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On the Characteristics of Mixed-Flow Propeller Pumps*

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In order to obtain the characteristic equation of mixed-flow propeller pumps, input head-capacity and hydraulic losses curves are investigated on the basis of experimental results by using various impellers and diffusion casings of different kinds and shapes. Both curves may be represented by flow conditions on the mean flow path, and hydraulic losses in impellers and diffusion vanes may be separated respectively into two parts, one being the friction or diffusion loss and the other shock loss. Each loss-capacity equation and loss coefficient are introduced.

Using these relations, the overall characteristic equation for a mixed-flow propeller pump with an open impeller is obtained. It is seen that the agreement between measured and calculated characteristics by this equation is good except in the backflow region in the impeller. The distribution of each hydraulic loss based on this equation is also discussed.

1. Introduction

It is yet impossible to accurately predict the pump characteristic and the effects of individual variables for both impellers and casings on the overall performance even in the centrifugal pumps which have been used from old, so long as the experimental result of similar pump is not obtained. The reason is probably non-existence of any adequate method by which the input head and hydraulic losses in the pump can be theoretically computed. So an attempt is made in this paper to obtain the calculating method of the performance of a mixed-flow propeller pump from its many experimental results.

2. Test equipment

Sectional elevation of the main part of the test pump is shown in Fig. 1. The driving torque was measured mechanically on a type of swinging-casing 5-HP electric dynamometer, whose speed was measured with a tachometer, and the overall pres-

Fig. 1 Sectional elevation of test pump

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sure rise was measured with a mercury-water U-tube for the higher pressure difference and with an inverted water U-tube for the lower. Small L-type static pressure tubes and cobra-type total pressure tubes with yawmeters, installed in each place shown in Fig. 1, were used for internal measurement.

The mechanical loss was measured by means of driving the impeller hub in water, while the shaft received the same axial thrust as in testing the pump which could be measured from the strain of the diaphragm warped by axial thrust because of touching the thrust bearing, and could be regulated by spring installed in the casing.

The used open impellers had various values of outlet angle and its distribution from hub to tip, tip-clearance, number of vanes, various vane thickness and surface roughness while the same basic design was maintained as far as possible. The outlet angle β_2 of each impeller is shown in Fig. 2, where R_2 is ratio of impeller outlet radius r_2 to its root mean

Fig. 2 Outlet angle distributions of test impellers

square value r_{2m} , and the figure in each impeller number shows the value of outlet angle at the radius r_{2m} ($R_2=1$), β_{2m} .

On the other hand, three kinds of diffusion casing having various inlet angles are used.

3. Input head-capacity characteristics

In order to establish the theoretical equation, which predicts the overall performance of a mixed-flow propeller pump, a calculating method of the actual theoretical head or input head-capacity and the hydraulic loss curves must be obtained.

For the former, some analytical investigations are made by means of potential flow theory⁽¹⁾, but it is so complicated to calculate by their methods the characteristic of a given impeller. Moreover it is necessary to take into consideration the effects of viscosity and tip-clearance in their results. For these reasons, the method based on potential

flow theory has not usually been adopted for obtaining the overall performance curve.

In this paper, therefore, the method of predicting the performance curve is introduced by using the blade characteristic on the particular radius (mean effective radius) of the impeller in the same way as on axial-flow impellers⁽²⁾. The mean effective radius, that is, the situation of a stream line which has the average energy head across the flow passage at the exit of various impellers is shown in both Fig. 3 and 4, where ρ_{2m} is ratio of the mean effective radius to the root mean square radius r_{2m} , and ϕ is flow coefficient ($=v_{2m}/r_{2m}\omega$, v_{2m} is the mean meridian velocity at impeller outlet, and ω the angular velocity).

It may be seen that ρ_{2m} is equal to unity except in the backflow region in each impeller with backward or radial vanes ($\beta_{2m} \leq 90^\circ$). The same results were also found in the experiments on other impellers with different numbers of vanes,

various vane roughnesses or tip-clearances, whose values were within the limits of values ordinarily used. Considering that these impellers are designed for high specific speed ones and consequently their width of flow passage is greater than usual, it may be suggested that the root mean square radius r_{2m} can be generally used as the mean effective radius.

On the other hand, since the meridian-velocity component near the impeller tip decreases under the influence of both tip-clearance and the boundary layer on the wall, the meridian velocity at the mean effective radius is somewhat larger than its mean. Therefore the input head-capacity curve calculated by flow

condition at this radius may be larger than the true one. For example, comparison between the measured input head H_{th} and the calculated one on the basis of the velocity at radius r_{2m} for the impeller M-39 Fo (whose outlet angle distribution has been illustrated in Fig. 2) is shown in Fig. 5, where τ_{th} is the input head coefficient [$\bar{\phi}_{th} = H_{th}/(r_{2m}^2 \omega^2/g)$, g the gravitational acceleration]. The solid line in this figure represents the measured $\tau_{th}-\phi$ curve obtained by subtracting the mechanical loss from input head, the broken line represents the calculated curve obtained by using the actual discharge angle β_{2m}' at the radius r_{2m} ($\cot \beta_{2m}' = 1.66$), and symbol $\circ$ represents $\bar{\tau}_{th}$ obtained by integrating the velocity measured by means of

Fig. 3 Mean effective radius at impeller outlet (effects of outlet angle distribution)

Fig. 4 Mean effective radius at impeller outlet (effects of outlet angle)

Fig. 5 Input head - capacity characteristics

pitot traverses, and symbol $\times$ which corresponds with symbol $\circ$ linked by dash-dot line represents $\bar{\phi}_{th}$ obtained from measured velocity at the radius r_{2m} . For reference, Euler's head $H_{th\infty}$ is also illustrated in this figure, where $\bar{\phi}_{th\infty} = H_{th\infty}/(r_{2m}^2\omega^2/g)$.

It will be seen from this figure that a modification of the discharge is required so as to represent the input head-capacity curve in terms of characteristic on the mean effective radius. Then, Fig. 6 shows the change of the ratio of actual capacity to the calculated one based on the velocity at the mean effective radius, c_ϕ , in the same value of $\bar{\tau}_{th}$ for three kinds of impellers having different outlet angle distributions. It may be seen that the calculating method of the input head-capacity curve using c_ϕ is inconvenient in practical use since c_ϕ varies with capacity. This was also found in other impellers with different numbers of vanes, various vane surfaces and tip-clearances.

On the other hand, it may be suggested that a method using the $\bar{\phi}_{th}$ at the mean effective radius

Fig. 6 Change in c_ϕ

(a) Change in k (effects of number of vanes and vane surface roughness)

(b) Change in k (effects of outlet angle)
Fig. 7

multiplied by k ($k < 1$) should be used in the calculation of $\bar{\phi}_{th}$ - ϕ curve. It is expected that k is roughly constant in a wide region of capacity, since the solid line in Fig. 5 coincides with the broken line at the $\bar{\phi}_{th} = 0$. The measured values of k are shown in Fig. 7. From these figures it may be seen that the change in k is considerably smaller than in c_ϕ , and k can be considered approximately as constant. Then the following relation holds between c_ϕ and k .

$$c_\phi = \phi \cot \beta_{2m}' / (1 - k + k \phi \cot \beta_{2m}') \quad \dots\dots\dots (1)$$

4. Hydraulic losses-capacity curves

4.1 Losses in impeller The losses in an impeller are composed of a profile loss in blade row at each stream surface, friction loss in the annular wall, secondary flow loss and tip-clearance loss, but it is difficult to determine these losses separately from experimental results, moreover there are few available data for rotating annular blade rows. Thus, for practical use, the method separating the total loss into shock loss at vane inlet and so-called friction loss composed of those which are proportional to the square of discharge should be used as well as in the input head-capacity curve.

(a) Friction loss For the expression of friction loss in an impeller, h_{if} , the following equations have been used⁽³⁾⁻⁽⁵⁾,

$$h_{if} = \zeta_{i1}(w_1^2/2g) \quad \dots\dots\dots (2)$$

$$h_{if} = \zeta_{i2}(w_2^2/2g) \quad \dots\dots\dots (2)'$$

$$h_{if} = \zeta_i(w_1^2 + w_2^2)/4g \quad \dots\dots\dots (2)''$$

where w_1 and w_2 are the relative velocities at the inlet to and outlet from an impeller, respectively, and ζ_{i1} , ζ_{i2} , ζ_i are the friction loss coefficients.

Each friction loss coefficient obtained from the measured values of losses at shockless point for impellers with different kinds of outlet angles is shown in Fig. 8, where $\zeta_{i2} > \zeta_i > \zeta_{i1}$ because of $w_1 > w_2$. However, it is not adequate to compare the three, since the solidity of these impellers is different from each other. Then the following drag

Fig. 8 Friction coeff. (effects of outlet angle)

coefficients c_{D1} , c_{D2} and c_D may be introduced,

$$c_{D1} = \zeta_{i1}/(l/t_1 \sin \beta_1) \dots\dots\dots (3)$$

$$c_{D2} = \zeta_{i2}/(l/t_2 \sin \beta_2) \dots\dots\dots (3)'$$

$$c_D = \zeta_i/(l/t_m \sin \beta_m) \dots\dots\dots (3)''$$

where l is the blade chord length, t the blade pitch, β the blade angle, and the subscripts 1, 2 and m refer to inlet to and outlet from the impeller vanes and their mean values, respectively.

Fig. 9 shows the ratio with each drag coefficient obtained from the loss coefficient illustrated in Fig. 8 to that of flat plate with equal length of the vane at mean radius, c_f . In this figure, taking vane thickness into consideration, c_{D1}' is obtained by means of using the minimum space across flow passage at the vane inlet in return for $t_1 \sin \beta_1$ in Eq. (3), and c_D , in the same manner, the mean value of the minimum spaces at vane inlet and outlet for $t_m \sin \beta_m$ in Eq. (3)''. It may be seen that c_{D2} is inconvenient for practical use as it changes remarkably with outlet angles, and both c_{D1} and c_{D1}' based upon the vane inlet profile decrease gradually with an increasing β_{2m} in the range less than 90° , though the larger the β_{2m} will be, the greater the blade chamber for the constant inlet angle. On the other hand, c_D is roughly

Fig. 9 Drag coeff. (effects of outlet angle)

Fig. 10 Friction loss coeff. (effects of outlet angle distribution)

constant for $\beta_{2m} < 60^\circ$, and its value is three and half times the value of c_f . Hence the method using c_D is convenient for estimating the friction coefficient. Hereafter let us consider only c_D and c_D' .

The results on the impeller with various outlet angle distributions ($\beta_{2m} = 39^\circ$) are shown in Fig. 10, where β_{2r} is the outlet angle at the tip. Taking into consideration that the outlet angle distribution of each test impeller is very different from every other distribution (see Fig. 2), it may be noted that difference of friction loss due to the outlet angle distribution or vane twisting is small in usual impellers. So it may be shown that the method using the flow conditions at the mean radius in the impeller is applicable for the calculation of pump characteristics.

The loss coefficients in the three impellers having different surface finishes are shown in Fig. 11, where abscissa k_s is the height of protrusion on the vane surface, and it may be seen that c_D/c_f remains roughly constant, although it is natural that ζ_i and c_D increase with an increasing surface roughness.

Fig. 12 shows an effect of the number of vanes on the loss coefficient, where abscissa t_m/l is a mean value of blade pitch/chord ratio. It is evident that c_D does not remain constant but increases with a decreasing number of vanes Z . The reason for this is probably that the rate of the annular wall friction loss in the total losses increases with a decreasing Z , so the following friction loss coefficient λ_i is introduced as well as in a pipe friction,

Fig. 11 Friction loss coeff. (effects of impeller surface roughness)

Fig. 12 Friction loss coeff. (effects of number of vanes)

$$\lambda_i = (l/4m) \dots\dots\dots (4)$$

where m is mean value of hydraulic mean depth of flow passage through the vane inlet and outlet. It will be seen from Fig. 12 that λ_i remains constant for various numbers of vanes, and its value is about 2.8 times the friction coefficient λ_p for a circular pipe whose diameter is equal to $4m$ ⁽⁶⁾. Hence it seems that the friction loss in an open impeller can be approximately treated like the pipe friction loss.

An effect of tip-clearance e is shown in Fig. 13, where B_2 is width of passage perpendicular to the stream line at the radius r_{2m} , and it is evident that both ζ_i and c_D increase with an increasing e . However, it may be improper to conclude that the value of ζ_i or c_D at $e=0$ extending the tendency of the experimental result in Fig. 13 is the value in the case of no tip-clearance, because an optimum clearance with maximum efficiency is in existence, and the hydraulic loss increases on the contrary when the clearance is less than this optimum value⁽⁷⁾. It may be expected that the optimum value in tip-clearance should be roughly equal to the thickness of the laminar sublayer of flow near the casing wall at the inlet of the impeller, thus its value for this experimental condition is illustrated by thin dash-dot line in the figure ($e/B_2 \approx 0.006$). In the above-mentioned experiments except in Fig. 13, the tip-clearance were held in this optimum value, so the effect of tip-clearance on hydraulic losses was left out of consideration.

(b) Shock loss For the shock loss at the inlet to an impeller, h_{is} , the following two equations are generally used,

$$h_{is} = \zeta_{is} \Delta v_{1u}^2 / 2g \dots\dots\dots (5)$$

$$h_{is} = \zeta_{is} \Delta v_{1u}^2 \sin^2 \beta_{1m} / 2g \dots\dots\dots (5')$$

where Δv_{1u} is shock velocity (see Fig. 20), ζ_{is} the shock loss coefficient. Although ζ_{is} may be varied with vane inlet angle and shape, vane thickness, pitch/chord ratio, etc., these effects on ζ_{is} are unknown. However, the change in ζ_{is} has little

Fig. 13 Friction loss coeff. (effects of tip-clearance)

effect on the calculated value of hydraulic loss in an impeller near the design point where $|\Delta v_{1u}|$ is very small; hence, in this paper, let us deal with $\zeta_{is}=1$ referring to the experimental result for the flat plate⁽⁸⁾.

Fig. 14 shows the comparison between experimental and calculated results for the total loss $h_i (=h_{if} + h_{is})$ in the impeller M-39 Fo, where $\Theta = h_i / (r_{2m}^2 \omega^2 / g)$, the solid line represents the calculated total loss curve using Eqs. (2)'' and (5), and the broken line using Eqs. (2)'' and (5)' for smooth surface ($k_s = 3 \times 10^{-3}$ mm, $\zeta_i = 0.08$). It is evident that the calculated curve of Θ_i using Eq. (5) for shock loss agrees with the experimental data (symbol $\circ$) in the lower point in the capacity region than the best efficiency point except in the backflow region, and similarly the curve calculated by Eq. (5)' agrees with the experimental value in a higher capacity region. It is about the same in other impellers, and the result for an impeller having a rough surface ($k_s = 0.3$ mm, $\zeta_i = 0.17$) is also shown in Fig. 14, where the dash-dot line represents the calculated value using either Eq. (5) or (5)' for the lower or higher capacity region, respectively, and so forth.

4.2 Losses in diffusion casing The measured values of losses in the diffusion casing whose inlet angle β_{d1} is 51.5° for various impellers are shown in Fig. 15, where $\Theta_{de} = h_{de} / (r_{2m}^2 \omega^2 / g)$ and h_{de} is a hydraulic loss in the diffusion

Fig. 14 Losses in impellers (comparison between calculated and measured values)

Fig. 15 Losses in diffusion casings (effects of impeller outlet angles)

vanes and exit passage. It may be noted that each θ_{de} curve makes contact with a line of $\theta_{def} = 0.247\phi^2$, and θ_{de} values increase before and behind the point of contact, that is, the shockless point. Hence, it may be suggested that h_{de} is composed of the loss which is proportional to the square of capacity and the shock loss at the entrance of diffusion vane.

The measured losses in the diffusion casing ($\beta_{d1} = 56^\circ$) with different impellers (M-39 Fo, M-39 Fr) are shown in Fig. 16, where the solid line is the values of θ_{de} calculated by the following equations for the shock loss at the entrance of diffusion vane, h_{ds} , in the same way as the impeller vane:

$$\text{For low capacity: } h_{ds} = \Delta v_{d1u}^2 / 2g \quad \dots\dots\dots (6)$$

$$\text{For high capacity: } h_{ds} = \Delta v_{d1u}^2 \sin^2 \beta_{d1} / 2g \quad \dots\dots\dots (6)'$$

where Δv_{d1u} is shock velocity (see Fig. 20), and dash-dot line θ_{def} is the part proportional to ϕ^2 in θ_{de} . Since it is necessary for calculating h_{ds} to know the actual discharge angle from an impeller, its measured value β_{2m}' shown in Fig. 16 has been used.

From Fig. 16 it is seen that there is little difference between the two measured values of θ_{de} .

Fig. 16 Loss in diffusion casing (effects of impeller outlet angle distributions)

Fig. 17 Loss in diffusion casing (effects of impeller design elements)

(symbols $\circ$ and $\times$) in spite of the impellers having different outlet angle distributions, and the agreement between theory and experimental results is good. Therefore the method using flow conditions at the mean effective radius is available for obtaining the characteristic curve.

Fig. 17 shows the results of using another diffusion vane ($\beta_{d1} = 51.5^\circ$) and different impellers. It may be seen that the measured values of θ_{de} are unchanged irrespective of impeller vane thickness and surface roughness, because of little difference in outlet flow conditions from an impeller, and the agreement between theory and experimental results is good except in the backflow region.

The loss proportional to ϕ^2 in diffusion vanes, h_{dc} , may be mainly composed of the expansion loss, therefore the following equations have been introduced

$$h_{dc} = \zeta_{dc} (v_{d1}^2 - v_{d2}^2) / 2 \quad \dots\dots\dots (7)$$

$$h_{dc} = \zeta_{dc}' (v_{d1} - v_{d2})^2 / 2 \quad \dots\dots\dots (7)'$$

where v_{d1} and v_{d2} are the mean velocity at the inlet to and outlet from the diffusion vane respectively, and ζ_{dc} , ζ_{dc}' the coefficients of expansion loss.

These coefficients calculated from h_{dc} , which is obtained from measured total loss at diffusion casing h_{de} by means of subtracting the presumed loss at outlet passage behind the diffusion vanes, are shown in Fig. 18, where θ_d is the equivalent divergent angle for the diffusion vanes. It may be seen that ζ_{dc} remains roughly constant ($\zeta_{dc} \approx 0.27$) in spite of different diffusion vanes being used, but ζ_{dc}' varies with β_{d1} , so it is convenient to use Eq. (7) for calculating the characteristics⁽⁹⁾.

5. Theoretical investigation on overall characteristics

5.1 Characteristic equation Using the results shown in the foregoing chapter, the overall characteristic equation for a given mixed-flow propeller pump with such an open impeller and a

Fig. 18 Expansion loss in diffusion vane

diffusion casing behind the impeller as illustrated in Fig. 19 is introduced. The total head H and hydraulic efficiency η_h are expressed by the following equations respectively

$$H = \left(\frac{p_e}{\gamma} + \frac{v_e^2}{2g} + Z_{es} \right) - \left(\frac{p_s}{\gamma} + \frac{v_s^2}{2g} \right) \\ = H_{th} - (h_s + h_i + h_d + h_e) \quad \dots\dots\dots (8)$$

$$\eta_h = H/H_{th} = 1 - (h_s + h_i + h_d + h_e)/H_{th} \quad \dots\dots\dots (9)$$

where H_{th} is the input head, γ the specific weight of liquid, p_s and p_e the static pressures, v_s and v_e the mean absolute velocities (subscripts s and e indicate pump entrance and exit respectively), Z_{es} the height from entrance to exit, h_s the loss in the passage from pump entrance to impeller inlet, h_i the loss within the impeller, h_d the loss within the diffusion vanes, and h_e the loss in the passage from outlet of diffusion vane to pump exit. Since it is improper to consider the flow condition near the tip in an open impeller to be leakage from outlet to inlet through narrow passage between impeller and casing in the same manner as the closed impeller, the so-called volumetric efficiency is included within η_h in above equations.

Input head may be expressed by the following equation using the result in chapter 3,

$$H_{th} = k r_{2m} \omega (r_{2m} \omega - v_{2m} \cot \beta_{2m}') / g \quad \dots\dots\dots (10)$$

Fig. 19 Layout of mixed-flow propeller pump

Fig. 20 Illustration of nomenclature

where v_{2m} is the meridian component of absolute velocity at the outlet from the impeller, and β_{2m}' actual discharge angle from the vane at the mean effective radius (see Fig. 20).

It may be expected that h_s and h_e are approximately proportional to the mean velocity head, hence they may be expressed as

$$h_s = \zeta_s (v_s^2 + v_1^2) / 4 \quad \dots\dots\dots (11)$$

$$h_e = \zeta_e (v_{d2}^2 + v_e^2) / 4 \quad \dots\dots\dots (12)$$

where v_1 and v_{d2} are the velocities at impeller inlet and diffusion vane outlet respectively, whose tangential component may be assumed to be zero. From the preceding chapter it is seen that h_i is composed of friction loss h_{if} and shock loss h_{is} , and h_d similarly expansion loss h_{de} and shock loss h_{ds} . So these may be expressed as follows (see Fig. 20)

$$h_{if} = \zeta_i (w_1^2 + w_2^2) / 4g \quad \dots\dots\dots (13)$$

$$h_{is} = \Delta v_{1u}^2 / 2g = v_1^2 (\cot \beta_{1m} - \cot \beta_{1m}') / 2g \\ \dots\dots\dots \text{for low capacity} \quad \dots\dots\dots (14)$$

$$= \Delta v_{1u}^2 \sin^2 \beta_{1m} / 2g = v_1^2 \sin^2 \beta_{1m} (\cot \beta_{1m} - \cot \beta_{1m}') / 2g \quad \dots\dots\dots \text{for high capacity} \quad \dots\dots\dots (14')$$

$$h_{de} = \zeta_{de} (v_{d1}^2 - v_{d2}^2) / 2g \quad \dots\dots\dots (15)$$

$$h_{ds} = \Delta v_{d1u}^2 / 2g = v_{d1m}^2 (\cot \beta_{d1} - \cot \beta_{d1}') / 2g \\ \dots\dots\dots \text{for low capacity} \quad \dots\dots\dots (16)$$

$$= \Delta v_{d1u}^2 \sin^2 \beta_{d1} / 2g = v_{d1m}^2 \sin^2 \beta_{d1} (\cot \beta_{d1} - \cot \beta_{d1}') / 2g \quad \dots\dots\dots \text{for high capacity} \quad \dots\dots\dots (16')$$

where v_{d1m} and β_{d1}' are the meridian components of velocity and its flow angle at inlet to diffusion vane, respectively.

Assuming the free vortex flow in the space from the impeller to the diffusion vane, β_{d1}' may be expressed by the following relation

$$\cot \beta_{d1}' = (r_{2m} / r_{d1}) (r_{2m} \omega - v_{2m} \cot \beta_{2m}') / v_{d1m} \quad \dots\dots\dots (17)$$

where r_{d1} is the mean effective radius at inlet to diffusion vanes.

Substituting above equations into Eqs. (8) and (9), and using next relations,

$$\phi = v_{2m} / r_{2m} \omega, \quad \psi = H / (r_{2m}^2 \omega^2 / g),$$

$$\bar{\phi}_{th} = H_{th} / (r_{2m}^2 \omega^2 / g), \quad \rho_{i1} = r_{1m} / r_{2m},$$

$$\rho_{d1} = r_{d1} / r_{2m}, \quad a_s = v_{2m} / v_s, \quad a_{i1} = v_{2m} / v_1,$$

$$a_{d1} = v_{2m} / v_{d1m}, \quad a_{d2} = v_{2m} / v_{d2}, \quad a_e = v_{2m} / v_e$$

and writing $\cot \beta_{1m} = \kappa_1$, $\cot \beta_{2m} = \kappa_2$, $\cot \beta_{2m}' = \kappa_2'$, $\cot \beta_{d1} = \kappa_{d1}$, the following non-dimensional equations for calculating the pump characteristic curves may be obtained

$$\psi = \bar{\phi}_{th} - (\theta_s + \theta_i + \theta_d + \theta_e) = k(1 - \phi \kappa_2') - (\theta_s + \theta_i + \theta_d + \theta_e) \quad \dots\dots\dots (18)$$

$$\eta_h = \psi / \bar{\phi}_{th} = 1 - (\theta_s + \theta_i + \theta_d + \theta_e) / \{k(1 - \phi \kappa_2')\} \quad \dots\dots\dots (19)$$

where

$$\theta_s = h_s / (r_{2m}^2 \omega^2 / g) = \zeta_s \{ (1/a_s^2) + (1/a_{i1}^2) \} (\phi^2 / 4) \quad \dots\dots\dots (20)$$

$$\Theta_i = h_i / (r_{2m}^2 \omega^2 / g) = \Theta_{if} + \Theta_{is} \dots \dots \dots (21)$$

$$\Theta_{if} = \frac{h_{if}}{r_{2m}^2 \omega^2 / g} = \zeta_{i1} \left(\frac{1 + \kappa_1^2}{a_{i1}^2} + 1 + \kappa_2^2 \right) \frac{\phi^2}{4} \dots (21)'$$

$$\Theta_{is} = \frac{h_{is}}{r_{2m}^2 \omega^2 / g} = \left(\frac{\rho_{i1}}{\phi} - \frac{\kappa_1}{a_{i1}} \right)^2 \frac{\phi^2}{2} \dots \dots \dots (21)''$$

$$= \frac{1}{1 + \kappa_1^2} \left(\frac{\rho_{i1}}{\phi} - \frac{\kappa_1}{a_{i1}} \right)^2 \frac{\phi^2}{2} \dots \dots \dots (21)'''$$

$$\Theta_d = h_d / (r_{2m}^2 \omega^2 / g) = \Theta_{dc} + \Theta_{ds} \dots \dots \dots (22)$$

$$\Theta_{dc} = \frac{h_{dc}}{r_{2m}^2 \omega^2 / g} = \zeta_{dc} \left(\frac{1 + \kappa_{d1}^2}{a_{d1}^2} - \frac{1}{a_{d2}^2} \right) \frac{\phi^2}{2} \dots (22)'$$

$$\Theta_{ds} = \frac{h_{ds}}{r_{2m}^2 \omega^2 / g} = \left\{ \frac{\kappa_{d1}}{a_{d1}} - \frac{1}{\rho_{d1}} \left(\kappa_2' - \frac{1}{\phi} \right) \right\}^2 \frac{\phi^2}{2} \dots \dots \dots (22)''$$

$$= \frac{1}{1 + \kappa_{d1}^2} \left\{ \frac{\kappa_{d1}}{a_{d1}} - \frac{1}{\rho_{d1}} \left(\kappa_2' - \frac{1}{\phi} \right) \right\}^2 \frac{\phi^2}{2} \dots \dots \dots (22)'''$$

$$\Theta_e = h_e / (r_{2m}^2 \omega^2 / g) = \zeta_e \left((1/a_{d2}^2) + (1/a_e^2) \right) (\phi^2/4) \dots \dots \dots (23)$$

The values of each angle β , radius ratio ρ and area ratio a in above equations are known for a given mixed-flow propeller pump, hence estimating the modified coefficient of input head k , each loss coefficient ζ , and actual discharge angle β_{2m}' or slip angle, the overall characteristic curve may be calculated. Then k and the principal loss coefficients can be obtained from the foregoing results, but the experimental results of slip angles are omitted from this paper because of space limitation⁽¹⁰⁾.

5.2 Comparison between theoretical and experimental characteristics Fig. 21 shows measured and calculated characteristics with impeller H-27, where each symbol represents the measured value, and solid lines the calculated curves using the value; $a_s = a_e = 1.608$, $a_{i1} = 0.986$, $a_{d1} = 1.016$, $a_{d2} = 1.309$, $\rho_{i1} = 0.800$, $\rho_{d1} = 1.085$ for the test pump

Fig. 21 Comparison between measured and calculated characteristics

casing dimensions, $\kappa_1 = 2.75$, $\kappa_2 = 1.96$, $\kappa_{d1} = -0.675$ for each vane angle, and $\kappa_2' = 2.90$ (estimating the slip angle as 8.0° ⁽¹⁰⁾), $k = 0.80$, $\zeta_s = 0.03$, $\zeta_e = 0.27$, $\zeta_i = 0.15$, $\zeta_{dc} = 0.27$ on the basis of the experimental results as mentioned above. It is seen that agreement between measured and calculated characteristics is good except in the backflow region.

The characteristic curves, each hydraulic loss and its ratio to the input head obtained from the above characteristic equations in the case of using the impeller H-32 ($\kappa_1 = 2.15$, $\kappa_2 = 1.60$, $\kappa_{d1} = -0.700$, $\kappa_2' = 2.19$, $\zeta_i = 0.10$, and other ζ , ρ , a are same as in Fig. 21) are shown in Fig. 22, where each symbol represents the experimental characteristics. From this figure and the other experiments, it is seen that agreement between theoretical and experimental characteristic curve is good except in $\beta_{2m} > 40^\circ$, and the following results are obtained by this theoretical calculation;

(1) hydraulic loss of flow through the impeller Θ_i is larger than the one through the diffusion vanes Θ_d as a rule, but it seems that actual values of them in low capacity region differ from the value in this figure because of appearance of the backflow;

(2) losses in the inlet and outlet pipe (Θ_s , Θ_e respectively) are considerably small;

(3) input head τ_{th} decreases linearly with an increasing capacity, thus the ratio of total friction loss $\Theta_f (= \Theta_s + \Theta_{if} + \Theta_{dc} + \Theta_e)$ to τ_{th} increases rapidly with an increasing capacity;

Fig. 22 Comparison between measured and calculated characteristics and distributions of losses

(4) minimum value of the total shock loss $\theta_u (= \theta_{is} + \theta_{ds})$ is in existence at somewhat larger capacity than the best efficiency point⁽¹¹⁾, and θ_u increases rapidly before and behind that point;

(5) about seventy per cent of total hydraulic loss $\theta_L (= \theta_f + \theta_u)$ is the friction losses which include the expansion or bend loss at the diffusion casing, and the rest is shock losses.

It seems that the results as mentioned above are applicable to the usual mixed-flow propeller pumps.

6. Conclusion

In this paper the author attempts to establish the calculating method of the characteristic of a mixed-flow propeller pump by presenting data on the effects of the various design elements of both impellers and diffusion casings. On the basis of the experimental results, both input head-capacity and hydraulic losses curves are investigated, and their calculating equations are expressed by using the flow conditions on the mean flow path. The general characteristic equations obtained by applying these relations for mixed-flow propeller pumps with an open impeller take very simple forms, and agreement between the experimental characteristic and the calculated one by the above equations is very good except in the backflow region. The distributions of each hydraulic loss are also discussed.

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