536.25:536.33

Temperature Distribution and Efficiency of a Single Sheet of Radiative and Convective Fin Accompanied by Internal Heat Source*

By Yoshizo OKAMOTO**

When the finned heat-transfer surface is used for heat exchangers, space radiators, or nuclear fuel elements under high temperature, there are some cases when the fin emits radiative and convective energy and are accompanied by internal heat source. There are some reports on the simple behavior of finned heat-transfer surface. But the mutual heat-transfer mechanism of fins has not yet been analysed. This report gives an analysis of the radiative and convective surface accompanied by an internal heat source. The steady state conduction equation is introduced, and solved numerically by the digital computer. Temperature distribution, efficiency and heat-transfer ratio are calculated. Various figures are introduced.

1. Introduction

Up to the present, finned surfaces have been used for many purposes to increase the heat-transfer rate, under high temperature condition. Heat exchangers, space-vehicle radiators, and fuel elements of nuclear reactor, under high temperature condition, require the use of finned surfaces as high-flux heat-transfer units. In these units, the finned surface emits a radiative and convective energy and is accompanied by an internal heat source.

But the theoretical analysis of such heat-transfer mechanism is not yet made. Gardner⁽¹⁾ solved the steady-state one-dimensional conduction equation for various fins (plate, annular, and spine fin). He introduced a strict solution of temperature and efficiency. Minkler⁽²⁾ analysed a convective plate fin accompanied by an internal γ heat-source. On the other hand, the design of space-vehicle radiators required the investigation of net radiation fins. Chamber⁽³⁾ and Eckert⁽⁴⁾ derived the conduction equation of annular fins where radiant energy was emitted into the vacuum space of constant temperature of absolute zero.

But, the mutual heat-transfer mechanism is not yet known.

This report shows the result on a single sheet of a radiative and convective fin accompanied by an internal heat-source.

The steady-state conduction equation of an arbitrary shaped fin is introduced, and solved numerically by the digital computer.

Temperature distribution, efficiency, heat-transfer ratio, and heat-transfer rate are calculated. Finally, various figures are introduced.

2. Basic equation

2-1 Several assumptions In order to establish the basic equation, several restrictions and assumptions are provided for simplification.

(1) The heat flow and temperature distribution throughout the fin are independent of time, i.e. the heat flow is steady.

(2) The fin material is homogeneous and isotropic.

(3) The fin thickness is so small compared with its height that the temperature gradient across the width may be neglected, i.e. the heat flow occurs only in the radial direction.

(4) The fin radiates heat from the fin surface to the other surface, according to Stefan Boltzmann's law.

(5) The heat-transfer coefficient and the radiative emissivity are constant over all the fin surface.

(6) The temperature of surrounding fluid, the fin-root surface, the reflector surface, and the heat-sink surface are uniform.

(7) The heat-transfer at the fin tip surface is negligible compared to that passing through the side.

* Received 28 th June, 1963.

** Research Member, Japan Atomic Energy Research Institute, Tokai-mura, Naka-gun, Ibaraki-ken.

$$\left. \begin{aligned} \Phi_c &= \frac{m \int_0^1 \{\theta(X)-1\} dX}{l(\theta_r^4-1)+m(\theta_r-1)}, \\ \varepsilon_r &= \Phi_r/\Phi, \quad \varepsilon_c = \Phi_c/\Phi, \\ Q &= -\frac{2\lambda_m y_b T_g}{x_f} \frac{d\theta(0)}{dX} \\ &= \frac{2\lambda_m y_b T_g \Phi}{x_f} \{l(\theta_r^4-1)+m(\theta_r-1)\} \end{aligned} \right\} \dots\dots\dots (8)$$

In the result, Φ , Φ_r , Φ_c , ε_r , ε_c and Q are calculated for various values of l , m , n and θ_r .

2.4 A single sheet of thin annular fin When a single sheet of the annular fin accompanied by internal heat source emits radiative and convective energy into the gas, the basic equation (1) is

$$\left. \begin{aligned} R &= \frac{r-r_f}{r_f-r_r} && : \text{dimensionless radius} \\ \theta(R) &= \frac{T(r)}{T_g}, \quad \theta(0)=\theta_r && : \text{dimensionless temperature} \\ l &= \frac{\sigma \varepsilon T_g^3 (r_f-r_r)^2}{\lambda_m y_b 10^8} && : \text{dimensionless radiative number} \\ m &= \frac{\alpha (r_f-r_r)^2}{\lambda_m y_b} && : \text{dimensionless convective number} \\ n &= \frac{q_i (r_f-r_r)^2}{\lambda_m T_g} && : \text{dimensionless internal heat generation number} \\ \rho &= \frac{r_r}{r_f-r_r} && : \text{dimensionless number} \end{aligned} \right\} \dots\dots\dots (11)$$

The dimensionless equation is represented by

$$\frac{d^2\theta(R)}{dR^2} + \frac{1}{R+\rho} \frac{d\theta(R)}{dR} = l\{\theta^4(R)-1\} + m\{\theta(R)-1\} - n \dots\dots\dots (12)$$

The dimensionless boundary condition is represented by

$$R=0; \quad \theta(0)=\theta_r, \quad R=1; \quad \frac{d\theta(1)}{dR}=0 \dots\dots\dots (13)$$

Solving Eqs. (12) and (13), the temperature distribution is introduced.

The fin efficiency Φ , the convective term of the efficiency Φ_c , the radiative term of the efficiency Φ_r , the convective heat-transfer ratio ε_c , the radiative heat-transfer ratio ε_r and the heat-transfer rate Q are represented by the definition.

$$\left. \begin{aligned} \Phi &= \frac{2}{1+2\rho} \frac{l \int_0^1 \{\theta^4(R)-1\} (R+\rho) dR + m \int_0^1 \{\theta(R)-1\} (R+\rho) dR}{l(\theta_r^4-1)+m(\theta_r-1)}, \\ \Phi_r &= \frac{2}{1+2\rho} \frac{l \int_0^1 \{\theta^4(R)-1\} (R+\rho) dR}{l(\theta_r^4-1)+m(\theta_r-1)}, \\ \Phi_c &= \frac{2}{1+2\rho} \frac{m \int_0^1 \{\theta(R)-1\} (R+\rho) dR}{l(\theta_r^4-1)+m(\theta_r-1)}, \\ \varepsilon_r &= \Phi_r/\Phi, \quad \varepsilon_c = \Phi_c/\Phi \\ Q &= -4\pi\lambda_m y_b T_g \frac{r_r}{r_f-r_r} \left(\frac{d\theta(R)}{dR} \right)_{R=0} = 2\pi(1+2\rho)\lambda_m y_b T_g \Phi \{l(\theta_r^4-1)+m(\theta_r-1)\} \end{aligned} \right\} \dots\dots\dots (14)$$

In the result, Φ , Φ_r , Φ_c , ε_r , ε_c and Q are calculated for various values of l , m , n , θ_r and ρ .

3. The temperature distribution and the efficiency for a single sheet of the plate fin

The solution (6) is found by application of the boundary condition (7). Thus the temperature distribution is introduced. The calculation is made by using a digital computer.

represented by

$$\frac{d^2T(r)}{dr^2} + \frac{1}{r} \frac{dT}{dr} = \frac{\sigma \varepsilon}{\lambda_m y_b 10^8} \{T^4(r)-T_g^4\} + \frac{\alpha}{\lambda_m y_b} \{T(r)-T_g\} - \frac{q_i}{\lambda_m} \dots\dots\dots (9)$$

where

$$\begin{aligned} \text{cross section area: } a &= 4\pi r y_b \\ \text{surface area: } A &= 2\pi(r^2-r_r^2) \end{aligned}$$

The boundary condition is given by

$$\left. \begin{aligned} r=r_r; \quad T=T_r \\ r=r_f; \quad \frac{dT}{dr}=0 \end{aligned} \right\} \dots\dots\dots (10)$$

Eqs. (9) and (10) are transformed into the dimensionless form.

3.1 The net radiative plate fin On the net radiative plate-fin, the convective term m and the term of the internal heat generation n become zero. The surface emits radiative energy in absorbing gas.

Fig. 2 shows the temperature distribution $\frac{\theta-1}{\theta_r-1}$ at the dimensionless radiative number $l=1$. The four curves are shown with the fin-root temperature ratio θ_r as a parameter. The greater θ_r is, the more steep the temperature gradient from fin-root to tip becomes. For reference, the temperature of the net convective plate fin at the dimensionless convective number $m=1$ is shown in dotted line. When l is equal to m , the temperature gradient of the radiation fin is more steep than the net convective fin.

Fig. 3 shows the relation between the fin efficiency of the net radiation fin and a root of the dimensionless radiative number with θ_r as a parameter. The relation between the fin efficiency of the net convective fin and the dimensionless convective number m is given by the dotted line. In the case where l is equal to m , the radiative efficiency is smaller than the convective efficiency.

Fig. 2 Temperature distribution of the net radiative plate fin

Fig. 3 The fin efficiency of the net radiative plate-fin

3.2 Radiative and convective plate fin For the cases when the radiative and convective emission coexist, the temperature distribution and the efficiency are calculated.

Fig. 4 shows the relation between the root of the dimensionless convective number m and the efficiency Φ , the radiative efficiency Φ_r , the convective efficiency Φ_c , the radiative heat-transfer ratio ϵ_r , and the convective heat-transfer ratio ϵ_c , when the dimensionless radiative number is 0.1. As shown in the Figure, the efficiency Φ approaches asymptotically to the curve of the net convective efficiency Φ_c , and the radiative heat-transfer ratio decays suddenly to zero.

Under this condition, the heat transfer from the surface is only net convection.

Based upon the result of Fig. 4, the relation between the root of the dimensionless convective number 1 and the root of the dimensionless convective number m on the condition of constant ϵ_r and constant Φ is shown in Fig. 5. Four figures at $\theta_r=1.2, 1.5, 2, 3$ are represented. When the variables l, m, θ_r are given, the efficiency Φ, Φ_r, Φ_c and the heat transfer ratio ϵ_r, ϵ_c can be evaluated.

Fig. 6 shows the curves of $\epsilon_r=0.1$, and $\epsilon_r=0.9$ with θ_r as parameter. The area between the longitudinal axis and the curves of $\epsilon_r=0.9$ is the range where the radiation is predominant, and can be considered as the net radiation. The area between $\epsilon_r=0.9$ curve and $\epsilon_r=0.1$ curve is the range where

Fig. 4 $\Phi, \Phi_r, \Phi_c, \epsilon_r, \epsilon_c$ of the radiative and convective plate-fin

the radiation and the convection coexist. The area between transverse axis and $\varepsilon_r=0.1$ curve is the range where radiation is predominant.

3.3 Radiative and convective plate-fin accompanied by internal heat generation For the case when the radiative and convective plate-fin generates the heat energy internally, the temperature distribution and the efficiency are analysed. As shown in Fig. 7, the character of the temperature distribution changes with the value of the heat

generation term n . In general, the temperature and the efficiency become larger with increase in the new dimensionless number

$$\frac{n}{l(\theta_r^4-1)+m(\theta_r-1)} = s.$$

On the limit of $s=1$, the temperature distribution becomes constant. If $s > 1$, the internal heat-generation rate becomes larger than surface heat transfer rate, and the temperature increase from the fin-root to the tip. The heat flows from

Fig. 5 Φ , ε_r constant lines ($\theta_r=1.2, 1.5, 2, 3$)

Fig. 6 Limiting zone of radiation or convection

Fig. 7 Temperature distribution of plate fin accompanied by internal heat generation

the fin into the fin-root.

As the result, the fin is not effective.

When the fin efficiency for the arbitrary value of l , m , θ_r and $n=0$ is given by Φ_0 , the efficiency Φ for the same values of l , m , θ_r including arbitrary value of n is given by the following equation;

$$\Phi = \frac{n}{l(\theta_r^4 - 1) + m(\theta_r - 1)} (1 - \Phi_0) + \Phi_0 \quad \dots (15)$$

Eq. (15) shows the relation between the efficiency with internal heat source and the efficiency with no heat source.

Fig. 8 shows the relation between the efficiency and the root of the dimensionless convective number at $l=1$, $\theta_r=1$.

4. Temperature distribution and the efficiency for a single sheet of the annular fin

The temperature distribution is introduced by solving Eqs. (12) and (13).

4.1 The net radiative annular fin In this case, the dimensionless convective term m and the internal heat term n are zero. Fig. 9 shows the fin efficiency at $\theta_r=2$ with ρ as a parameter. The

Fig. 8 The efficiency of radiative and convective plate fin with heat source ($l=1.0$, $\theta_r=1.2$)

Fig. 10 Φ , Φ_r and ϵ_r , ϵ_c of the radiative and convective annular fin ($l=0.1$, $\rho=1.0$)

smaller ρ is, the more steep the temperature gradient from fin-root to tip becomes. For reference, the curve between the efficiency of the net convective annular fin and the root of the dimensionless convective number m is given by the dotted line with ρ as a parameter.

In the case where l is equal to m , the radiative efficiency is extremely smaller than the convective efficiency.

4.2 Radiative and convective annular fin

For the cases when the radiative and convective emission coexist, the efficiency and the heat-transfer ratio are calculated.

Fig. 10 shows the relation between the root of the dimensionless convective number l and the efficiency Φ , Φ_r , Φ_c and the heat-transfer ratio ϵ_r , ϵ_c , when the dimensionless radiative number l is 0.1 and ρ is 1.0. As shown in the Figure, the efficiency Φ approaches asymptotically to the curve of the net convective efficiency $\Phi_c^{(1)}$. Under this condition, the heat-transfer from the surface is only net convection.

Fig. 9 The efficiency of the net annular fin ($\theta_r=2$)

Fig. 11 shows the curves of constant Φ and constant ε_r .

When variables l , m , θ_r , ρ are given, the efficiency Φ , Φ_r , Φ_c and the heat transfer ratio ε_r , ε_c can be evaluated from Fig. 11.

The behavior of the emission for various parameters l , m , θ_r , ρ is determined from Figs. 5 and 11. The next relation equation is introduced.

- (1) The range where the radiation is predominate. $\varepsilon_c < 0.1$, $\varepsilon_r > 0.9$

$$\frac{l}{m(1-0.3\theta_r)^2} \frac{1+\rho}{\rho} > 2.41, \quad \frac{\sigma \varepsilon T_g^3 T_r^2 r_f}{\alpha (T_r - 0.3 T_g)^2 r_r} > 2.41 \times 10^8 \quad (16)$$

- (2) The range where the radiation and convection coexist. $0.1 < \varepsilon_r < 0.9$, $0.1 < \varepsilon_c < 0.9$

$$0.0325 < \frac{l}{m(1-0.3\theta_r)^2} \frac{1+\rho}{\rho} < 2.41, \quad 3.25 \times 10^6 < \frac{\sigma \varepsilon T_g^3 T_r^2 r_f}{\alpha (T_r - 0.3 T_g)^2 r_r} < 2.41 \times 10^8 \quad (17)$$

- (3) The range where the convection is predominate. $\varepsilon_c > 0.9$, $\varepsilon_r < 0.1$

$$\frac{l}{m(1-0.3\theta_r)^2} \frac{1+\rho}{\rho} < 0.0325, \quad \frac{\sigma \varepsilon T_g^3 T_r^2 r_f}{\alpha (T_r - 0.3 T_g)^2 r_r} < 3.25 \times 10^6 \quad (18)$$

5. Conclusions

Several conclusions were made.

- (1) For the case when the fin surface emits the radiative and convective heat energy and is accompanied by the internal heat source, the one-dimensional conduction equation is obtained.

The total fin efficiency Φ , the radiative efficiency Φ_r , the convective efficiency Φ_c , the radiative heat-transfer ratio ε_r , the convective heat-transfer ratio ε_c , and the heat-transfer rate Q can be defined. These nonlinear equations are solved numerically by the digital computer.

- (2) In the simplified case of generalized equations, non-dimensional conduction equations are solved for a single sheet of plate and annular fins. The temperature and the fin efficiency are calculated for

Fig. 11 Φ , ε_r constant lines

various values of l , m , n , θ_r , ρ .

(3) Under three conditions of (1) net radiation, (2) radiation and convection, and (3) with internal heat source, the characters of the temperature θ_r , the efficiency Φ , Φ_r , Φ_c , and the heat-transfer ratio ε_r , ε_c were analysed.

(4) When the dimensionless radiative number l is equal to the dimensionless number m , the net radiative efficiency is always smaller than the net convective efficiency.

(5) The fin efficiency Φ , Φ_r , Φ_c and the heat-transfer ratio ε_r , ε_c can be evaluated from Figs. 5, 11.

(6) The fin efficiency with internal heat

source was formulated from the fin efficiency with no internal heat source. Eq. (15) shows these relations.

The author is indebted to Dr. F. Tachibana for his advice, and to M. Akanuma, K. Namatame, and S. Negoya for assistance in the work presented.

References

- (1) K.A. Gardner: *Trans. ASME*, Vol. 67, No. 8 (1945), p. 621.
- (2) W.S. Minkler: *Nucl. Sci. Engng.*, Vol. 8, No. 7 (1960), p. 400.
- (3) R.L. Chamber and E.V. Somers: *Trans. ASME*, Ser. C, Vol. 81, No. 4 (1959), p. 327.
- (4) E. R. G. Eckert and others: *Jour. Amer. Rocket Soc.*, Vol. 30, No. 7 (1960), p. 644.