_v} \left( 1 - \frac{l_{ij}}{L} \right) + V_j \right\}, \quad (j=1,2) \quad \text{(1)} \]

where
\[ H_j = \int_j^{e_j} f_i(x)dx, \quad (j=1,2) \quad \text{(2)} \]
\[ \frac{l_{ij}}{L} = -\frac{\gamma_i h_i}{Q_i} V_j, \quad (j=1,2) \quad \text{(3)} \]
\[ f_{s_i} = \frac{f_{s_i}}{f_{s_i} + \frac{\gamma_i h_i}{\gamma_j h_j} - \frac{1-f_{s_i}}{\gamma_j h_j} + \frac{1-f_{s_i}}{\gamma_i h_i}}, \quad (j,1,2) \quad \text{(4)} \]

and \( j \) is a suffix representing the channel I or II. The path of integration \( e_j \) in the right hand side of Eq. (2) is taken from A to B along the channel I or II. Eq. (1) is rewritten as
\[ A \frac{dH_j}{dt} = -\Phi_j(V_j, Q_j, \Delta h) \quad (j=1,2) \quad \text{(1)'} \]

where
\[ \Phi_j(V_j, Q_j, \Delta h) = f_{s_i} \left\{ \frac{Q_i}{\gamma_i h_i} - \frac{\gamma_i h_i}{\gamma_j h_j} V_j + V_j \right\} - \left[ \frac{Q_j}{\gamma_j h_j} - \frac{\gamma_j h_j}{\gamma_i h_i} V_j \right] \quad \text{(5)} \]

then \( \Phi_j \) is a function of \( Q_j, V_j \) and \( \Delta h \).

In Fig. 1 let \( p \) and \( p_0 \) be the pressure heads at A and B, and \( z_0 \) be the height between A and B, then from the law of conservation of momentum the equations governing the change of flow rates are obtained
\[ m_i \frac{dV_i}{dt} = p_0 - p - T_d(V_i) \quad \text{(6)} \]
\[ m_i \frac{dV_j}{dt} = p - p_0 - z_0 + H_j - T_d(V_j, Q_j, \Delta h), \quad (j=1,2) \quad \text{(7)} \]

where \( m_i \) and \( m_d \) are the inertia constants\((1,8)\) of the channels I, II and the downcomer, respectively.

From the continuity of flow at point A, Eq. (8) is obtained
\[ V_1 + V_2 = V_d \quad \text{(8)} \]

Eliminating \( p - p_0 - z_0 \) and \( V_4 \) from Eqs. (6), (7) and (8) yields
\[ (m_1 + m_2) \frac{dV_1}{dt} + m_d \frac{dV_2}{dt} = H_1 - T_d(V_1, Q_1, \Delta h) - T_d(V_1 + V_2) \quad \text{(9)} \]

Eqs. (1)’ and (9) are the basic equations to discuss the hydrodynamic instability in two parallel boiling channels. When a pump is included in the basic equations, these are obtained by replacing \( T_d(V_1 + V_2) \) in Eq. (9) by \( T_d(V_1 + V_2) - F(V_1 + V_2) \)

where \( F(V_1 + V_2) \) is the pump characteristics.

3. The equilibrium of a hydrodynamic system and its stability

At steady state \( \frac{dV_1}{dt}, \frac{dV_2}{dt}, \frac{dH_1}{dt} \) and \( \frac{dH_2}{dt} \) are all equal to zero. Then, letting \( \Delta p = p - p_0 - z_0 \), which is the pressure drop between A and B, these conditions are represented as
\[ \Delta p = H_1 - T_d(V_1, Q_1, \Delta h) = H_2 - T_d(V_2, Q_2, \Delta h) = T_d(V_1 + V_2) \quad \text{(10)} \]

and
\[ \Phi(V_1, Q_1, \Delta h) = 0, \quad \Phi(V_2, Q_2, \Delta h) = 0 \quad \text{(11)} \]

where \( V_1 \) and \( V_2 \) are the flow rates of the channels I and II at steady state. Eq. (10) shows that at steady state the pressure drops in the channels I, II and in the downcomer are all equal. Eq. (10) is rewritten
\[ H_1 = T_d(V_1, Q_1, \Delta h) + T_d(V_1 + V_2) \]
\[ H_2 = T_d(V_2, Q_2, \Delta h) + T_d(V_1 + V_2) \quad \text{(12)} \]

which means that the driving head in each channel is equal to the total pressure losses in the circulation loop. Eq. (11) shows that at steady state the void volume evaporating in each channel per unit time is equal to that going
out of each channel.

In order to examine whether the equilibrium is stable or not, the behavior of the system in the neighborhood of the equilibrium is considered. As mentioned in the first paper, $Q_j$ and $\Delta h$ may be assumed to be constant without losing the generality of the hydrodynamic instability, so that $\Phi_j$ and $\Tilde{\Phi}_j$ may be considered to be the functions of only the flow rate $V_1$ or $V_2$.

Now putting $v_1 = V_2 - V_{10}$, $v_2 = V_2 - V_{20}$, Eqs. (1)' and (9) are rewritten for the small disturbances of the flow rates $v_1$ and $v_2$. The right hand side of Eq. (1)' is approximated by a linear function of $v_j$, so $\Phi_j$ is expressed as

$$\Phi_j(v_1 + v_j, Q_j, \Delta h) = k_j v_j, \quad k_j = f_{j1} + (1 - f_{j1}) \frac{\partial \Phi_j}{\partial v_j}, \quad (j = 1, 2) \tag{13}$$

$k_j$ is here considered to be constant, since small disturbances near the equilibrium are considered. Expanding $\Phi_j$ and $\Phi_d$ in Taylor series near the equilibrium, $\Phi_j(V_j) + \Phi_d(V_1 + V_2)$ is represented as a function of $v_1$ and $v_2$

$$\Phi_j(v_1 + \alpha v_1 + \beta v_2) + \Phi_d(v_1 + \gamma v_1 + \delta v_2) = \Phi_j(v_1) + \Phi_d(v_1) + a_1 v_1 + a_2 v_2 + b_1 v_1 v_2 + b_2 (v_1 + v_2), \tag{14}$$

where $a_j = \frac{\partial \Phi_j}{\partial v_j} |_{V_j = V_{10}}$, $\gamma = h_j = h_j(v_1, v_2)$ is a polynomial of $v_1$ and $v_2$ including only the higher order terms of $v_1$ and $v_2$. Putting $h_j = H_j - \Phi_j(v_1) - \Phi_d(v_1) - \Phi_d(v_1 + v_2)$, basic equations are represented as

$$\begin{align*}
(m_1 + m_2) v_1 + m_3 v_2 &= h_1 - a_1 v_1 + a_2 v_2 + b_1 (v_1 + v_2) + a_3 (v_1 + v_2)^2 - a_4 (v_1 v_2) \\
(m_1 + m_2) h_2 &= h_1 - a_1 v_1 + a_2 v_2 + b_1 (v_1 + v_2) + a_3 (v_1 + v_2)^2 - a_4 (v_1 v_2) \\
A h_1 &= -k_1 v_1 \\
A h_2 &= -k_2 v_2
\end{align*} \tag{15}$$

For the simplicity of the expression new parameters are introduced. Let us denote, assuming $m_1 = m_2 = m$,

$$\mu = \frac{m_3}{m_1 + m_2}, \quad \alpha_j = a_j \sqrt{\frac{A}{(m_1 + m_2) k_1}} (j = 1, 2), \quad \beta = b \sqrt{\frac{A}{(m_1 + m_2) k_1}},$$

$$\gamma = \frac{h_2}{h_1}, \quad \Omega = \frac{h_2 \sqrt{h_1}}{h_1}, \quad \Omega \tau = \tau \tag{16}$$

and by the linear transformation

$$v_1 = \frac{y_1}{1 - \mu^2} - \frac{\mu y_2}{1 - \mu^2}, \quad v_2 = -\frac{\mu y_1}{1 - \mu^2} + \frac{y_2}{1 - \mu^2}, \quad h_1 = \frac{A}{(m_1 + m_2) k_1} y_1, \quad h_2 = \frac{A}{(m_1 + m_2) k_1} y_2 \tag{17}$$

transform variables $(v_1, v_2, h_1, h_2)$ into new variables $(y_1, y_2, y_3, y_4)$. Then, in vector-matrix notation, Eq. (15) may be written compactly,

$$\bar{Y}' = B \bar{Y} \tag{18}$$

where $\bar{Y}$ is a column vector whose components are $y_1, y_2, y_3$ and $y_4$, and $'$ denotes a derivative with respect to $\tau$. $B$ is a matrix,

$$B = \begin{bmatrix}
1 & \frac{\beta}{1 - \mu^2} & 1 & \frac{\beta}{1 - \mu^2} \\
\frac{\beta}{1 - \mu^2} & 1 & \frac{\beta}{1 - \mu^2} & 0 \\
\frac{\beta}{1 - \mu^2} & 1 & \frac{\beta}{1 - \mu^2} & 0 \\
\frac{\beta}{1 - \mu^2} & 1 & \frac{\beta}{1 - \mu^2} & 0
\end{bmatrix} \tag{19}$$

higher order terms of $y_1$ and $y_2$, $\epsilon$ being a small parameter. When the behavior of the system near the equilibrium is considered, the second term in the right hand side of Eq. (17) is negligible comparing with the first.

In order that the equilibrium in a hydrodynamic system may be stable, that is, any small disturbance near the equilibrium diminishes with time and dies out after the lapse of sufficiently long time (asymptotical stability in Lyapunov's definition) the real parts of the characteristic roots of matrix $B$ must be all negative. The characteristic equation is written as

$$A_4 S^4 + A_3 S^3 + A_2 S^2 + A_1 S + A_0 = 0 \tag{20}$$

so that the condition of stability is expressed as

$$A_0 A_3 A_4 > 0, \quad A_1 A_2 A_4 > 0, \quad A_1 A_2 A_3 > 0, \quad A_0 A_2 A_4 + A_1 A_3 A_4 > 0 \tag{21}$$

where

$$\begin{align*}
A_0 &= \gamma \\
A_1 &= (\alpha_1 + \beta) \gamma + \alpha_2 + \beta \\
A_2 &= (\alpha_1 + \beta) (\alpha_2 + \beta) + 1 + \gamma \\
A_3 &= \alpha_1 + \alpha_2 + 2 \beta (1 - \mu) \\
A_4 &= 1 - \mu^2
\end{align*} \tag{22}$$
Relations (19) are not so simple, but the stable region is graphically shown in Fig. 2 with respect to coordinates $\alpha_i, \alpha_2, \beta$ being a parameter.

It is found from Eq. (19) and Fig. 2 that the equilibrium is always stable in the region where $\alpha_i > 0, \alpha_2 > 0$ and always unstable in the region where $\alpha_i < 0, \alpha_2 < 0$, and in the region where $\alpha_i < 0, \alpha_2 > 0$ there exist stable regions near the coordinates. In other words, when the equilibrium of a hydrodynamic system lies in the range of positive inclination of the characteristic curve for each channel, any disturbance near the equilibrium diminishes and dies out with time, so that the system is stable. On the other hand when the equilibrium lies in the range of negative inclination of the characteristic curve, any small disturbance near the equilibrium grows with time, so that the system is unstable. The origin $(0,0)$ in Fig. 2 lies on the boundary line between stable and unstable region.

In order to examine in detail what is stated above, a symmetric system is considered as a special case. Assume that $\alpha_1 = \alpha_2 = \alpha$ and $\gamma = 1$. By the linear transformation

$$
\begin{align*}
y_1 + y_2 &= 2a_1 \\
y_2 + y_3 &= 2a_2 \\
y_2 - y_3 &= 2a_3 \\
y_2 - y_4 &= 2a_4
\end{align*}
$$

(21)

vector $\tilde{Y}$ is transformed into vector $\tilde{Z}$. Applying the linear transformation (21) to Eq. (17) and neglecting the higher order terms of $\xi_i$ yields

$$
\tilde{Z}' = \tilde{D}\tilde{Z}
$$

(22)

where $\tilde{Z}$ is a column vector whose components are $(\xi_1, \xi_2, \xi_3, \xi_4)$ and $\tilde{D}$ is a matrix,

$$
\begin{pmatrix}
-\alpha + 2\beta & 1 & 0 & 0 \\
\frac{1}{1+\mu} & 0 & 0 & 0 \\
-\frac{1}{1+\mu} & 0 & 0 & 1 \\
0 & 0 & -\frac{\alpha}{1-\mu} & 1 \\
0 & 0 & -\frac{1}{1-\mu} & 0
\end{pmatrix}
$$

As before, in order that the system may be stable, the real parts of the characteristic roots of matrix $\tilde{D}$ must be all negative. The characteristic roots are given by the following two quadratic equations of $s$

$$
\begin{align*}
s^2 + \frac{\alpha + 2\beta}{1+\mu} s + \frac{1}{1+\mu} &= 0 \quad (E_1) \\
s^3 + \frac{\alpha}{1-\mu} s^2 + \frac{1}{1-\mu} &= 0 \quad (E_2)
\end{align*}
$$

(23)

Since $\beta$ is regarded to be positive from its physical sense, the condition that the system is stable is represented as

$$
\alpha > 0
$$

(24)

From Eqs. (16), (21) and (23), it is found that for the characteristic roots $s_i, s_2$ given by $(E_1)$ of Eq. (23), the ratio

$$
\frac{v_1}{v_2} = 1
$$

which means that the channels I and II vibrate in the same phase, and for $s_b, s_4$ given by $(E_2)$, the ratio

$$
\frac{v_1}{v_2} = -1
$$

which means that the channels I and II vibrate in the opposite phase.

It is now concluded from the results obtained above as follows.

(1) In the system having two parallel boiling channels there are two normal modes of vibration. The first is a mode in which each channel vibrates in the same phase ... in-phase mode. The second is a mode in which each channel vibrates in the opposite phase ... opposite-phase mode. Natural angular frequencies of the normal modes are given with respect to the time $\tau$ as

$$
\begin{align*}
\xi_1 &= \frac{1}{\sqrt{1+\mu}} \quad \text{(in-phase mode)} \\
\xi_2 &= \frac{1}{\sqrt{1-\mu}} \quad \text{(opposite-phase mode)}
\end{align*}
$$

(25)

Then, the period of vibration of the in-phase mode is longer than that of the opposite-phase mode.

(2) As long as $\alpha$ is negative, the opposite-phase mode is always unstable. As $\beta$ is not included in $(E_2)$, a throttle or a pump in the downcomer is not effective to cause positive damping to the opposite-phase mode. This fact contradicts the results obtained in the single boiling channel system in which a throttle or a pump in the downcomer causes positive damping to the system. As long as $\alpha < 0, \alpha_2 < 0$, the opposite-phase mode is always unstable so that the hydrodynamic oscillation is generated and each channel is oscillating in opposite phase, respectively.

(3) When $\beta$ is very small comparing with $|\alpha|$,
two modes of vibration may occur according to initial conditions, but when $\beta$ is large enough to damp the in-phase mode of vibration, only the opposite mode is able to occur. In other words, the in-phase mode is expected to occur only when the resistance in the downcomer is very small, and when it is very large, only the opposite mode may occur.

(4) As mentioned in the previous paper, it is, of course, important for the stabilization of the system to decrease inlet subcooling as far as possible, in order that the characteristic curve may be a monotonically increasing function of the flow rate $V_1$ or $V_2$. In addition, it is desirable to pay attention to the devices to stop the opposite mode of vibration, in particular, in the system having parallel boiling channels. Then if throttles are used for the stabilization, they must be set in each channel, respectively. This fact justifies the method taken in the design of boiling channels in BWR. In view of construction some considerations are needed for the design of such header that has a large resistance to the opposite-phase mode.

4. Existence of more than one equilibrium and non-uniform distributions of flow rates

In the system having parallel boiling channels, it may well happen that there are many equilibriums under some conditions, among which stable and unstable equilibriums are included. In such a case, even if the system exists in the unstable equilibriuim at the outset, it departs from the unstable equilibrium with time and tends to a stable one, so that after the lapse of sufficiently long time, it settles down there and does not show oscillatory behaviors.

In order to examine such behaviors, let us obtain equilibriums graphically by illustrating Eqs. (10). Denote in Eq. (10)
\[
\begin{align*}
\Delta p_1 &= \bar{T}_1 - H_1 \\
\Delta p_2 &= \bar{T}_2 - H_2 \\
\Delta p_3 &= -\bar{T}_4
\end{align*}
\]
and illustrate three equations in the space as the coordinates of $V_1$, $V_2$, and $\Delta p$, then the equilibriums may be obtained as the intersections of three surfaces in the space. In this paper, however, as this method is not applicable, they are obtained as the intersections of the combined curve $\Delta p_1$ of $\Delta p_1$ and $\Delta p_2$ at steady state with that of $\Delta p_3$ in the plane as the coordinates of $V_1 + V_2$ and $\Delta p$.

When a pump is included in the downcomer, $\Delta p_3$ is represented as $\Delta p_3 = F - \bar{T}_4$. In this paper $V_1$ and $V_2$ are regarded as positive, which means that water flows upward in each channel.

Several cases are possible according to the relations between $H_1$ and $\bar{T}_1$. When $H_1$ and $\bar{T}_1$ are characterized as shown in Fig. 3, $\Delta p_1$ is a monotonically increasing function of $V_1$ as shown in Fig. 4, in which the part of dotted line shows the region where $\bar{T}_1$ has a negative slope. Combining such $\Delta p_1$ and $\Delta p_2$ results in also a monotonically increasing function of $V_1 + V_2$ as shown in Fig. 6. If $\bar{T}_4$ and $F$ are such curves as shown in dotted line in Fig. 5, $\Delta p_3$ is represented as a solid line in Fig. 5.
for the natural circulation or for the forced circulation, respectively. Then, in general, an equilibrium exists under $V_1 + V_2$ axis for the natural circulation and above it for the forced circulation. Illustrating $\Delta p_1$ and $\Delta p_2$ in the same plane as shown in Fig. 6, equilibriums are obtained as the intersections of two curves. In the case as shown in Fig. 6 there exists only one equilibrium for a given $\Delta p_1$. Accordingly, what can be expected to occur is, as described in the previous paper, either that the system is stable so that the steady flow with boiling is established, or that the system is unstable so that it shows oscillatory behaviors. In Fig. 6 it is expected that in cases of (i) and (iv) the system is stable and that in cases of (ii) and (iii) the system is unstable.

Now, when the pressure losses are very large comparing with the driving head as in once-through boilers and the relation between $H_1$ and $\Psi_1$ is characterised as shown in Fig. 7, $\Delta p_1$ is not a monotonically increasing function of $V_1$ and has the characteristics as shown in Fig. 8. Combined characteristics of such $\Delta p_1$ and $\Delta p_2$ are very much complicated. For example, if $\Delta p_1$ and $\Delta p_2$ have such characteristics as shown in Fig. 9 (a), $\Delta p_2$ is plotted as shown in Fig. 9 (b). Then, under some conditions many equilibriums may exist for a given $\Delta p_1$. Theoretically speaking, nine equilibriums can exist for a given $\Delta p_1$ in the case shown in Fig. 9.

In Fig. 9 a part of the dotted line in the curve of $\Delta p_1$ corresponds to the region where $\alpha_1<0$, $\alpha_2<0$, and a part of the chain line to the region where $\alpha_1 \alpha_2<0$.

According to the above discussions it is found as follows. In the case (i) in Fig. 9 (b), the system is stable, so that a steady flow is established in each channel, but evaporation is expected to be small, since the flow rate is large. In the case (ii) three equilibriums (a), (b) and (c) exist for a given $\Delta p_1$. The equilibriums (a) and (c) are stable, but (b) is unstable. Then, if it happens that the system lies in the equilibrium (b) at the outset, it departs

Fig. 6

Fig. 7

Fig. 8

Fig. 9
from (b), since the equilibrium (b) is unstable. However, as there exist stable equilibriums (a) and (c) in this case, the system tends to (a) or (c) and settles down there after sufficiently long time, so that the system does not show oscillatory behaviors. In equilibrium (a) or (c), however, it is expected that in one channel the flow rate is large, so that evaporation hardly occurs, but in the other channel the flow rate is small, so that violent evaporation may occur. Though the system is stable in the sense of dynamics, but such situation is not desirable for boiling channels because a possibility of burnout exists in the channel in which the flow rate is small. In the case (iii) only one equilibrium is obtained and it is unstable, so that the system is oscillatory. In the case (iv) the system is stable, so that steady flows are established in both channels. But as the absolute value of flow is small, violent evaporation is expected to occur.

For the stabilization of the system it is most important to improve the characteristic curves $\mathcal{F}_1$ and $\mathcal{F}_2$ so as not to have negative slopes.

5. Conclusions

An investigation is carried out on the hydrodynamic instability in the system having two parallel boiling channels. Following facts are found.

(1) The system is always stable when the equilibrium of the hydrodynamic system lies in the range of positive inclination of the characteristic curve for each channel, and always unstable when the equilibrium is in the range of negative inclination of the characteristic curve for each channel.

When the equilibrium lies in the range of negative inclination for one channel and positive inclination for the other channel, the system is either stable or unstable.

(2) In the system having two parallel boiling channels, there are two modes of vibration. The first is an in-phase mode in which each channel vibrates in the same phase, and the second is an opposite-phase mode in which each channel vibrates in the opposite phase.

(3) When the inclinations of the characteristic curves at equilibrium are both negative, the opposite-phase mode of vibration is always unstable. In this situation, a throttle or a pump in the downcomer is not effective to cause positive damping to the opposite-phase mode.

(4) When the resistance in the downcomer is small, both modes of vibration may occur, but when it is large, the in-phase mode is damped, so that only the opposite-phase mode may occur.

(5) For the stabilization of the system, it is, of course, important to improve the characteristic curves so as not to have negative inclination. In addition, it is desirable to pay attention to the design of such system that has a larger resistance to the opposite-phase mode, specially in the design of parallel boiling channels.

(6) In the system with parallel boiling channels, it happens that there are many equilibriums, so that non-uniform distributions of flow rates may occur.

**Nomenclature**

- $A$: channel cross-sectional area $\text{m}^2$
- $f(x)$: void volume fraction at $x$
- $f_0, f_1$: exit void volume fraction and equivalent exit void volume fraction
- $F(V)$: pump characteristics $\text{m}$
- $\Delta h$: subcooling of inlet water $\text{kcal/kg}$
- $h_0$: latent heat per unit weight of steam $\text{kcal/kg}$
- $l_s$: length of non-boiling region in heated region $\text{m}$
- $L$: length of heated region $\text{m}$
- $m$: inertia constant of pipe $\text{sec}^3/\text{m}^2$
- $p$: pressure head $\text{m}$
- $\Delta p$: pressure drop $\text{m}$
- $Q$: heat power supplied to the channel $\text{kcal/sec}$
- $V$: flow rate of water at the channel inlet $\text{m}^3/\text{sec}$
- $\gamma$: specific weight of saturated water and steam $\text{kg/m}^3$
- $\gamma$: slip ratio between steam and water
- $\mathcal{F}$: pressure losses (or characteristic curve) $\text{m}$
- $\Phi$: void volume characteristics $\text{m}^3/\text{sec}$
- $x$: length along the channel $\text{m}$
- $t$: time $\text{sec}$

Suffix $1, 2$ and $d$ show the channels I, II and the downcomer, respectively.

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