the tests which affects the ϕ -value for $\eta_{\max}$ point and the different v_{m0} -values along the impeller inlet edge under actual flow conditions.

The value of $k_d \min$ was smaller for the axial-flow impeller than for the mixed-flow type, as shown in Fig. 15. This is connected also with the velocity distribution along the impeller inlet edge, and further with the expression of cavitation parameter as

$$k_d = \frac{H_{sv} - v_{m0}^2/2g}{w_{0a}^2/2g}$$

where the second term of the numerator is given with the mean velocity along impeller inlet edge.

In a strict sense, therefore, the hydrofoil section should be modified by the two- or three-dimen-

sional method, as pointed out by Doctor Murai.

Eq. (9) is developed under the assumption of

$$\beta_{1a} = \tan \beta_{1a} = \sin \beta_{1a}, \quad 3D_{1a}^2 \gg d_n^2$$

Usually $\beta_{1a} \leq 20^\circ$ so that the error under the assumption does not exceed +2% or -4%; d_n/D_{1a} on the other hand is about 0.3 and $d_n^2/3D_{1a}^2$ becomes 0.03. Thus, the errors are considered to be negligible.

As for the shut-off horsepower, it should be mentioned that it varied in proportion to the square of inlet diameter within the range of this study. The shut-off horsepower is affected by many factors as is wellknown, and its correlation with inlet diameter should be adjusted for pumps with different N_s .

621.694.3

Study of Prototype 2-Stage Jet Type Pump*

(1st Report, Characteristic Tests)

By Mitsuhiro HOSHI**, Iwao YOSHIKAWA***, and Koshiro SAKAI**

The jet pump works by means of tendency of a stream or jet of fluid to drive or carry contiguous particles along with it. In a jet pump the maximum possible realized suction capacity Q_s will be twice the driving capacity Q_0 , because of limitations in area ratio between nozzle and throat.

In the case of the newly designed 2-stage jet pump, the suction openings of the 1st and 2nd stage are connected on an even water level. Further, suction water of the 1st stage is also utilized for driving the 2nd stage. One primary objective of designing this trial pump was to achieve $Q_s/Q_0=4$.

Changes in construction design, based on experimentation, were incorporated in the test pump unit on which test results were obtained, as follows:-

- (1) $Q_s/Q_0 > 4$ was obtained as anticipated; further efficiency is favorable.
- (2) The pump without a diffuser in the first stage outlet showed higher performance than that of a pump having a diffuser.
- (3) The function of the 1st stage was not influenced by opening or closing the discharge valve. The performance of the pump was, however, affected critically by operation of the suction valve of the 1st stage.

1. Preface

In the jet pump it is known that the suction capacity Q_s does not exceed 2 times the driving

capacity Q_0 , because of limitation in area ratio between nozzle and throat. Therefore, an increase in suction capacity is held within close limitations for a given driving head, even though the driving head is much higher than the total jet pump head H_T . We could not increase suction capacity beyond these definite limitations, so we had to reduce the

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H_0 factor at the inlet of the jet pump to a suitable value. This method is not the most desirable because we do not realize the full benefit of the H_0 factor. We designed and manufactured a 2-stage jet type pump on a new concept for utilizing these H_0 factors. In this pump the suction openings of the 1st and 2nd stages were connected to the unit on an even water level. Further, suction water of the 1st stage was utilized for driving the 2nd stage with $Q_s/Q_0=4$ as a target.

2. Nomenclature

- H_0 : driving head m
 H_s : suction head m
 H_d : discharge head m
 Q_0 : driving capacity l/min
 Q_s : suction capacity l/min
 $Q_d = Q_0 + Q_s$: discharge capacity l/min
 $R = \frac{\text{Nozzle area}}{\text{Throat area}}$ non-dimensional

Suffixes 1 and 2 to these symbols indicate 1st stage and 2nd stage, respectively.

For performance of the 1st stage individually⁽¹⁾⁽²⁾:-

$$M_1 = \frac{Q_{s1}}{Q_{01}} \dots\dots\dots (1)$$

non-dimensional capacity ratio

$$N_1 = \frac{H_{d1} - H_{s1}}{H_{01} - H_{d1}} \dots\dots\dots (2)$$

non-dimensional head ratio

$$\eta_1 = \frac{Q_{s1}(H_{d1} - H_{s1})}{Q_{01}(H_{01} - H_{d1})} = M_1 N_1 \dots\dots\dots (3)$$

pump efficiency

For performance of the 2nd stage individually:-

$$M_2 = \frac{Q_{s2}}{Q_{02}} \dots\dots\dots (4)$$

non-dimensional

$$N_2 = \frac{H_{d2} - H_{s2}}{H_{02} - H_{d2}} \dots\dots\dots (5)$$

non-dimensional

$$\eta_2 = \frac{Q_{s2}(H_{d2} - H_{s2})}{Q_{02}(H_{02} - H_{d2})} = M_2 N_2 \dots\dots\dots (6)$$

pump efficiency

For performance of the 2-stage jet type pump unit:-

$$\bar{M} = \frac{Q_{s1} + Q_{s2}}{Q_{01}} \dots\dots\dots (8)$$

non-dimensional capacity ratio

$$\bar{N} = \frac{H_{d2} - H_s}{H_{01} - H_{d2}} \dots\dots\dots (9)$$

cf. $H_{s1} = H_{s2} \equiv H_s$ non-dimensional head ratio

$$\bar{\eta} = \frac{Q_{s1}(H_{d2} - H_{s1}) + Q_{s2}(H_{d2} - H_{s2})}{Q_{01}(H_{01} - H_{d2})} \dots\dots\dots (7)$$

pump efficiency

3. Explanation of the 2-stage jet type pump employed in the prototype development

We designed this pump on the following rating:-

$$H_0 = 34 \text{ m} \quad Q_0 = 80 \text{ l/min}$$

$$H_d - H_s = 0.5 - (-0.5) = 1 \text{ m}$$

$$Q_s = 320 \text{ l/min}$$

Fig. 1 shows the pump assembly cross section. The throat of the 1st stage serves as nozzle of the 2nd stage.

Fig. 2 shows the revised type which consists of 2 original jet pumps connected in series and the hatched parts in Fig. 2 are used in common with those of Fig. 1.

Principal dimensions are as follows:-

- Inlet diameter of driving water: 25 mm bore
 Suction diameter of 1st stage : 38 mm bore
 Suction diameter of 2nd stage : 65 mm bore
 Discharge diameter : 75 mm bore
 1st stage nozzle diameter :
 7 mm, 8.2 mm, 9 mm (3 kinds)
 Throat diameter of 1st stage = nozzle diameter of 2nd stage: 18 mm

① First stage casing, ② First nozzle, ③ Second stage casing, ④ Second nozzle, ⑤ Throat sleeve, ⑥ Throat, ⑦ Conical increaser

Fig. 1 Assembly cross section of 2-stage jet pump without diffuser

① First stage casing, ② First nozzle, ③ Diffuser, ④ Second nozzle holder, ⑤ Second stage casing, ⑥ Throat sleeve, ⑦ Throat, ⑧ Conical increaser, ⑨ Second nozzle

Fig. 2 Assembly cross section of 2-stage jet pump with diffuser

4. Test apparatus and procedure

4.1 Test apparatus Fig. 3 shows an outline of the test apparatus. The driving water for the jet pump was fed by a 4-stage volute pump of 40 mm bore. Driving water capacity was regulated by a valve and a bypass, and capacities were measured by venturi-meters arranged between the 4-stage volute pump and the jet pump. The suction of the 2nd stage was piped downward through a venturi-meter into the suction pit. In a practical arrangement the suction of the 1st stage must be connected to this suction pit, too, but we connected it to a separate suction tank through a venturi-meter and valve. The separate suction tank was used in the tests to permit change and measurement of Q_{s1} , and to determine full and detailed characteristics of pump. The separate suction tank was set above its pump to avoid cavitation in the venturi-meter. A discharge valve was located on the discharge side of jet pump so that Q_d could be adjusted.

Fig. 4 is a photograph of the test apparatus.

4.2 Test procedure Characteristics of all venturi-meters for measuring Q_{01} , Q_{s1} and Q_{s2} were

calibrated prior to test by the weighing tank method. Driving head H_0 was measured by a calibrated Bourdon tube pressure gage and the other heads, i.e., H_{s1} , H_{s2} , H_{d1} , H_{d2} , were measured by water and mercury manometers, respectively. Velocity head at measuring point was taken into consideration. As to the measurements, we measured H_{s1} , H_{s2} , H_{d2} , vs. Q_d for several Q_{s1} , keeping $H_0 = 34$ m constant. It was necessary to adjust the 1st stage suction valve slightly in order to keep Q_{s1} constant when changing the opening of the discharge valve.

5. Test results

Four series of tests were conducted, as shown in Table 1. Figs. 5 to 8 show the results of these tests. Fig. 9 shows the characteristics of four pumps measured under full opening of the 1st stage suction valve for the sake of comparison. Furthermore, we tested each stage of the jet pump individually as a single stage pump. The head-capacity and efficiency curves expressed by non-dimensional coefficient are shown in Figs. 10 and 11. Fig. 10 is a curve of the 1st stage and Fig. 11 one of the 2nd stage.

Table 1 Dimensions of 2-stage jet pump

Pump	Test No.	Nozzle dia. at 1st stage mm	Throat dia. at 1st stage mm	Diffuser	Nozzle dia. of 2nd stage mm	Throat dia. of 2nd stage mm	Fig. No.
8.2-18D-44	T-5	8.2	18	With	18	44	5
7-18L-44	T-8	7.0	18L*	Without	18L*	44	6
8.2-18L-44	T-7	8.2					7
9-18L-44	T-16	9.0					8

* Constructed as a single unit

Fig. 3 Outline drawing of test apparatus

Fig. 4 Photograph of test apparatus

6. Study of test results

6.1 Aspects of operation (A) When the opening of 1st stage suction valve was held constant, operating the discharge valve exerted little influence on Q_{s1} .

That is, it is apparent that the working point of the 1st stage was independent of Q_d .

(B) It was found that operating the 1st stage suction valve produced major changes in the jet

Fig. 5 Results of test series No. 8.2-18D-44

Fig. 6 Results of test series No. 7-18I-44

characteristics. That is, as shown in Fig. 5 to 8, each Q - H curve is formed individually corresponding to each different Q_{s1} . Each Q - H curve shows the same aspects as a single stage jet pump, driven with different driving heads. The above-mentioned properties (A) and (B) do not include the effects of diffuser. Since we could measure H_{d1} in the case of a pump having a diffuser, we developed

Fig. 7 Results of test series No. 8.2-18L-44

Fig. 8 Results of test series No. 9-18L-44

M - N curves for each stage. These curves are shown with the performance curves of each stage, obtained individually as a separate pump (Fig. 12). As a result of this comparison, we found that both

Fig. 9 Performance curves in case of full opening of 1st stage suction valve

Fig. 10 Dimensionless performance curves of 1st stage pump

Fig. 11 Dimensionless performance curves of 2nd stage pump

curves coincide closely. In other words, in the case of a pump having a diffuser, an M_2 - N_2 curve is drawn individually corresponding to an arbitrary point on an M_1 - N_1 curve.

6.2 Study on suction capacity and pump efficiency The characteristic curves of a pump working under $(H_{d2}-H_{s2})=(H_{d2}-H_{s1})\equiv(H_{d2}-H_s)$ conditions were plotted from the above-mentioned test records and are shown in Fig. 13. The values of Q_s/Q_0 and $\bar{\eta}$ at $H_{d2}-H_s=1$ m for 4 pumps are tabulated in Table 2. In comparing these data with the performance of an ordinary single stage jet pump, which is designed under the same specifications, we find that the former is superior. When we design an ordinary single stage jet pump the minimum value of R should be 0.165 for a practical pump⁽⁸⁾, therefore the following two cases should be adopted for such a high driving pressure.

Case A: The pump was designed so as to

Fig. 12 Performance curves for each stage of 2-stage pump. M_1 - N_1 and M_2 - N_2 curves denote performance of each stage of 2-stage pump (8.2-18D-44), and are compared with individual test of (8.2-18) and (18-44), respectively

Fig. 13 Performance curves of 2-stage jet pump at $(H_{d2}-H_{s1})=(H_{d2}-H_{s2})$

operate with best efficiency and to get maximum pumping capacity by reducing H_0 to suitable head value by means of a pressure reducing valve. In this case, assuming $\eta = 24\%$ and $M = 2.0$, N becomes 0.12. So if Q_0 were given as 80 l/min, Q_s should be 160 l/min and H_0 should be 8.82 m. Namely, the efficiency of this pump may be 24%, but if we take all losses into consideration, the total efficiency may be cut down to 5.8%.

Case B: We used H_0 as original and made the pump operate at a point apart from the b.e.p. In this case available N is 0.029 and M becomes 2.3, then we will get $Q_s = 184$ l/min and $\bar{\eta} = 6.7\%$. In both cases, Q_s/Q_0 and $\bar{\eta}$ are much less favorable than the values shown in Table 2.

6.3 On the non-dimensional expression In considering the efficiency of a 2-stage jet pump, efficiency may be expressed in a similar manner to the ordinary type pump,

$$\bar{\eta} = \frac{Q_{s1}(H_{d2} - H_{s1}) + Q_{s2}(H_{d2} - H_{s2})}{Q_{01}(H_{01} - H_{d2})} \quad (7)$$

And in case of $H_{s1} = H_{s2} = H_s$, Eq. (7) becomes

Table 2 Comparison of characteristics at condition $H_{d2} - H_s = 1$ m

Pump	Test No.	Q_{01} l/min	$Q_s = Q_{s1} + Q_{s2}$ l/min	Q_s/Q_{01}	η %	Fig. No.
8.2-18D-44	T-5	80	90+266=356	4.45	14	5
7-18L-44	T-8	58	100+160=260	4.48	15	6
8.2-18L-44	T-7	80	105+243=348	4.35	13	7
9-18L-44	T-16	100	110+306=416	4.16	14	8

Table 3 Comparison of $\bar{M}$ and $\bar{N}$ at $\bar{\eta}_{\max}$ in Fig. 14

Pump	Test No.	$\bar{\eta}_{\max}$ %	$\bar{M}$	$\bar{N}$
8.2-18D-44	T-5	13.3	3.8	0.035
7-18L-44	T-8	14.5	4.2	0.034 5
8.2-18L-44	T-7	14.9	3.3	0.045
9-18L-44	T-16	16.1	3.0	0.053 6

Fig. 14 Dimensionless performance under conditions $H_{s1} = H_{s2}$

$$\bar{\eta} = \frac{(Q_{s1} + Q_{s2})(H_{d2} - H_s)}{Q_{01}(H_{01} - H_{d2})} \quad (7')$$

In order to transform Eq. (7)' into non-dimensional form, the following substitutions are made.

$$\frac{Q_{s1} + Q_{s2}}{Q_{01}} = \bar{M} \quad (8)$$

$$\frac{H_{d2} - H_s}{H_{01} - H_{d2}} = \bar{N} \quad (9)$$

then Eq. (7)' becomes

$$\bar{\eta} = \bar{M}\bar{N} \quad (10)$$

with this equation, efficiency is calculated by a formula similar to that for the ordinary type single stage jet pump. Fig. 14 shows the relationship between N and M , calculated according to Eqs. (8) and (9), using indicated values of H_{01} , H_{d2} , Q_{s1} , Q_{s2} and Q_{01} at the same $H_{s1} = H_{s2}$ value. The maximum value of $\bar{\eta}$ for each pump are tabulated in Table 3. However, since a valve was not located at the 2nd stage suction nozzle, data concerning variable driving heads were not be obtained, so we could obtain only one M - N curve for one set of nozzles and throats in this paper. Therefore, we could not confirm that the relationship between M , N and $\bar{\eta}$ can be defined as one M - N curve for several conditions.

6.4 On the effects of a diffuser Generally speaking, the characteristics of a pump designed without a diffuser is better than one with a diffuser. In the case of a pump designed with a diffuser, the 2nd stage is driven by the 1st stage discharge head (see Item 6.1). In the case of the pump without a diffuser H_{d1} could not be measured due to structural characteristics of the unit. Therefore, we computed H_{d1} and H_{02} as follows.

(Computation of H_{d1})

From Eq. (2),

$$H_{d1} = \frac{N_1 H_0 + H_{s1}}{1 + N_1} \quad (11)$$

Using the relation of M_1 - N_1 shown in Fig. 10, N_1 value is determined for a given M_1 , and then we can compute H_{d1} from Eq. (11). In this case the loss with a diffuser has to be taken into account. Therefore, at throat end of the 1st stage, the discharge head H_{d1}' is

$$H_{d1}' = H_{d1} + L_c \quad (12)$$

where L_c : loss for a diffuser.

Diffuser loss is given as follows⁽⁴⁾.

$$L_c = 0.1 \frac{C_{d2}^2 - C_{d1}^2}{2g} \quad (13)$$

where C_{d1} and C_{d2} are the velocities in m/sec at diffuser inlet and outlet section.

(Computation of H_{02})

From Eq. (5)

$$H_{02} = \left(\frac{H_{d2} - H_{s2}}{N_2} \right) + H_{d2} \quad (14)$$

Fig. 15 2nd stage driving head (calculated) versus 1st stage discharge head (calculated)

Using Eq. (14), Eq. (4) and the relationship of $M_2 - N_2$ shown in Fig. 11, we can compute H_{02} . The head at the nozzle end has to be less than H_{02} due to the loss in the straight pipe L_s . Then

$$H_{02}' = H_{02} - L_s \quad (15)$$

Here pipe loss L_s is determined by Reference⁽⁵⁾, then

$$L_s = 0.022 \frac{1}{d} \frac{C^3}{2g} \quad (16)$$

where C denotes velocity in the pipe in m/sec. Fig. 15 shows the relation between H_{d1}' and H_{02}' in case of tests 7-18L-44, 8.2-18L-44 and 9-18L-44. The reason why H_{02}' is larger than H_{d1}' in the case of a pump having no diffuser, as shown in Fig. 15, will be one of our subsequent themes of study. There should be some good effects at the junction of 1st and 2nd stages when designed without a diffuser.

7. Conclusion

We obtained results from the experiments on a 2-stage jet type pump having a high driving head which indicates that design and construction are more critical than experienced with an ordinary single stage jet pump.

Results of the tests are as follows:-

- (1) That the prototype jet pump could utilize

a sufficiently high driving head to realize the objective of attaining the extremely high capacity ratio of $Q_s/Q_0=4$. Moreover, it was proved that the efficiency of a 2-stage jet type pump is considerably better than an ordinary single stage type.

- (2) That a jet pump designed without diffuser was better than one with a diffuser, in both characteristics and pump size.

- (3) Some aspects of operation were as follows.

- (a) Operating the discharge valve produced no influence on the operating conditions of the 1st stage.

- (b) Characteristics varied widely when the valve of the 1st stage suction was operated. Individual $Q-H$ performance curves were developed for arbitrary Q_{s1} values.

In view of non-programming of pilot production of this prototype two stage jet type pump we wanted to investigate further and clarify several questions regarding the relationship of Q_s/Q_0 , effects of diffuser and the relationship of driving head to pump operation. Though we could solve some of these questions by these current experiments, we propose that additional points should be investigated in the future, as follows:-

- (1) To determine the general characteristics of multistage jet pumps, and to indicate with a non-dimensional factor.

- (2) To clarify fully the relationship between 2-stage type and single stage jet type pumps.

- (3) To investigate further the influence of nozzle area ratio R_1 to R_2 on the characteristics of a 2-stage jet type pump.

- (4) To investigate how to determine the 1st stage throat diameter required for a 2-stage jet pump.

References

- (1) A.J. Stepanoff: *Centrifugal and Axial Flow Pumps*, (1957), Chap. 18, p. 404.
- (2) Ref. (1) p. 405.
- (3) Ref. (1) p. 406.
- (4) T. Ueda: *Trans. Japan Soc. Mech. Engrs.*, Vol. 20, No. 89 (1954), p. 30.
- (5) *Mechanical Engineering Handbook* (1954), Chap. 8, p. 20, Japan Soc. Mech. Engrs.

Discussion

T. ASANO: (1) The annular passage areas around the first and second stage nozzles look different; one is considerably narrow and the other considerably broad. The discussor wants to know how the shapes of these areas were decided.

- (2) The authors' conclusion about the better

operating characteristics of a pump without a diffuser seems to be based on the frictional loss arising from acceleration under decreased pressure at the second nozzle following deceleration under increased pressure at the diffuser. The discussor thinks that the former process of deceleration is needless; it

would be more reasonable to allow a necessary mixing length.

(3) What do the authors think of the conventional multi-stage jet pump with the first stage and second stage suction holes consolidated into one?

K. TABUSHI: (4) It seems that there is no need to insert a diffuser between the first stage throat and the second stage nozzle.

(5) When H_0/H_s is very high, the operation becomes unstable due to cavitation. A branch pipe, for example, connecting the first stage suction and the pump exit may control such cavitation.

(6) In the case stated above, Q_s is obtained from flow quantities measured at pump exit, pressure water inlet and second stage suction.

K. TAJIMA: (7) In the performance comparison of jet pumps with and without a diffuser, the effect of the diffuser will be made more clear, if a curve is plotted with $Q_{01} + Q_{s1}$ on the abscissa against, say, the efficiency difference for constant Q_s . In this connection, if the coefficients in Eqs. (13), (16) are calculated from the experimental results with, for instance, the conception of $H_{02}' = H_{d1}'$, it will provide useful reference.

Authors' closure

(1) The annular passage area around the 2nd stage nozzle was determined by the same method as for the 1st stage nozzle. Similar nozzles with appropriate annulation were used for both 1st and 2nd stage in test 8.2-18D-44 (Fig. 5) and in test 8.2-18L-44 (Fig. 7). The shapes of water annular passages around all nozzles were similar in general to those of a standard single stage jet type pump.

(2) (i) We forecast that the operating characteristics of a pump having a diffuser would be lower than those of a pump without a diffuser, because of added losses without compensating the gain in operating efficiency. However, we made a 2-stage pump with a diffuser to test and to determine the effect of a diffuser. The primary study was made on a pump without a diffuser for the purpose of examining the relationship between a 2-stage type pump and the ordinary single stage type pump which made up the 2-stage type pump unit.

(ii) We calculated H_{02}' and H_{d1}' by taking into consideration the diffuser loss of the 1st stage diffuser and the straight pipe loss of 2nd stage inlet, as shown in Eq. (11) to (16). The comparison between these values and measured value is shown in Fig. 15. In this diagram we can find $H_{d1}' > H_{02}'$, i. e. $H_{d1}' \neq H_{02}'$. This means that the 2nd stage jet

pump was driven with a higher driving head than the expected value at the 1st stage discharge. This appeared to be unreasonable and we considered at first that there was some question in regard to calculation of losses. But upon examination we found that the jet spray from the 2nd nozzle of the pump without a diffuser was wider than that from the jet of a pump with a diffuser, and this phenomenon would have an influence on the 2nd stage jet pump characteristics. This assumption is presumed on the experienced fact that a nozzle having a jet regulated by a needle nozzle has lower efficiency than a nozzle not regulated. For these reasons we consider that a jet pump without a diffuser is superior if suitable length for mixing is provided in the nozzle. But determination of optimum length for mixing is an important question to be solved later, as stated in "conclusions".

(3) In 1879, Greathead published data on a pump as shown in Append-Fig. 1. There are several different points of design between this pump and the 2-stage jet type pump made by us, as follows:-

(i) Our purpose for developing the 2-stage jet type pump was to gain pumping capacity when we have a higher driving head; but it appears that Greathead wanted to decrease the collision loss because low collision loss produces good pump efficiency. On this point, our pump has suitable mixing length in the 1st stage throat, but Greathead's pump seems not to have such length.

(ii) As Greathead's pump has only one suction nozzle, it must have only one Q - H curve for constant driving head and capacity conditions. But our pump gives many characteristic curves by changing Q_{s1} under constant driving head and capacity conditions.

(4) Please see the answer to Prof. T. Asano.

(5), (6) The present series of studies and

Append-Fig. 1 Greathead's pump published in 1879

tests did not specifically cover characteristics of suction and suction head. The question of suction is considered of sufficient importance to warrant further investigation, and will be covered in a subsequent report.

(7) Rearranging these data on diffusers as suggested may be effective in pointing out the

reasons of whether the difference in characteristic between existence and non-existence of diffuser is caused chiefly by diffuser loss or another reason, but we do not have sufficient data on the subject at this time to "pin-point" the reasons. Further investigation of the diffuser question will be conducted and included in a future report.

536.252:532.5.011

Heat Transfer from an Inclined Thin Flat Plate by Natural Convection*

By Itaru MICHIOYOSHI**

This paper deals with a theoretical analysis of heat transfer from an inclined thin flat plate by natural convection, by using same analytical method as the author⁽¹⁾ did for the heat transfer from a horizontal thin flat plate.

If the laminar boundary layer exists all around the plate, the local heat transfer coefficient increases at a place near the lowest point of plate, and the difference of the coefficient between the upper and lower surface decreases as the inclined angle from the horizontal plane is increased. The average heat transfer coefficient of the lower surface is larger than that of the upper surface, and the average coefficient increases as the plate approaches to the vertical position.

1. Introduction

The heat transfer by natural convection from a horizontal thin flat plate was previously analyzed by present author⁽¹⁾, and it was found that the distribution of local coefficient of heat transfer on the plate agreed very well with Weise's⁽²⁾ experimental results in the case of laminar convection. And the relationship among the Nusselt, Prandtl and Grashoff numbers was derived.

This paper deals with a theoretical analysis of heat transfer from such a thin flat plate that is inclined from the horizontal plane, when the Prandtl number of fluid is close to unity, by using the mathematical procedure analogous to the previous report.

2. Theoretical analysis

In this paper, an ellipsoid, of which the cross section is a thin ellipse as shown in Fig. 1 and whose length is infinity in the direction normal to both

major and minor axes, is assumed to be a thin flat plate which has certain finite thickness and side length. This assumption is analogous to that of previous paper.

When the temperature of such plate is higher than the surrounding fluid, the natural convection occurs around the plate. If we assume that the convective flow is two dimensional in steady state and the fluid flows along the circumference of the ellipsoid and thus the thickness of laminar boundary layer around the plate is very thin, the following fundamental equations will be obtained, which are similar to those derived by Hermann⁽³⁾ for the natural convection outside the horizontal cylinder:

Fig. 1 Co-ordinate

* Received 4th April, 1963.

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