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Study on Sand Pump*

(1st Report, On the Trajectories of Solid Particles in the Pump Impeller)

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When solid particles flow through the impeller of a volute pump with water, the behavior of particles is not always clear. The writers recorded photographically the trajectories of solid particles in the impeller using a high speed camera. In this paper the experimental results are cited, especially those on the relative velocity and its direction at the impeller exit. Moreover the numerical method of calculating the trajectories is demonstrated.

1. Introduction

Recently there is not a few chance to handle the two-phase flow which contains solid particles in water in the volute pump. In such cases the performance characteristics of the volute pump are influenced by solid particles and some experimental results are reported already as to its effect. Though it is thought to be very important to know the behavior of solid particles in the pump impeller, this influence has not always been clear up to this time. The writers recorded the trajectories of solid particles in the pump impeller using a high speed camera. In this paper the experimental results, especially, the relative velocities and its directions at the impeller exit are shown, and also the numerical method of calculating the trajectories is shown.

2. Experimental equipment

The experimental equipment is shown schematically in Fig. 1. The test pump is a single suction, single stage, 75 mm-volute pump and one side of the pump casing is made of a synthetic resin plate. The impeller is an open, perfect radial flow type. The outlet diameter of the impeller is 200 mm, the inlet diameter is 115 mm, the vane width is 30 mm, the number of vane is 7 and the outlet angles of the vanes of the test impellers are four kinds, that is, 15°, 25°, 35°, and 45° respectively. The solid particles used in this experiment are three kinds of glass balls of relatively large diameter, whose diameter and specific weight are given in Table 1.

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The pump was driven by a direct current motor at a constant speed of $1000 \text{ rpm} \pm 1$ or 2 rpm in the experiment. Water and glass balls are mixed in the mixing tank and made to flow into the pump. The mixed flow discharged from the pump was sorted out into glass balls and water by the screen filter and returned to each storage tank.

The flow quantities of water were measured by an inlet nozzle which was provided in the water storage tank and the flow rate of the solid particles was measured by the weighing method, that is, the flow from the belt feeder was received into a vessel and the flow rate was calculated from the weight and time. The moving course of solid particles in the impeller was recorded by a high speed camera of standard 16 mm frame size. The film speed used in the experiment was about 1000 pps, when the shutter time was about $1/5000$ second, and the rotating angle of the impeller in the shutter time was about 1°. The precise framing rate was decided by the rotating angle of the impeller which was recorded in each frame and we did not use the timing mark of the camera. The head-discharge curves for the four kinds of impeller are shown in Fig. 2.

3. Experimental results

In the experiment the pump discharge valve was opened fully and the moving courses of the solid particles were traced. The discharge of water was about 17 l/sec and the flow rate of the solid particles was about 0.5 l/sec. Under such a little

Table. 1

Mean diameter mm	12.57	8.82	5.19
Specific weight g/cm ³	2.52	2.90	2.87

flow late of the particles the pump head did not change so much. In Fig. 3 the pictures which were recorded by a high speed camera are shown for example. From such a series of pictures the relative position of the particles to the impeller was obtained. The examples showing the trace of particles are shown in Fig. 4. The relative velocity and its direction were obtained from these diagrams. The results are shown in Fig. 5 and Fig. 6. From these results the following relations may be suggested, although the data are scattered considerably. The influence of the diameter of the particles on the relative velocity and its direction is not so distinct, rather the velocity and its direction may be regarded as nearly constant in the range of the experiment.

The influence of the outlet angle of the impeller is as follows: the relative velocity of the particles has a tendency to decrease slightly with an increase of the outlet angle of the impeller, but the tendency is not clear. The outlet angle of the particles at the impeller exit is nearly equal to that of the impeller in the range of 15° to 35° , but in the case of 45° the solid particles almost clash with the surface wall of the impeller vane and the trajectories become very irregular. In the range of 15° to 35° such clashing is not so much.

As the inlet condition of the particles influences the subsequent course extremely, it is thought to be very important to know the inlet condition. However precise data about the inlet condition were

Fig. 1

Fig. 2

Exit angle of impeller 25° ,
Diameter of particle 5 mm

Fig. 3

Fig. 4

not obtained owing to the suction pipe condition in this experiment. But by presuming from the partially recorded data and the flow condition following the impeller inlet, it was known that the particles flow into the impeller at a nearly constant speed (4.5 to 5 m/sec) and an angle (30° to 50°) without influence of the diameter of the particles or the outlet angle of the impeller. This inlet condition differs considerably from those of water. It is reasoned that the particles which move with water in the suction pipe in the axial direction collide once against the inner surface of the impeller eye and change the moving direction to the radial, and there obtain a velocity component of peripheral direction. As it is thought that the velocity component is different depending on the location of the particle collision, it is thought that this is the largest cause of scattering in the experimental data.

4. Numerical method of calculating the trajectories

Although it is thought to be very difficult to

obtain the trajectories of the particles in the impeller analytically, the approximate course is calculated as follows.

Nomenclature

- w_q : relative velocity of solid particle in impeller
- w_f : relative velocity of water in impeller
- u : peripheral velocity
- r : radius from impeller center
- ω : angular velocity of impeller
- β : angle between flow direction of water and peripheral velocity
- $\Delta\beta$: angle between w_q and w_f
- C_D : resistant coefficient of solid particle
- p : pressure
- γ_f : specific weight of water
- γ_q : specific weight of solid particle
- A : sectional area of solid particle
- V : volume of solid particle
- m : mass of solid particle
- R : radius of curvature of solid particle trajectory

The coordinate is taken in the impeller and the

Fig. 5

Fig. 6

moving direction of the solid particle is denoted by S and the normal direction by N.

The forces shown in Fig. 7 act on the solid particle which moves in water. F_r is the centrifugal force which is caused by the rotation of the impeller. The direction is radial and the magnitude is represented as

$$F_r = mr\omega^2$$

F_R is the centrifugal force which is caused by the curvature of the trajectory of the solid particle and the magnitude is given by

$$F_R = mw_q^2/R$$

The direction is N-direction. F_c is Coriolis' force and the magnitude is

$$F_c = 2m\omega w_q$$

The direction is N-direction. F_{ps} and F_{pN} are the forces which are caused by the pressure gradient of water in the directions S and N respectively. Assuming that the pressure gradient is constant in the range of the particle size, the magnitudes are

$$F_{ps} = V\partial p/\partial S$$

$$F_{pN} = V\partial p/\partial N$$

respectively. F_f is the viscous resistance which is caused by the velocity difference between particle and water. Putting the magnitude of the difference of the velocity vector between particle and water Δw , the magnitude of F_f is

$$\frac{dw_q}{dt} = -\frac{g}{\gamma_q} \frac{\partial p}{\partial S} - \frac{1}{2} C_D \frac{A}{V} (w_f \cos \Delta\beta - w_q)^2 + r\omega^2 \sin(\beta + \Delta\beta) \dots\dots\dots (1)$$

$$\frac{w_q^2}{R} - 2\omega w_q - \frac{g}{\gamma_q} \frac{\partial p}{\partial N} + r\omega^2 \cos(\beta + \Delta\beta) - \frac{1}{2} C_D \frac{A}{V} \frac{\gamma_f}{\gamma_q} (w_f \sin \Delta\beta)^2 = 0 \dots\dots\dots (2)$$

respectively in S- and N-direction. It is necessary to note that the sign of the term of viscous resistance changes depending on the magnitudes w_q and w_f .

Though it is thought very difficult to obtain analytically a strict solution of Eqs. (1) and (2), it is possible to trace numerically the approximate course of the solid particle as follows. Putting $\kappa = (1/2)C_D(A/V)(\gamma_f/\gamma_q)$ in Eq. (1) and using the relation of $dw_q/dt = w_q(\partial w_q/\partial S)$, $\sin(\beta + \Delta\beta) = \partial r/\partial S$ and $r\omega^2 \sin(\beta + \Delta\beta) = u(\partial u/\partial S)$ etc., Eq. (1) becomes

$$w_q \frac{\partial w_q}{\partial S} - u \frac{\partial u}{\partial S} + \frac{g}{\gamma_q} \frac{\partial p}{\partial S} = -\kappa (w_f \cos \Delta\beta - w_q)^2 \dots\dots\dots (3)$$

Integrating it from $S=S_a$ to S_b , along the course of particle

$$\left[\frac{1}{2} w_q^2 - \frac{1}{2} u^2 + \frac{g}{\gamma_q} p \right]_{S_a}^{S_b} = - \int_{S_a}^{S_b} \kappa (w_f \cos \Delta\beta - w_q)^2 dS$$

Considering a minute section of $\Delta S = S_b - S_a$, it may be put approximately as

$$\begin{aligned} & \frac{1}{2} (w_{qb}^2 - w_{qa}^2) - \frac{1}{2} (u_b^2 - u_a^2) + \frac{g}{\gamma_q} (p_b - p_a) \\ & \approx - \frac{1}{2} \kappa \Delta S \{ (w_{fb} \cos \Delta\beta_b - w_{qb})^2 + (w_{fa} \cos \Delta\beta_a - w_{qa})^2 \} \dots\dots\dots (4) \end{aligned}$$

Subscripts a and b denote the respective values at the points S_a and S_b , and accordingly

$$\begin{aligned} w_{qb} = w_{qa} \frac{1}{1 + \kappa \Delta S} & \left[\kappa \Delta S \frac{w_{fb}}{w_{qa}} \cos \Delta\beta_b + \sqrt{\left(\kappa \Delta S \frac{w_{fb}}{w_{qa}} \cos \Delta\beta_b \right)^2 + (1 + \kappa \Delta S)} \right] \\ & \times \left[1 + \frac{u_b^2 - u_a^2}{w_{qa}^2} - \frac{2g}{\gamma_q} \frac{p_b - p_a}{w_{qa}^2} - \kappa \Delta S \left(\frac{w_{fb}}{w_{fa}} \cos \Delta\beta_b \right)^2 - \kappa \Delta S \left(\frac{w_{fa}}{w_{qa}} \cos \Delta\beta_a - 1 \right)^2 \right] \dots\dots\dots (5) \end{aligned}$$

Fig. 7

$$F_f = \frac{1}{2} C_D \frac{\gamma_f}{g} A \Delta w^2$$

Dividing it into S-direction component and N-direction one, it becomes

$$F_{fS} = \frac{1}{2} C_D \frac{\gamma_f}{g} A (w_f \cos \Delta\beta - w_q)^2$$

$$F_{fN} = \frac{1}{2} C_D \frac{\gamma_f}{g} A (w_f \sin \Delta\beta)^2$$

Using these formulas, the equations of motion of particle are

By this formula the particle speed w_{qb} at the point S_b is calculated from the speed w_{qa} at the point S_a . On the other hand the curvature at the point S_a is calculated from Eq. (2). Assuming that the curvature is constant in this minute section ΔS , the trajectory in this minute section is decided, and consequently the condition of the particle at the point S_b is decided from the condition at the point S_a . Therefore, if the condition of the particle at the impeller inlet is known, the whole course is obtained by repeating the above mentioned calculation successively. In the practical calculation, assuming that the flow condition of water in the impeller is uniform and putting $\partial p / \partial N = 0$ in Eq.

Fig. 8

(2), the curvature is

$$\frac{1}{R} = \frac{2\omega}{w_q} - \frac{r\omega^2}{w_q^2} \cos(\beta + \Delta\beta) + \frac{1}{2} \kappa \left(\frac{w_f}{w_q} \sin \Delta\beta \right)^2 \quad \dots\dots\dots(2)'$$

Moreover the distance l between the impeller center and the curvature center, the radius r_b at the point S_b , the angle $\Delta\zeta$ against ΔS at the impeller center, $\Delta\beta_b$ etc. are calculated from the following geometrical relations.

$$\begin{aligned} l^2 &= R^2 + r^2 - 2Rr \cos(\beta + \Delta\beta) \\ \Delta S &= R_a \left\{ \cos^{-1} \left(\frac{R_a^2 + l_a^2 - r_b^2}{2l_a R_a} \right) - \cos^{-1} \left(\frac{R_a^2 + l_a^2 - r_a^2}{2l_a R_a} \right) \right\} \\ \Delta\zeta &= \frac{\Delta S}{R_a} + \left\{ \cos^{-1} \left(\frac{r_b^2 + R_a^2 - l_a^2}{2r_b R_a} \right) - \cos^{-1} \left(\frac{r_a^2 + R_a^2 - l_a^2}{2r_a R_a} \right) \right\} \\ \Delta\beta_b &= \Delta\beta_a + \Delta\zeta - \frac{\Delta S}{R_a} \end{aligned}$$

In Fig. 9 the calculated examples of the trajectories are shown, where $\beta = 25^\circ$ and the flow condition of water in the impeller is assumed as uniform. In this figure the solid line (a) represents the results which are calculated using the practically observed condition at the impeller inlet and the dotted lines do the experimentally obtained trajectories. The line (a) coincides with the experimental results fairly well as shown in Fig. 9 and shows especially good agreement when the initial condition is same. On the other hand the solid line (b)

Fig. 9

Fig. 10

shows the curve which is calculated using the initial condition similar to the water condition, and does not coincide with experimental results. In Fig. 10 the calculated results of the relative velocity and direction of a 9 mm diameter particle at the impeller exit are shown.

In practical calculation the value of the resistance coefficient C_D is important. As the value of C_D is nearly constant in the range of Reynolds' number $R_e = 10^3 \sim 2 \times 10^5$ and in this experiment R_e is contained in this range, C_D is treated as nearly constant ($=0.5$). Moreover it is significant how long the minute distance ΔS is to be selected. As a practical example, when $\Delta S = 2$ mm and 5 mm for the whole course $S = 100$ mm, the difference of the relative velocity is less than 1% and that of the outlet angle is less than 1° . It is thought enough for the practical purpose to select as the value of ΔS such a ratio. Although the uniform flow condition of water in the impeller is assumed in this paper, the actual flow in the impeller is never uniform. It is thought that this also is one of the reason why the data are scattered considerably.

5. Summary

The writers recorded photographically the behavior of the solid particles of relatively large diameters (5 to 13 mm) which flowed through the impeller of a volute pump with water, and confirmed that the relative velocity and direction of the particle at the impeller exit did not change so much by the diameter of the particle in the range of this experiment, also the relative velocity did not change so much by the outlet angles of the impeller, and the direction was near the outlet angles of the impellers in the range of 15° to 35° . The value of the slip coefficient, calculated in the same way as water, was about 0.30 to 0.33.

Moreover the writers demonstrated the numerical method calculating the trajectory and how the calculated results coincided with the experimentally recorded courses fairly well, if the inlet condition of the particle was known.

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