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Fatigue Strength of Plain and Notched Bars under Pulsating Load Varying in Two Stages*

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Fatigue lives of plain and notched specimens of a heat treated carbon steel are investigated under pulsating load varying in two stages. The experiments cover the cases where the stress value of the first stage is both higher and lower than that of the second stage, the stress difference between the first and second stages is varied and the stress cycle ratio in the first stage is varied.

According to the result of the experiment, Miner's hypothesis seems to be effective, if scatter of fatigue lives is taken into consideration, although some exceptions are found in which cumulative cycle ratio is increased to some extent. For these exceptions, however, Miner's hypothesis gives lives on the safe side in design calculations.

1. Introduction

Although many experimental researches on the fatigue lives of metals have been made under a completely reversed load varying in two stages⁽¹⁾, a few have been made under a pulsating load⁽²⁾. It was reported that the fatigue life of a notched specimen is prolonged remarkably if the specimen is initially stretched by one cycle, and is shortened if it is compressed by one cycle. In these cases, Miner's law⁽³⁾ can not be applied⁽⁴⁾. These results seem to have been caused by the residual stresses induced in the bottom of the notch.

The author made a fatigue testing on plain and notched specimens turned from a heat treated carbon steel, under a pulsating load varying in two stages, in order to investigate the validity of Miner's law in this case.

2. Specimens and experimental method

Fatigue tests were carried out with a Losenhhausen type pulsator of 5 tons, under a frequency of 2000 cycle per minute. Test specimens were turned as Fig. 1 from heat treated 0.45% carbon

steel bars. Heat treatment was as follows: A bar was quenched from 850°C in oil, and then tempered from 600°C in furnace. Chemical compositions and mechanical properties are shown in Table 1.

The experiments were carried out under a pulsating load keeping the minimum stress constant at 3.5 kg/mm², and varying only the maximum stress, as shown in Fig. 2. An ordinary S-N curve

Fig. 1 Dimensions of specimens

Table 1 Chemical compositions and mechanical properties of test material

Material	C %	Si %	Mn %	P %	S %	Tensile strength kg/mm ²	Yield point kg/mm ²	Elongation %	Area contraction %
S45C	0.50	0.24	0.67	0.041	0.046	80	51	21	53

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was obtained for plain and notched specimens respectively. Then fatigue tests were carried out varying the maximum stress in two stages as shown in Fig. 2. Primary stress was repeated till the cycle ratio became 10%, 28% or 60%, and then secondary stress was repeated till the specimen fractured. Thus cumulative cycle ratios were investigated.

3. Experimental results and considerations

3.1 Constant stress amplitude fatigue test

Ordinary fatigue test results under constant stress amplitude are shown in Fig. 3 for plain and notched specimens. Fatigue limits for plain and notched specimens were 50.5 kg/mm² and 34.5 kg/mm² respectively. Through the middle of the

experimental points, a curve is drawn. On the right and left sides of this curve, two curves of $\pm 2\sigma$ are drawn, where σ is a standard deviation. These curves are drawn assuming that the standard deviation of fatigue life is equal for all stress levels. The value of the standard deviations thus calculated are shown in Table 2.

3.2 Fatigue testing under pulsating load varying in two stages The experimental results of two stage loading for plain and notched specimens are shown in Tables 3 and 4 respectively. In these Tables, primary stress σ_1 , secondary stress σ_2 , number of cycles at primary stress n_1 , number of cycles till fracture at secondary stress n_2 and cumulative cycle ratio $\sum(n/N)$ are shown. Cumulative cycle ratio should be unity, if linear damage law, i.e., Miner's law is valid. According

Table 3 Two stage load fatigue test results of plain specimens

Test piece	Hardness BHN	σ_1 kg/mm ²	σ_2 kg/mm ²	$\frac{n_1}{N_1} = \alpha$	$n_1 \times 10^5$	$n_2 \times 10^5$	$\alpha N_2 + n_2 \times 10^5$	$\frac{n_2}{N_2}$	$\sum \frac{n}{N}$
13	243	63.5	59.5	0.28	0.53	3.10	4.00	0.95	1.23
24	237	59.5	55.5	0.10	0.33	6.41	7.04	1.02	1.12
10	244	"	"	0.28	0.90	>147	>149	>23	>23.3
29	229	"	"	0.60	1.95	2.27	6.05	0.36	0.96
31	238	55.5	51.5	0.28	1.75	>100	>107	>4.00	>4.30
27	237	51.5	55.5	"	6.93	10.39	12.14	1.65	1.93
12	242	55.5	59.5	"	1.75	4.43	5.33	1.36	1.64
32	233	59.5	63.5	0.10	0.33	1.11	1.30	0.59	0.69
11	243	"	"	0.28	0.90	1.20	1.73	0.63	0.91
30	229	"	"	0.60	1.95	2.31	3.45	1.21	1.81
28	237	63.5	55.5	0.28	0.53	3.05	4.80	0.49	0.77
26	234	55.5	63.5	"	1.75	1.25	1.78	0.66	0.94

Table 2 Standard deviation of life in constant stress amplitude fatigue test

Test piece	Standard deviation* σ
Plain	0.0420
Notched	0.0495

$$* \sigma = \sqrt{\frac{\sum (\log N - \log N_m)}{n}}$$

Fig. 2 Two stage load fatigue test

Fig. 3 S-N curve for plain and notched specimens

to the results obtained here, the values of cumulative cycle ratio are scattered around unity. Figs. 4 and 5 show the test results at two stage loading when the cycle ratio of primary stress n_1/N_1 is 0.28, taking the value of secondary stress σ_2 in ordinate, and the value $\alpha N_2 + n_2$ i. e. converted

value of primary stress cycles for secondary stress added by n_2 in abscissa. These experimental points should be on the S-N curves for one stage testing shown in these Figures, if Miner's law is valid. Considering the scatter of the experimental results, however, we can conclude that the Miner's

Table 4 Two stage load fatigue test results of notched specimens

Test piece	Hardness BHN	σ_1 kg/mm ²	σ_2 kg/mm ²	$\frac{n_1}{N_1} = \alpha$	$n_1 \times 10^5$	$n_2 \times 10^5$	$\alpha N_2 + n_2 \times 10^5$	$\frac{n_2}{N_2}$	$\sum \frac{n}{N}$
34	239	47.3	43.5	0.10	0.08	1.76	1.94	0.98	1.08
8	219	"	"	0.28	0.22	0.91	1.41	0.51	0.79
30	242	"	"	0.60	0.48	1.28	2.36	0.71	1.31
33	244	45.5	41.5	0.10	0.12	3.74	4.01	1.36	1.46
32	230	"	"	0.28	0.33	1.99	2.75	0.72	1.00
31	228	"	"	0.60	0.40	0.40	2.05	0.15	0.75
26	230	43.5	39.5	0.10	0.18	4.12	4.54	0.98	1.08
9	239	"	"	0.28	0.50	0.96	2.12	0.23	0.51
27	240	"	"	0.60	1.08	64.80	67.30	15.40	16.00
12	228	39.5	35.5	0.28	1.16	9.39	12.83	0.76	1.04
40	239	47.5	39.5	0.10	0.08	3.31	3.73	0.79	0.89
5	233	35.5	39.5	0.28	3.34	5.25	6.41	1.25	1.53
29	227	39.5	43.5	0.10	0.42	1.26	1.44	0.70	0.80
1	237	"	"	0.28	1.16	2.35	2.85	1.30	1.58
28	238	"	"	0.60	2.52	0.81	1.89	0.45	1.05
35	232	41.5	45.5	0.10	0.28	0.91	1.03	0.76	0.86
36	228	"	"	0.28	0.76	1.42	1.75	1.18	1.46
37	230	"	"	0.60	1.65	0.40	1.12	0.33	0.93
38	230	43.5	47.5	0.10	0.18	0.79	0.87	0.99	1.09
6	217	"	"	0.28	0.50	0.73	0.95	0.91	1.19
39	234	"	"	0.60	1.08	0.69	1.17	0.86	1.46

Fig. 4 Two stage load fatigue test results of plain specimens (1)

Fig. 5 Two stage load fatigue test results of plain specimens (2)

Fig. 6 Two stage load fatigue test results of plain specimens (3)

Fig. 7 Two stage load fatigue test results of plain specimens (4)

law is valid if the experimental points lie in the range of $\pm 2\sigma$ curves. It is also said that cumulative cycle ratio exceeds unity if the experimental points lie in the right side of the $+2\sigma$ curve, and that it is less than unity if they lie in the left side of -2σ curve.

Fig. 4 shows such results that the difference between primary and secondary stress is 4 kg/mm^2 . According to these results cumulative cycle ratio $\sum (n/N)$ exceeds unity if the secondary stress approaches to the fatigue limit. Fig. 5 shows the result when the primary stress σ_1 is constant and the secondary stress σ_2 is increased or decreased. Figs. 6 and 7 represent the relation between $\{1 - (n_2/N_2)\}$ and (n_1/N_1) , for the case when σ_1 is smaller than σ_2 , and greater than σ_2 respectively. We find, from these Figures, that cumulative cycle ratio sometimes increases greatly at any cycle ratio of primary stress. Figs. 8 and 9 show the test results for notched specimens, corresponding to Figs. 4 and 5 for plain specimens. Figs. 10 and 11 show the relation between $\{1 - (n_2/N_2)\}$ and (n_1/N_1) for notched specimens.

3.3 Consideration on the results In the two stage fatigue testing, the fatigue life comes into

Fig. 8 Two stage load fatigue test results of notched specimens (1)

Fig. 9 Two stage load fatigue test results of notched specimens (2)

Fig. 10 Two stage load fatigue test results of notched specimens (3)

Fig. 11 Two stage load fatigue test results of notched specimens (4)

question. In such a case, scatter of the life can not be neglected. As is clear from Fig. 3 or Table 2, twice the standard deviation of life 2σ is 0.084 and 0.099 for plain and notched specimens respectively. Accordingly, the ratio of life corresponding to 2σ to the mean life at any stress level becomes as follows:

for plain specimen $N/N_m = 1.21$

for notched specimen $N/N_m = 1.26$

If we assume in Miner's law, $\sum (n/N) = 1$, that N scatters as much as 2σ from N_m , then cumulative cycle ratio $\sum (n/N)$ will also scatter, in place of unity, as follows:

0.83~1.21 for plain specimens

0.80~1.26 for notched specimens

In other words, Miner's law is said to be valid if cumulative cycle ratio scatters within this range. The value of safety factor for fatigue life, which is necessary in calculating fatigue life with Miner's law in machine design, will also be presumed from this range of scatter.

According to the present experiment, Miner's law may be used for predicting the fatigue life for plain and notched specimens so long as the type of load application is the same as with the present

experiment. There are a few exceptions of cumulative cycle ratio increasing remarkably, but this would be on the safe side in machine design.

4. Conclusion

Fatigue tests were carried out with plain and notched specimens turned from a heat treated 0.45% carbon steel under a pulsating load varying in two stages. The value of primary stress and the cycle ratio of primary stress were varied for the cases where the primary stress was higher and lower than the secondary stress. The result of this experiment showed that Miner's law nearly holds with a few exceptions, if the scatter of the fatigue life is taken into consideration. In a few cases, the cumulative cycle ratio increased remarkably above unity. This allows Miner's law to be used in life estimation because the scatter of the fatigue life in this side is conservative.

These experiments should be continued here-

after, but using at least 10 specimens for the same experiment, in order that the statistical evaluation can be made.

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Fatigue Tests on Small-Sized Springs*

(Development of Small-Sized Spring Fatigue Testing Machine and Some Experiments of Electrical Apparatus Springs)

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In presenting this article, it is our desire to describe the development of a fatigue testing machine for bent wire springs and shaped flat springs in sizes ranging from 0.1 mm to 1 mm diameter or thickness, and some consideration on the repeated bending fatigue strength of corrosion resistant spring materials; nickel silver, phosphor bronze, and 18-8 stainless steel.

The results indicate that when a cantilever spring is vibrated by applying force to it at the point 74.5% of the whole length from the fixed end, the effect of circular frequency of excitation in the fatigue tests is the smallest.

The fatigue strength of an 18-8 stainless steel spring is notably increased as the section size of the spring is decreased, but in the case of nickel silver, the size effect on the fatigue strength is small.

1. Introduction

Structural design of an electrical apparatus is based on the fatigue strength of movable springs.

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In modern times, there are several corrosion resistant spring materials in which bent wire springs with a diameter of 0.4~1 mm, and shaped flat springs with a thickness of 0.1~0.6 mm are usually used. Considering the construction of their spring systems, the load of the spring is in the form of repeated bending. When parts of the electrical apparatus are designed, the internal stress of the