that the contributions from the integrations along $A'D'$ and AD vanish as R goes to infinity. In this manner, one can pass from the representation (12) for the temperature in the cylinder to Eq. (13), which is more suitable for numerical work.

If $\varphi(p)$ involves the delta function, or $\varphi_+(w)$ or $\varphi_-(w)$ contains terms which are not regular in the domains (15) and (16), the contributions due to these terms have to be evaluated separately.

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Fatigue Damage of 7-3 Brass Specimens*

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The nature of the fatigue damage of 7-3 brass specimens is quite the same as what occurs in 6-4 brass or in low carbon steel specimens. The damage is confined within a thin surface layer of the specimen and cannot be removed by stress relief annealing. The thickness of the damaged layer becomes greater when the applied stress is high and the number of the stress repetitions is large. The fatigue stressing hardens the material but the hardening can be removed by stress relief annealing.

By taking X-ray diffraction patterns of the surface as well as of the portions beneath the surface of the specimens, it was revealed that the distortion and fragmentation of the grain in fatigued specimens decrease as the distance of the irradiated grain from the surface increases and it was concluded that, as the thickness of the severely damage layer reaches some threshold value of one or two grains thickness, fatigue cracks develop there which, in most cases, eventually cause the plain specimens to break down.

1. Introduction

By annealing and electro-polishing procedure it was revealed that;

1) The damage done by the alternating stressing is confined within a thin surface layer of the specimens. The thickness of the damaged layer increases when the applied stress is high and the number of stress repetitions is large and cannot be removed by stress relief annealing.

2) The thickness of the damaged layer seems

to increase grain by grain.

3) The fatigue stressing hardens the material beneath the surface but this hardening can be removed by stress relief annealing.

Above results were obtained using 6-4 brass⁽¹⁾ and low carbon steel (0.35% C) specimens⁽²⁾. Since those materials are not single phased materials it was desirable to make the same kind of experiments with single phased materials. The results of experiments reported here were obtained with 7-3 brass specimens (single phased material) using annealing and electro-polishing procedure as well as taking X-ray diffraction patterns of damaged specimens.

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2. Material used and method of testing

The material used was 7-3 brass (69.85% Cu, Pb trace, 0.01% Fe, balance Zn; mean grain size

Specimen No. 39 ($\sigma=0$, $N=0$)

Specimen No. 24, This specimen was stressed to 14.5 kg/mm^2 for 13.451×10^6 repetitions and failed. It was annealed 18 times during these repetitions

Specimen No. 26, This specimen was broken under the stress of 14.5 kg/mm^2 after 19 repetitions of annealing. Stress repetition to failure was about 1.74×10^6

Fig. 1 Micrographs of the materials, $\times 200$

$0.03 \sim 0.04 \text{ mm}$) which was offered by Dr. K. Takeuchi of the Sumitomo Light Alloy Mfg. Co., Japan, in annealed state ($470 \sim 475^\circ\text{C}$, 2 hr.). All specimens were turned out from bars of 17 mm in diameter.

Fig. 1 (A) shows the micrograph of the original state (annealed state), while Fig. 1 (B) shows the surface of the specimen subjected to rotary bending stress of 14.5 kg/mm^2 repeated 13.451×10^6 times, during which the specimen was annealed 18 times at 400°C for 30 min regularly [c. f. Fig. 4 (a)]. The micrograph of Fig. 1 (C) shows the part near the surface of the specimen which was broken under the stress of 14.5 kg/mm^2 after 19 repetitions of annealing (400°C , 30 min). The stress repetitions to failure was about 1.74×10^6 .

The mechanical properties of the materials (mean value of 14 specimens) are: proof stress for 0.2% set was 13.2 kg/mm^2 , tensile strength was 38.5 kg/mm^2 , actual stress at fracture was 104.2 kg/mm^2 , area contraction was 74%, elongation was 66% and fatigue limit was 13.0 kg/mm^2 .

Endurance curve shown in Fig. 2 was obtained with electro-polished specimens each about 11 mm in dia. and the rate of stress repetitions was about 2500 per minute. (By taking X-ray diffraction patterns of the specimens it was revealed that when the process of turning the specimens from the original bars was severe, the region of the severely deformed part near the surface of the specimens was somewhat thicker than 30μ , but the specimen used for the determination of the endurance curve in Fig. 2 was electro-polished to take off the surface layer of about 30μ thick.)

For detailed discussion of the fatigue life of the material itself, it will be necessary to test more specimens than shown in Fig. 2 but, as will be stated below, since localization of the damage is so evident that the conclusions reached would not be misleading because of the scattering of the endurance lives, so we did not make detailed discussions

Number in the bracket is the specimen No.

Fig. 2 Endurance curve of the specimens with electro-polished surface

about the scattering of the endurance lives of the material.

Experimental procedures (in the following this is abbreviated as A.P. procedure) are as follows; After a specimen is subjected to a rotary bending stress σ for N_1 repetitions, the surface of the specimen is polished off mechanically by an amount of Δr_1 with 04 emery paper followed by pot-annealing for 30 min at 400°C avoiding oxidation of the surface. The specimen is then electro-polished to remove the surface layer to a thickness of Δr_2 . When the layer to be removed is thin, mechanical polishing is omitted and only electro-polishing is done. The thickness Δr_2 of the layer removed by electro-polishing was about 30 ± 3 microns in all cases of the experiments.

In fatigue tests, for one specimen, σ , $\Delta r (= \Delta r_1 + \Delta r_2)$ and N_1 were kept constant and the fatigue life of the specimen was measured. The specimen was, therefore, subjected to repeated polishing and annealing after every N_1 cycles of stress repetitions until it failed under the applied stress σ . For another specimen, σ , or Δr or N_1 was changed and its fatigue life was measured, and so forth.

3. Results of tests and discussions

In Fig. 3 to Fig. 5 numerals such as (8), (3), for example, represent the specimen number. Life of a specimen is given by the abscissa of the black point at the end of each horizontal line. Black point with an arrow shows that the specimen did not fail. At the number of stress repetitions corresponding to the abscissa of the vertical dotted lines, stress relief annealing (400°C, 30 min) and polishing for removal of the layer were carried out to a thickness of Δr . N_0 is the life of specimen stressed without intermediate A.P. procedure.

a) Case of $\sigma = 13.5 \text{ kg/mm}^2$ (Fig. 3) Although the specimen (11) did not fail and survived 25.922×10^6 repetitions of stress without intermediate A.P. procedures as shown in Fig. 3 the life of the specimen under the stress 13.5 kg/mm^2 was regarded to be $N_0 = 12.05 \times 10^6$ which was nearly equal to the life of the specimen (4) (Fig. 2).

Fig. 3 Case of $\sigma = 13.5 \text{ kg/mm}^2$

Specimen (28) survived 9 repetitions of A.P. procedures ($N_1 = 3 \times 10^6 \cong 0.25 N_0$, $\Delta r = 40 \mu$), and after the last A.P. procedure it endured 18.053×10^6 repetitions of stress, this number of stress repetitions is a little larger than the life of the specimen (4), showing that the thickness of a severely damaged layer during $N_1 = 3 \times 10^6$ under the stress of 13.5 kg/mm^2 is less than 40μ . Fig. 3 shows also that the thickness of the damaged surface layer is thicker than 80μ when the same stress is repeated to about one half the expected normal life ($N_1 \cong 0.5 N_0$) of the specimen.

b) Case of $\sigma = 14.5 \text{ kg/mm}^2$ (Fig. 4) The expected normal life of the specimen under the stress of 14.5 kg/mm^2 is about 5.5×10^6 as can be seen from Fig. 2. Now referring to Fig. 4 we see that the total life of the specimen (24), which, after surviving 18 repetitions of A.P. procedures ($\Delta r = 40 \mu$), was broken at 3.241×10^6 stress repetitions, was more than two times the expected normal life of the specimen. It seems, therefore, that the thickness of the severely damaged surface layer during the repetitions $N_1 = 0.1 N_0$ is about 40μ . But since the number of stress repetitions (3.241×10^6) which the specimen (24) endured in 18 times of A.P. procedures is about one half the normal

Fig. 4 Case of $\sigma = 14.5 \text{ kg/mm}^2$

expected life of the specimen under the stress of 14.5 kg/mm^2 , it may be concluded that the material suffered in some way from the repetitions of A. P. procedures. That this damage will be due to the effect of annealing in A. P. procedure can be seen from the results of the specimen (15), (26) and (29) as will be explained below.

The fact that the life of a fatigued specimen can be prolonged by removal of its surface layer means that the damage due to fatigue stressing is confined mainly to the surface of the specimen. Fig. 4 (b) and (c) show that the thickness of the surface layer to be removed to prolong the life becomes greater as the number of stress repetitions increases. Further Fig. 4 (d) shows that the thickness of the damaged surface layer developed in the specimen during stress repetitions of one half of its expected normal life reaches more than 160μ (this corresponds to the magnitude of 4 grain size).

c) Case of stress 15.5 kg/mm^2 (Fig. 5) The normal expected life of the specimen under the stress of 15.5 kg/mm^2 is about 3×10^6 as can be seen from Fig. 2. Fig. 5 shows the results of tests. The thickness of the damaged layer developed during stress repetitions of one tenth of its normal life is about 40μ . Specimen (15) after surviving 16 repetitions failed after about 1.52×10^6 repetitions of the stress. This number of repetitions is about one half of the expected normal life of the specimen. About this, the same thing as about the test piece (24) may be said.

Fig. 5 shows that the thickness of the severely damaged layer developed during $1/4$ of its normal

Fig. 5 Case of $\sigma = 15.5 \text{ kg/mm}^2$

life is about 100μ . The thickness will increase to about 200μ when the stress repetition is continued to about 43% of the normal life of the specimen.

Above results suggest that the main fatigue damage is initially confined within a thin surface layer of the specimen and as the stress repetition continues the thickness of the damage layer increases.

It is also true, as the above experiments show, that even if the damaged layer is removed the life of the specimen can not be prolonged infinitely. For these facts, the following two factors, viz. the detrimental effect of repeated annealing and the hardening effect of coxing of the stress, may be responsible.

To see the detrimental effect of the repeated annealing, specimens (26) and (29) were tested. Specimen (26) was annealed (400°C , 30 min) 19 times repeatedly in a stainless steel pipe to avoid surface oxidation and then the surface was electro-polished to remove the surface layer of about 31μ thick and then it was tested under the stress of 14.5 kg/mm^2 . The life of this specimen was about 1.74×10^6 . This life was about one third of the expected normal life of the specimen in as received condition. The specimen (29) was annealed 16 times repeatedly, and then the surface layer of about 31μ thick was removed by electro-polishing. Then it was tested under the stress of 15.5 kg/mm^2 . The life was about 75% (2.33×10^6) of the expected normal life ($N_0 = 3.0 \times 10^6$). Although the figure 75% may lie in the region of scattering of the fatigue lives, above two facts together with the results of the specimens (24) and (15) suggest that repeated annealing (400°C , 30 min) seems to shorten the life of the specimens. No evidence, however, was observed about the grain growth in the annealed specimens (c. f. Fig. 1).

Coxing effect will strengthen the inner part of the specimen. In our experiments the part at the depth $n\Delta r$ beneath the surface of the specimen comes eventually to the surface of the specimen after n -times repetitions of polishing (and annealing), since by each polishing the surface layer of thickness Δr is removed. In a specimen of $2a$ dia. under bending stressing, this part will undergo the stress levels $\sigma(1 - n\Delta r/a)$, $\sigma(1 - (n-1)\Delta r/a)$, ..., $\sigma(1 - \Delta r/a)$ until it comes to the surface in such way that at each stress level the stress is repeated N_1 times and so due to this coxing effect the fatigue strength of that part may change. It will be natural to conclude that if the stress relief annealing (400°C , 30 min) nullifies the strain hardening due to repeated stressing, the strengthening due to coxing will also be removed or become

very small after annealing.

To confirm this we examined the distribution of the hardness over the cross section of a fatigued specimen before and after annealing. The results are shown in Figs. 6 and 7. Fig. 6 shows the microhardness over the section (several millimeters apart from the fractured section) of the specimen (9) which was tested under the stress of 14.5 kg/mm^2 . This specimen as shown in Fig. 4 (b) endured 10 repetitions of A.P. procedures, so Fig. 6 can be seen to give the hardness of the specimen after stress repetitions. Mean-while, Fig. 7 shows the distribution of hardness over a section of the same specimen after annealing (400°C , 30 min). These Figures show that the strain hardening due to stress repetitions disappears after such annealing. We concluded, therefore, that the coxing effect will also disappear by such stress relief annealing.

Fig. 8 shows the relations between the thickness Δr_0 of the damaged layer, as determined by the method stated above, and the magnitude of applied stress or number of stress repetitions. The upward arrow in the Figs. shows that the thickness of the damage layer is larger than that shown by the point and the point with downward arrow shows that the damaged layer is thinner than that shown by the point.

4. X-ray investigation

As the surface damage due to fatigue seems mainly due to the appearance of extrusions and

Fig. 6 Micro-hardness distribution over the cross section of specimen No. 9 [c.f. Fig. 4 (b)]. This section is electro-polished before measurement

Fig. 7 Micro-hardness distribution over the cross section of specimen No. 9 in annealed state. This section is electro-polished before measurement

intrusions, the grain on the surface of the specimen will suffer the damage first. In order that the damage may proceed inward, extrusions and intrusions of the material must appear at the grain boundary such as $A'B'$ [Fig. 9(a)] after the surface grain $ABB'A'$ has been damaged by stressing. But since the boundary $A'B'$ is not a free surface as AB is, the occurrence of extrusions and intrusions will be more difficult than on the surface AB . Occurrence of extrusions and intrusions will be more difficult on the grain boundary $A'B'$, and so forth. In spite of this the damage proceeds into the interior of the specimen as our experiment shows, therefore some kind of damage, e.g. microcrack formation must take place at a time when the thickness of the damaged part reaches some extent.

It is not ascertained that microcrack formation can be detected by X-ray investigation but some evidences about the inward penetration of fatigue damage will be obtained if the X-ray diffraction patterns from various parts beneath the damaged surface of fatigued specimens are examined, since the sharpness of the diffraction spots will reveal something about the crystalline state of the fatigued

Numbers in (a) are the ratio of N_1/N_0
Fig. 8 Relations between the thickness Δr_0 of damaged layer and applied stress or its number of repetitions

Fig. 9 Schematic diagram. (a) The thick lines represent slip lines. (b) d is the dia. of specimen

specimen.

In the first place we examined whether the thickness of the damaged layer determined by A. P. procedure will or will not be the same as that determined by X-ray method. Fig. 10 shows the diffraction patterns (Co radiation with Fe filter, dia. of the beam was 0.86 mm) of various parts beneath the surface of the specimen subjected to the stress of 15.5 kg/mm^2 for $\sim N_0/4 = 0.75 \times 10^6$. In this Fig., (17-2) is the pattern from the surface and (17-4) is one from about 1 grain (49μ) below and (17-5) is about 2 to 3 grains (100μ) below and (17-10) is about 4 to 5 grains (184μ) below the surface of the specimen respectively. The spots in the Figure become sharper as the diffracting part deepens into the interior of the specimen.

This is an evidence of the fact that large plastic strain is confined within 2 or 3 grains beneath the surface. This thickness coincides with the thickness of damaged layer obtained by A. P. procedure approximately, which was about 100μ in this case [c. f. Fig. 5 (b)].

$\sigma = 15.5 \text{ kg/mm}^2$, $N_1 = 0.75 \times 10^6$
Fig. 10 Diffraction patterns of specimen No. 17

Fig. 11 shows the patterns of the specimen subjected to repeated stress of 15.5 kg/mm^2 for about 43% of the expected normal life of the specimen. In this Fig., (16-2) is the pattern from the surface and (16-3) is one from 49μ below and (16-5) is one about 100μ below the surface of the specimen; (16-7) and (16-9) are the patterns from the layers about 140μ and 180μ below the surface respectively. From this Fig. we see that the expanded spots of the pattern (16-2) are also expanded in (16-9) showing that even 180μ below the surface there remains some disturbance in crystal planes. This fact coincided approximately with the result obtained by A. P. procedure [c. f. Fig. 5 (c)]. Above results lead us to conclude that

$\sigma = 15.5 \text{ kg/mm}^2$, $N_1 = 1.3 \times 10^6$
Fig. 11 Diffraction patterns of specimen No. 16

the thickness of the damaged layer can be determined by the sharpness of the diffraction spots.

Fig. 12 are the patterns (no filter was used, dia. of beam was 1.08 mm) of plain specimen (43) (c. f. Fig. 2) which failed under the stress 14.0 kg/mm^2 . Since the fatigue limit of the material was about 13 kg/mm^2 as shown in Fig. 2 and the dia. of the specimen was about 11 mm, the part of the specimen which was subjected to the stress above its fatigue limit reached about 400μ below the surface. In Fig. 12 the blurring of the spots decreases as the diffracting part deepens. The spots from the layer about 450μ beneath the surface are

sharp, showing that the deformation of crystal planes nearly stopped there. The diffraction spots of the specimen (32) (c. f. Fig. 13) tested under the stress of 14.5 kg/mm^2 show also the same result. In the latter case, however, the layer about 580μ beneath the surface was subjected to a stress above its fatigue limit (13 kg/mm^2). We see, from Fig. 13, that the diffraction patterns from the layer about 600μ beneath the surface is sharp and so the deformation of crystal planes ceased near this part.

Those results suggest that the expansion of the diffraction spots takes place only in the layer

$\sigma = 14.0 \text{ kg/mm}^2$, $N_1 = 5.6439 \times 10^6$, broken

Fig. 12 Diffraction patterns of specimen No. 43

subjected to the stress above its fatigue limit.

The results of A.P. procedure and of X-ray investigation tell us that the damage of the part below CD in Fig.9 must be very small compared with the damage in ABCD. In spite of this the specimens fail when the magnitude of the fibre stress is larger than its fatigue limit. To reconcile those discrepancies it is natural to think that when the thickness of the damaged layer reaches a certain value, microcracks will develop there and propagate into the specimen and causing eventual failure of the specimen as the stress repetition is continued.

According to Petch⁽³⁾ (in static loading) and Okubo et al.⁽⁴⁾ (in fatigue loading) the strain concentration in large grain is larger than in smaller grain, so in specimens subjected to the stress above its fatigue limit, the larger grain in the surface layer will be damaged first. In Fig.9 let the grain ABB'A' be the grain damaged first then, as a consequence, internal stress of this grain will relax. If, however, fatigue crack initiation is mainly due to the formation of extrusion and intrusion, initial microcracks must develop in this grain. On the other hand, if the microcrack formation is due mainly to the pile-up of dislocation,

$\sigma = 14.5 \text{ kg/mm}^2$, $N_1 = 6.5947 \times 10^6$, broken

Fig. 13 Diffraction patterns of specimen No. 32

initial crack will develop in the inner grains such as A'B'B''A'' or A''B''DC etc.

In both cases the first fatigue crack will develop in the surface or near the surface of the specimen. In our experiments we can not ascertain the location of the grain in which the first microcrack developed.

Watanabe⁽⁵⁾ and Wadsworth⁽⁶⁾ reported that they found fatigue crack in the surface of the specimen stressed to their fatigue limit and they concluded that fatigue limit was a stress under which fatigue crack did not propagate at all.

We infer that the initial crack will develop in the layer stressed above a stress σ_{w_0} (Fig. 9) which is nearly equal to the fatigue limit of a plain specimen. Even in the plain specimen, exactly speaking, fatigue limit is expressed by the fibre stress AB in Fig. 9 (b) and so it does not equal to σ_{w_0} exactly.

The condition that fatigue crack will start on or near the surface of the specimen seems to have intimate relations with the thickness of the damaged layer. On the other hand we know that fatigue limit of notched specimen can be obtained by considering a thin surface layer at the root of the notch. This and above fact suggest that in order that a crack may develop in damaged layer a certain thickness of the damaged layer is necessary.

5. Conclusion

About fatigue damage of 7-3 brass specimens under rotary bending stress with repeated annealing and polishing procedure, as well as by taking X-ray diffraction patterns, the author reached the following conclusions.

a) Fatigue damage due to stress above fatigue limit concentrates near the surface of the specimen.

The damage can not be removed by stress relief annealing.

b) Fatigue damage preceeds grain by grain from the surface into the interior of the specimen. The thickness of the damaged layer is a function of the magnitude of applied stress and its number of repetitions.

c) Most part of the fatigue damage in the interior part of the specimen is of strain hardening character and can be removed by stress relief annealing though not perfectly.

d) By taking X-ray diffraction patterns of fatigued specimens it is concluded that when the thickness of the damaged layer reaches a certain value, microcracks will develop there and their propagation causes an eventual fracture of the specimen.

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