where Π_0 is the constant defined by the roll dimensions.

According to the experimental results and the theoretical, it has become clear that the cold-roll forming may be treated as the biaxial bending problem provided that the forming curvature is not too large.

Moreover, the analytical method in this paper will be applicable to the second roll and so on.

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Effect of Tensile Force in Stretch-Forming Process on the Springback*

By Akijiro BABA** and Yasuhisa TOZAWA***

In the case of forming a sheet into cylindrical surface by stretching the sheet in circumferential direction under various conditions, the effects of time and magnitude of stretching are studied experimentally and theoretically. The experimental results are well explained by the results of theoretical calculations. The obtained results reveal that the springback decreases with an increase of the tensile force, and the amount of it is minimized when the sheet is only stretched after bending. It is also found that the difference in the springback between stretching after bending and stretching with bending is small for materials having a remarkable Bauschinger effect and large for materials having little such effect. Furthermore, it is found that if it is necessary to stretch the sheet during bending, in order to decrease the springback it is desirable that the tensile force be maintained as small as possible during bending and finally a large tensile force is worked after release of the first force.

1. Introduction

Stretch-forming is a process of forming a sheet or a profile into a desired surface or contour by stretching the sheet or profile. One of the major merits of the process is reduction of springback.

In case of inconstancy of tension during the forming, any small portion of the work is wrapped around the die possessing the desired contour under constant tension. Therefore it is all right that the effect of tensile force on the springback is investigated for time and magnitude of stretching, and directions of stretching for the sheet. Researches on springback for bending with tension in consideration of the time of tension are few^{(1)~(3)}. In this report in the case of forming a sheet into cylindrical surface by stretching the sheet in circumferential direction, the effects of time and

magnitude of stretching on the springback are studied experimentally and theoretically.

2. Experiment

2.1 Experimental apparatus and materials tested The apparatus is shown in Fig.1. The sample clamped by the jaws is stretched by the action of oil pressure, and wrapped around the die by the swing of the arms on which the tension cylinders are supported. The arms are connected

Fig. 1 Experimental apparatus

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by links to a cross-head which travels along the center line of the apparatus by the revolution of a spindle. A relief valve controls its pressure to a constant value during the wrapping. Maximum pulling force at 60 kg/cm² (oil pressure) is 2 tons. Mechanical properties of materials tested are listed in Table 1.

2.2 Experimental procedure Effect of the tensile force in circumferential direction on the springback is tested under seven conditions for the time of stretching, namely (Fig. 2):

- (1) Stretching after bending ($M-T_2$)
- (2) Bending with constant tensile force (T_1-M)
- (3) Stretching and bending after unloading (T_1-O-M)
- (4) Stretching, bending after unloading, and secondary stretching ($T_1-O-M-T_2$)
- (5) Bending with constant tensile force, and final stretching (T_1-M-T_2)
- (6) Bending with constant tensile force, and secondary stretching after unloading ($T_1-M-O-T_2$)
- (7) Bending with constant tensile force, stretching finally and stretching secondarily after unloading ($T_1-M-T_2-O-T_3$)

The radius of curvature of the sample after springback is estimated by

$$R' = \frac{a^2}{8\delta} + \frac{\delta}{2}$$

where a and δ are shown in Fig. 3.

The amount of springback is expressed by

$$\Delta \left(\frac{t_0}{2R} \right) / \left(\frac{t_0}{2R} \right) = 1 - \frac{R}{R'}$$

where t_0 : sheet thickness

R : die radius

The tensile stress due to stretching is represented by the mean value of this nominal stress as follows;

$$\sigma_1 = T_1/A_0 \quad \text{for } T_1$$

$$\sigma_{2,3} = (T_{2,3}/A_0)e^{-\mu\theta} \quad \text{for } T_2 \text{ or } T_3$$

where A_0 : cross-sectional area of sheet

μ : coefficient of friction between die and sheet

θ : mean angle between normal to outermost element of sheet in contact with die and normal to element of sheet where curvature is measured

e : 2.718, base of natural logarithms

The die radius used is 41.0 mm, therefore ($t_0/2R$) is 0.010 for 75 SO, and 0.018 for Al.

The width of sample is 25 mm for 75 SO, 30 mm for Al, and 45 mm in case where data at especially low tensile stress are required. The die surface is lubricated with machine oil. The coefficient of friction measured is 0.19 for 75 SO, and 0.26 for Al.

2.3 Experimental results The results obtained under conditions (1) to (4), Fig. 2, are shown in Fig. 4 for 75 SO and in Fig. 5 for Al. Under conditions ($M-T_2$) and (T_1-M), the springback decreases with an increase of the tensile force, and the effect is more remarkable for ($M-T_2$) than for (T_1-M). The difference between these two conditions is larger for Al than for 75 SO. For (T_1-O-M), the springback increases with an increase of the tensile force because of work-hardening. The condition ($T_1-O-M-T_2$) corresponds to the condition ($M-T_2$) for work-hardened sheet. The amount of springback saturated for a hardened sheet is almost the same as for a

Table 1 Mechanical properties of materials tested

Materials	Thickness mm ²	Proof stress kg/mm ²	Tensile strength kg/mm ²	Elongation %	E kg/mm ²	$\sigma = F\epsilon^{1/n'}$	
						F kg/mm ²	n'
Duralumin (75 SO)	0.82	11.2	21.3	14.4	7 300	40	4.6
Pure aluminium (Al)	1.50	2.3	7.8	46.1	6 400	17.9	3.0

Fig. 2 Conditions for the time of stretching (abscissa is step of change of force)

Fig. 3 Measurement of radius of curvature

soft sheet. This result is valuable for production. The results obtained under condition (5), (T_1-M-T_2) , are shown in Fig. 6. When the value of $(\sigma_2-\sigma_1)$ is small, springback decreases a little, and its value is larger than for (T_1-M) . When $(\sigma_2-\sigma_1)$ reaches a certain value, springback decreases remarkably, and approaches the value for $\sigma_1=0$ or $(M-T_2)$. In other words the springback for a certain maximum stress under the condition (T_1-M-T_2) is minimum when $\sigma_1=0$, and maximum when σ_1 is moderate. This is shown in Fig. 7.

For $(T_1-M-O-T_2)$, as shown in Fig. 8, springback decreases a little until σ_2 attains to σ_1 , and thereafter decreases remarkably. Comparison between (T_1-M-T_2) and $(T_1-M-O-T_2)$ shows that the springback for $(T_1-M-O-T_2)$ is smaller than for (T_1-M-T_2) . It is interesting to note that the springback is affected by the unloading process. Effect of the unloading process is examined in $(T_1-M-T_2-O-T_3)$. The results are shown in Fig. 9.

Fig. 4 Change of springback due to stretching [stretch condition (1) to (4)]

Fig. 5 Change of springback due to stretching [stretch condition (1) to (4)]

The experimental results mentioned above have revealed the following. In the case of forming a sheet into cylindrical surface by stretching the sheet in circumferential direction, the amount of springback is minimized when the sheet is stretched after bending. If the bending with stretching is necessary, in order to decrease the springback it is desirable that the tensile force be maintained

Fig. 6 Change of springback due to stretching [stretch condition (5)]

Fig. 7 Change of springback due to stretching [effect of σ_1 under the condition (5)]

Fig. 8 Change of springback due to stretching [stretch condition (6)]

as small as possible during bending and finally a larger tensile force be worked after release of the first force.

3. Theoretical analysis

3.1 Fundamental equations and assumptions

After the work⁽⁴⁾ of one of the authors a mathematical analysis was made. In the analytical treatment of the problems, the Levy-Mises equations together with the assumption of volume constancy were used, and the Bauschinger effect was taken into consideration. For the bent sheet r -, θ -, and z -axis were chosen as shown in Fig. 10. The stresses and strains in each axial direction were represented by suffix letters. If the plane strain

$$d\varepsilon_z = 0 \quad (1)$$

is assumed in order to simplify the mathematical procedure, the volume constant condition becomes

$$d\varepsilon_\theta = -d\varepsilon_r, \quad (2)$$

where $d\varepsilon_z$, $d\varepsilon_\theta$, $d\varepsilon_r$ = total logarithmic strain-increment in each direction respectively. From Eq. (1), equivalent stress is

$$\sigma_{eq} = \pm \frac{\sqrt{3}}{2} (\sigma_\theta - \sigma_r) > 0 \quad (3)$$

When the stress-strain curve obtained by the tensile test is represented as follows

$$\varepsilon = (\sigma/F)^{n'} \quad (4)$$

in the case of $d\sigma_{eq} > 0$ (loading), one has

$$d\varepsilon_\theta = \pm \frac{\sqrt{3}}{2} n' F^{-n'} \sigma_{eq}^{n'-1} d\sigma_{eq} \quad (5)$$

provided that, in the case of $d\sigma_{eq} < 0$ (unloading) and the elastic state,

$$d\varepsilon_\theta = \frac{3}{4E} (d\sigma_\theta - d\sigma_r) \quad (6)$$

where E : Young's modulus.

3.2 Stretching after bending ($M-T_2$)

3.2.1 Bending For moderate bending,

$$\sigma_r = 0 \quad (7)$$

is assumed, therefore Eq. (3) yields

$$\sigma_{eq} = \pm \frac{\sqrt{3}}{2} \sigma_\theta \quad (8)$$

Furthermore, if one assumes that individual elements in the sheet have been progressively extended

or compressed, the strain of the plane that has a distance of η (taking the outside direction for positive value) from the central plane of the sheet is represented by

$$\varepsilon_{\theta A} = \ln \left(1 + \frac{t_0}{2r_m} \lambda \right) \doteq \frac{t_0}{2r_m} \lambda \quad (9)$$

provided that

$$\lambda = 2\eta/t_0$$

and the suffix A shall be added for the state after bending. From Eqs. (5) and (8), one gets

$$\sigma_{eqA} = F \left(\frac{2}{\sqrt{3}} |\varepsilon_{\theta A}| \right)^{1/n'} \quad (10)$$

$$\sigma_{eqA} = \pm F \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} |\varepsilon_{\theta A}|^{1/n'} \quad (11)$$

where the order of $\pm$ follows the order of $\varepsilon_{\theta A} \geq 0$, i. e. $\lambda \geq 0$.

3.2.2 Stretching The strain-increment due to stretching is expressed as

$$d\varepsilon_\theta = \sqrt{(1+d\varepsilon_{\theta m}) \left\{ \left(1 + \frac{t_0}{2r_m} \lambda \right)^2 + d\varepsilon_{\theta m} \right\} - \left(1 + \frac{t_0}{2r_m} \lambda \right)}$$

where $d\varepsilon_{\theta m}$ is the strain-increment at $\lambda = 0$. Usually $(t_0/2r_m) \ll 1$, and $|\lambda| \leq 1$, so one gets

$$d\varepsilon_\theta \doteq d\varepsilon_{\theta m} \quad (12)$$

In the part of $\lambda \geq 0$, where $d\sigma_{eq} > 0$, there continuously occurs a plastic flow. From Eqs. (5) and (3), one obtains

$$\begin{aligned} \sigma_\theta - \sigma_r &= F \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} (\varepsilon_{\theta m} + \varepsilon_{\theta A})^{1/n'} \\ &\equiv f_1(\varepsilon_{\theta A}, \varepsilon_{\theta m}) \end{aligned} \quad (13)$$

In the part of $\lambda < 0$, where $d\sigma_{eq} < 0$, it shows an elastic behavior. From Eq. (6), one obtains

$$\begin{aligned} \sigma_\theta - \sigma_r &= \frac{4E}{3} \varepsilon_{\theta m} - F \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} |\varepsilon_{\theta A}| \\ &\equiv f_2(\varepsilon_{\theta A}, \varepsilon_{\theta m}) \end{aligned} \quad (14)$$

If the Bauschinger effect is disregarded, Eq. (14) is applicable until $\varepsilon_{\theta m}$ attains to $\varepsilon_{\theta mB}$ represented by

$$\varepsilon_{\theta mB} = \frac{2F}{E} \left(\frac{2}{\sqrt{3}} \right)^{(1-n')/n'} |\varepsilon_{\theta A}|^{1/n'} \quad (15)$$

When $\varepsilon_{\theta m} > \varepsilon_{\theta mB}$, there occurs again a plastic flow.

Fig. 9 Change of springback due to stretching [stretch condition (7)]

Fig. 10 Definitions of directions and dimensions

One obtains similarly to Eq. (13)

$$\sigma_\theta - \sigma_r = F \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} (\varepsilon_{\theta m} - \varepsilon_{\theta m B} + |\varepsilon_{\theta A}|)^{1/n'} \quad (16)$$

Because $\varepsilon_{\theta m B}$ is a function of λ , Eq. (16) is held true for $0 > \lambda > -\lambda_B$, Eq. (14) for $-\lambda_B > \lambda > -1$, where $-\lambda_B$ is taken for the λ that allows $\varepsilon_{\theta m} = \varepsilon_{\theta m B}$, obtained from the following equation,

$$\varepsilon_{\theta m} = \frac{2F}{E} \left(\frac{2}{\sqrt{3}} \right)^{(1-n')/n'} \left(\frac{t_0}{2r_m} \right)^{1/n'} \lambda_B^{1/n'} \quad (17)$$

If one assumes that the r_m is constant during stretching, the equilibrium equation of the radial forces is

$$\frac{d\sigma_r}{d\lambda} = \frac{\sigma_\theta - \sigma_r}{(2r_m/t_0) + \lambda} = (\sigma_\theta - \sigma_r) \frac{t_0}{2r_m} \quad (18)$$

The solution of this equation which satisfies the conditions, $\sigma_r = 0$ at $\lambda = 1$ and continuity of σ_r at $\lambda = 0$ and $\lambda = -\lambda_B$, gives the following expression for the pressure at inner surface $\lambda = -1$, where $\sigma_r = -p_i$:

$$p_i = \frac{t_0}{2r_m} \left[\int_{-1}^{-\lambda_B} f_2 d\lambda + \int_{-\lambda_B}^0 f_3 d\lambda + \int_0^1 f_1 d\lambda \right] \quad (19)$$

The mean tensile stress $\sigma_{\theta m}$ under the stretching force T_2 becomes

$$\sigma_{\theta m} = \frac{T_2}{bt_0(1-\varepsilon_{\theta m})} = \frac{[1-(t_0/2r_m)]}{(1-\varepsilon_{\theta m})} \frac{p_i}{t_0/r_m} \quad (20)$$

3.2.3 Springback At first, the curvature is kept constant and the stretching force is removed alone (suffixing the letters with the sign ').

In this case where all parts of the sheet present elastic behavior, one obtains

$$\sigma_{\theta}' - \sigma_r' = \frac{4E}{3} (\varepsilon_{\theta m}' - \varepsilon_{\theta m}) + (\sigma_\theta - \sigma_r) \quad (21)$$

The equilibrium equation for this state similar to Eq. (18) is substituted by Eq. (21) and solved

$$\frac{t_0}{2r_m''} = \frac{\frac{8E}{9} (1-\varepsilon_{\theta m}') \frac{t_0}{2r_m} - \int_{-1}^{-\lambda_B} f_2 \lambda d\lambda - \int_{-\lambda_B}^0 f_3 \lambda d\lambda - \int_0^1 f_1 \lambda d\lambda}{\left(\frac{t_0}{2r_m''} \right)^{-1} \int_{-1}^1 \sigma_r'' \lambda d\lambda + \frac{8E}{9} (1-\varepsilon_{\theta m}')} \quad (24)$$

The amount of decreasing of bending due to springback is represented as follows,

$$\Delta \left(\frac{t_0}{2r_m} \right) = \frac{t_0}{2r_m} - \frac{t_0}{2r_m''} \quad (25)$$

Therefore, the relation between the mean tensile stress $\sigma_{\theta m}$ due to stretching and the amount of springback $\Delta(t_0/2r_m)/(t_0/2r_m)$ is obtained by using $\varepsilon_{\theta m}$ for as parameter. A result of the calculation is shown in Fig. 11.

3.2.4 Effect of the inner pressure If the inner pressure p_i caused by stretching is neglected, the mean tensile stress will be expressed by

$$(\sigma_{\theta m})_{p_i=0} = \frac{1}{2} \left[\int_{-1}^{-\lambda_B} f_2 d\lambda + \int_{-\lambda_B}^0 f_3 d\lambda + \int_0^1 f_1 d\lambda \right] \quad (26)$$

Comparing it with Eq. (20), one gets

$$\sigma_{\theta m} = \frac{[1-(t_0/2r_m)]}{(1-\varepsilon_{\theta m})} (\sigma_{\theta m})_{p_i=0}$$

The value of $\varepsilon_{\theta m}$ is very small until the springback becomes very small, therefore $\sigma_{\theta m}$ is about $(1-t_0/2r_m)$ times $(\sigma_{\theta m})_{p_i=0}$.

If the pressure is neglected, the curvature after springback is represented as follows,

$$\left(\frac{t_0}{2r_m''} \right)_{p_i=0} = \frac{t_0}{2r_m} - \frac{9}{8E(1-\varepsilon_{\theta m}')} \left[\int_{-1}^{-\lambda_B} f_2 \lambda d\lambda + \int_{-\lambda_B}^0 f_3 \lambda d\lambda + \int_0^1 f_1 \lambda d\lambda \right] \quad (27)$$

Fig. 11 Result of calculation [stretch condition (1)]

under the boundary condition. One obtains

$$\varepsilon_{\theta m}' = \varepsilon_{\theta m} - \frac{3}{4E} \frac{p_i}{t_0/r_m} \quad (22)$$

Secondly, the change in curvature is allowed (with the suffix sign ''), all external force being removed, so one gets

$$\sigma_{\theta}'' - \sigma_r'' = \frac{4E}{3} (1-\varepsilon_{\theta m}') \lambda \left(\frac{t_0}{2r_m''} - \frac{t_0}{2r_m} \right) + f_{(1,2,3)} - \frac{p_i}{t_0/r_m} \quad (23)$$

The equilibrium equation is again substituted by Eq. (23), and solved under the similar condition for σ_r'' . The stress σ_r'' in any part of the sheet, which is the residual stress, is obtained as the homogeneous linear function about $(t_0/2r_m'')$. From Eq. (23) and

$$\int_{-1}^1 \sigma_{\theta}'' \lambda d\lambda = 0, \quad \int_{-1}^1 \frac{p_i}{t_0/r_m} \lambda d\lambda = 0,$$

one obtains

If it is compared with Eq. (24), when

$$\int_{-1}^1 \sigma_r'' \lambda d\lambda = 0,$$

Eq. (24) becomes the same as Eq. (27). A result of calculation shows that the first term is a very small value as compared with the second term in the denominator of right side of Eq. (24).

The relation between Eqs. (26) and (27) is given in Fig. 11, with dotted line. From this figure it is found that the effect of pressure p_i is very small. Therefore, if the pressure is neglected, the labour of calculation will be saved and a good approximate value will be obtained. Consequently the effects of amount of bending ($t_0/2r_m$) and the material constants (E, F, n') on the relation between springback and stretching are presumed from the results obtained previously⁽⁴⁾.

3.3 Bending with constant tensile force ($T_1 - M$)

3.3.1 Stretching The suffix T shall be added for the stretched state. One obtains the following,

$$T_1 = bt_T \sigma_{\theta T} \dots \dots \dots (28)$$

$$\epsilon_{\theta T} = \left(\frac{\sqrt{3}}{2} \right)^{1+n'} \left(\frac{\sigma_{\theta T}}{F} \right)^{n'} \dots \dots \dots (29)$$

$$t_T = t_0 e^{-\epsilon_{\theta T}} \dots \dots \dots (30)$$

3.3.2 Bending In this article one assumes that $\sigma_r = 0$ in all parts of the sheet. When the radius of curvature of the plane which has the same strain as before bending is r_n , the strain of the plane at $\lambda (=2\gamma/t_0)$ caused by bending is represented by

$$\epsilon_{\theta R} = \ln \left(1 + \frac{t_0}{2r_n} \lambda \right) \doteq \frac{t_0}{2r_n} \lambda \dots \dots \dots (31)$$

If one assumes that the elements of $\lambda > 0$ have been progressively extended, while the elements of $\lambda < 0$ have been progressively compressed during bending, in the part of $\lambda > 0$ one obtains from Eq. (5)

$$\sigma_{\theta} = F \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} (\epsilon_{\theta T} + \epsilon_{\theta R})^{1/n'} \doteq g_1(\epsilon_{\theta T}, \epsilon_{\theta R}) \dots \dots \dots (32)$$

in the part of $\lambda < 0$, from Eq. (6)

$$\sigma_{\theta} = \frac{4E}{3} \epsilon_{\theta R} + \sigma_{\theta T} \doteq g_2(\epsilon_{\theta T}, \epsilon_{\theta R}) \dots \dots \dots (33)$$

If the Bauschinger effect is disregarded, Eq. (33) is applicable until $\epsilon_{\theta R}$ attains to $\epsilon_{\theta RB}$ represented by

$$\epsilon_{\theta RB} = -\frac{3}{2E} \sigma_{\theta T} \dots \dots \dots (34)$$

Because $\epsilon_{\theta R}$ is a function of λ , if one puts λ where $\epsilon_{\theta R} = \epsilon_{\theta RB}$ as $-\lambda_B$ one gets following.

$$\lambda_B = \left(\frac{t_0}{2r_n} \right)^{-1} \frac{3\sigma_{\theta T}}{2E} \dots \dots \dots (35)$$

Therefore, Eq. (33) is held true for $0 > \lambda > -\lambda_B$.

In the part of $-\lambda_B > \lambda > -\lambda_i$, there occurs again a plastic flow, and one gets

$$\sigma_{\theta} = -F \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} (\epsilon_{\theta T} + \epsilon_{\theta RB} - \epsilon_{\theta R})^{1/n'} \doteq g_3(\epsilon_{\theta T}, \epsilon_{\theta R}) \dots \dots \dots (36)$$

The tensile force is maintained constant, so one gets

$$2\sigma_{\theta T} \frac{t_T}{t_0} = \int_{-\lambda_i}^{-\lambda_B} g_3 d\lambda + \int_{-\lambda_B}^0 g_2 d\lambda + \int_0^{\lambda_0} g_1 d\lambda \dots (37)$$

and from the volume constant condition one obtains

$$\frac{t_0}{2r_n} \frac{t_T}{t_0} = \frac{1}{4} \left\{ \left(1 + \frac{t_0}{2r_n} \lambda_0 \right)^2 - \left(1 - \frac{t_0}{2r_n} \lambda_i \right)^2 \right\} \dots (38)$$

From Eqs. (37) and (38), λ_0 and λ_i , therefore $t = (t_0/2)(\lambda_0 + \lambda_i)$, is obtained.

When the tensile force is large and the bending is moderate, the part where stress is represented by Eq. (36) does not exist. At the limit of existence

$$\lambda_i = \lambda_B = \left(\frac{t_0}{2r_n} \right)^{-1} \frac{3\sigma_{\theta T}}{2E} \dots \dots \dots (39)$$

If one puts Eq. (39) into Eqs. (37) and (38), one obtains the relation between $\sigma_{\theta T}$ (or $\epsilon_{\theta T}$) and $t_0/2r_n$ at the limit of application of Eq. (37). In the case where the tensile force is large above the limit, one substitutes the following for Eq. (37).

$$2\sigma_{\theta T} \frac{t_T}{t_0} = \int_{-\lambda_i}^0 g_2 d\lambda + \int_0^{\lambda_0} g_1 d\lambda \dots \dots \dots (40)$$

3.3.3 Springback When the external force is released, the curvature of the sheet changes after the tensile force has become almost zero. One is apt to consider the process of the release of external force to be elastic, but the plastic flow occurs partially.

Fig. 12 Symbols ($T_1 - M$)

Fig. 13 Stress distribution in the sheet for ($T_1 - M$)

For $-\lambda_C > \lambda > -\lambda_B$

$$\sigma_{\theta 2} = \frac{4E}{3} \varepsilon_{\theta 2} + g_2 \quad (57)$$

where $-\lambda_C$, the boundary between these ranges, is represented by

$$\lambda_C = \left(\frac{t_0}{2r_n} \right)^{-1} \varepsilon_{\theta 2} \quad (58)$$

For $-\lambda_B > \lambda > -\lambda_i$

$$\sigma_{\theta 2} = \frac{4E}{3} \varepsilon_{\theta 2} + g_3 \quad (59)$$

When the tensile force becomes large, it becomes $-\lambda_C < -\lambda_B$, where λ_C satisfies the following equation.

$$\varepsilon_{\theta 2} = \frac{3F}{2E} \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} \left(\varepsilon_{\theta T} - \frac{3\sigma_{\theta T}}{2E} + \frac{t_0}{2r_n} \lambda_C \right)^{1/n'} \quad (60)$$

In the part of $-\lambda_B > \lambda > -\lambda_C$, there occurs a plastic flow, and one gets

$$\sigma_{\theta 2} = F \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} \left\{ \varepsilon_{\theta 2} - \frac{3\sigma_{\theta T}}{2E} - \varepsilon_{\theta R} - \frac{3F}{2E} \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} \left(\varepsilon_{\theta T} - \frac{3\sigma_{\theta T}}{2E} - \varepsilon_{\theta R} \right)^{1/n'} \right\}^{1/n'} \quad (61)$$

The mean tensile stress $\sigma_{\theta T_2}$ is obtained from the following equation,

$$2\sigma_{\theta T_2} \frac{t_{T_2}}{t_0} = (1 - \varepsilon_{\theta 2}) \int_{-\lambda_i}^{\lambda_0} \sigma_{\theta 2} d\lambda \quad (62)$$

where $\sigma_{\theta 2}$ is substituted by Eqs. (56), (57), (59) and (61) for each range corresponding to $\varepsilon_{\theta 2}$.

3.5.2 Springback When the tensile force is removed alone from the state where $-\lambda_C > -\lambda_B$, a small strain change $-\Delta\varepsilon_{\theta 2} (>0)$ shows an elastic behavior in all parts of the sheet, and one gets

$$\sigma_{\theta 2}' = \frac{4E}{3} \Delta\varepsilon_{\theta 2} + \sigma_{\theta 2} \quad (63)$$

When it becomes $-\Delta\varepsilon_{\theta 2} \geq \varepsilon_{\theta 2}$, in the part of $-\lambda_B' > \lambda > -\lambda_i$ there occurs a plastic flow, and one obtains

$$\sigma_{\theta 2}' = -F \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} (\varepsilon_{\theta T} + \Delta\varepsilon_{\theta 2B} - \Delta\varepsilon_{\theta 2})^{1/n'} \quad (64)$$

where

$$\Delta\varepsilon_{\theta 2B} = - \left(\frac{3}{2E} \sigma_{\theta T} + \varepsilon_{\theta R} + \varepsilon_{\theta 2} \right) \quad (65)$$

and

$$\lambda_B' = \left(\frac{t_0}{2r_n} \right)^{-1} \left(\frac{3\sigma_{\theta T}}{2E} + \varepsilon_{\theta 2} + \Delta\varepsilon_{\theta 2} \right) \quad (66)$$

When the tensile force is removed alone from the state where $-\lambda_B > -\lambda_C$, it shows an elastic behavior in all parts of the sheet.

The $\Delta\varepsilon_{\theta 2}$ is obtained from one of the following equations; in the case where all parts of the sheet present elastic behavior,

$$\Delta\varepsilon_{\theta 2} = - \frac{3}{4E} \sigma_{\theta T_2} \quad (67)$$

and in the case when there occurs a plastic flow

in any part,

$$\int_{-\lambda_i}^{\lambda_0} \sigma_{\theta 2}' d\lambda = 0 \quad (68)$$

The springback is obtained similarly to Eq. (49), and represented by

$$\Delta \left(\frac{t_0}{2r_m} \right) / \left(\frac{t_0}{2r_m} \right) = \frac{9}{8E(1 - \Delta\varepsilon_{\theta 2} - \varepsilon_{\theta 2})} \left(\frac{t_0}{2r_m} \right)^{-1} \left(\frac{t}{t_0} \right)^{-3} \int_{-\lambda_i}^{\lambda_0} \sigma_{\theta 2}' \lambda d\lambda \quad (69)$$

3.6 Bending with constant tensile force, and secondary stretching after unloading ($T_1-M-O-T_2$) The previous process to secondary stretching is discussed in the sections 3.3.1 to 3.3.3.

3.6.1 Stretching after unloading When the strain due to stretching, $\varepsilon_{\theta 2}$, is small, it shows an elastic behavior in all parts of the sheet, and one gets

$$\sigma_{\theta 2} = \frac{4E}{3} \varepsilon_{\theta 2} + g_{(1,2,3)}' \quad (70)$$

When it becomes $\varepsilon_{\theta 2} > -\Delta\varepsilon_{\theta}$, there occurs a plastic flow in the part of $0 > \lambda > -\lambda_C$, and one gets

$$\sigma_{\theta 2} = F \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} (\varepsilon_{\theta T} + \varepsilon_{\theta R} + \Delta\varepsilon_{\theta} + \varepsilon_{\theta 2})^{1/n'} \quad (71)$$

where $-\lambda_C$, the boundary of the range, is represented by

$$\lambda_C = \left(\frac{t_0}{2r_n} \right)^{-1} (\varepsilon_{\theta 2} + \Delta\varepsilon_{\theta}) \quad (72)$$

When the stretching is larger, it becomes $\varepsilon_{\theta 2} > (-3/2E)g_3'$, and in the part of $-\lambda_B' > \lambda > -\lambda_C$ the following is given.

$$\sigma_{\theta 2} = F \left(\frac{2}{\sqrt{3}} \right)^{(n'+1)/n'} (\varepsilon_{\theta T} + \Delta\varepsilon_{\theta B} - \Delta\varepsilon_{\theta} + \varepsilon_{\theta 2} + \frac{3}{2E} g_3')^{1/n'} \quad (73)$$

The mean tensile stress $\sigma_{\theta T_2}$ is obtained from the following equation,

$$2\sigma_{\theta T_2} \frac{t_{T_2}}{t_0} = (1 - \varepsilon_{\theta 2} - \Delta\varepsilon_{\theta}) \int_{-\lambda_i}^{\lambda_0} \sigma_{\theta 2} d\lambda \quad (74)$$

where $\sigma_{\theta 2}$ is substituted by Eqs. 70), (71) and (73) corresponding to $\varepsilon_{\theta 2}$.

3.6.2 Springback When the external force is released, all parts present elastic behavior. The strain due to the release of tensile force is expressed

Fig. 15 Stress distribution in the sheet for ($T_1-M-O-T_2$)

as follows

$$\Delta \varepsilon_{\theta 2} = -\frac{3}{4E} \sigma_{\theta T 2} \dots (75)$$

$$\therefore \sigma_{\theta 2}' = \sigma_{\theta 2} - \sigma_{\theta T 2} \dots (76)$$

The springback is obtained similarly to Eq. (49), and represented by

$$\Delta \left(\frac{t_0}{2r_m} \right) / \left(\frac{t_0}{2r_m} \right) = \frac{9}{8E(1-\varepsilon_{\theta 2}-\Delta \varepsilon_{\theta}-\Delta \varepsilon_{\theta 2})} \left(\frac{t_0}{2r_m} \right)^{-1} \left(\frac{t}{t_0} \right)^{-3} \int_{-\lambda_i}^{\lambda_0} \sigma_{\theta 2}' \lambda d\lambda \dots (77)$$

3.7 Results calculated and discussion The result of calculation for $(M-T_2)$ and (T_1-M) is given in Fig. 16, where the full lines represent the case without regard to the Bauschinger effect and the dotted lines the case with the Bauschinger effect considered. In every case, the springback decreases against an increase of the tensile force, and the influence of the tensile force is more remarkable for $(M-T_2)$ than for (T_1-M) , in agreement with experimental results. The results also have revealed that the springback decreases more in the case without regard to the Bauschinger effect than in the case with the Bauschinger effect considered for $(M-T_2)$ and the reverse is the case for (T_1-M) . Therefore, the difference between $(M-T_2)$ and (T_1-M) is small for materials having a remarkable Bauschinger effect. This is in good agreement with the results obtained for 75 SO having remarkable Bauschinger effect and for Al having little such effect.

In the case of (T_1-M) if one assumes that it shows an elastic behavior in all parts of the sheet when the tensile force is released, one obtains the improper result that the springback increases with the tensile force, as shown with the chain line in Fig. 16. Therefore, it is proved that this assumption is not appropriate.

Fig. 16 Result of calculation [stretch conditions (1) and (2)]

The result of calculation for (T_1-M-T_2) and $(T_1-M-O-T_2)$ is given in Fig. 17. In each case the springback is larger than in the case of (T_1-M) when $(\sigma_{\theta T 2} - \sigma_{\theta T})$ is small, and decreases rapidly towards the amount in case of $(M-T_2)$ when $(\sigma_{\theta T 2} - \sigma_{\theta T})$ becomes large. But the effect of unloading before final stretching is not remarkably shown in the result.

4. Conclusion

In the case of forming a sheet into cylindrical surface by stretching the sheet in circumferential direction under various conditions, the effects of tensile force on the springback are revealed by experiments and theoretical analyses. And it is found that the optimum condition of stretching with a reduced springback is stretching after bending.

In practice the friction between die and sheet is a matter of importance. When the tensile force is sufficient to decrease the springback at any element of the sheet in contact with the die, the tensile stress is excessive at the element free from friction, and there is a fear of fracture. The fear is strong for the sheet having a small work-hardening rate.

In the case of stretching in circumferential direction, the inner pressure caused by the stretching reduces the springback but its effect is very small, and the frictional resistance grows. Furthermore, in this case the springback is not sufficiently reduced for materials having a remarkable Bauschinger effect. Therefore, one may consider stretching the sheet in its lateral direction, where the influences of the Bauschinger effect and the friction are small.

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Fig. 17 Result of calculation [stretch conditions (5) and (6)]

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On the Accuracy of Sheared Blanks*

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The bending deformation of material under the punch at the shearing time causes a deflection of sheared blanks. Moreover, because of the spring back of the bending, the diameter of sheared blanks is not always equal to the diameter of the die hole. The authors, in this paper, analyzed the bending moment and the axial force, which cause the deflection of the material, and made clear the effect of spring back on the dimensional accuracy. These analytical results were also confirmed by experimental results.

1. Introduction

It is well known that the shearing by means of punches and dies is the most suitable for production of precision machine parts. But judging from the point of view of accuracy, sheared blanks have the following defects: The broken sheared surface, the deflection or dishing and the dimensional change. And it is considered that these defects could be removed with proper shearing conditions. Therefore the authors discussed, in this paper, the deflection and the dimensional change of sheared blanks.

In the blanking operation, a blanking tool provided with a die hole of desired dimensions is usually employed. However, in practice, the resulting sheared blanks are not always exactly the same as the die hole in dimensions. Besides, they commonly result in deflection. This might be caused by the great deformation of materials themselves at the time of blanking as well as that of the die. From this fact, it is considered that the dimensional

accuracy and the deflection of sheared blanks differ according to the different kind of tools and materials and to blanking conditions. As for the influence of these factors, there are several former researches^{(1)~(4)}. In these works one of the authors, T. Maeda, analyzed the external forces of the material under the punch and introduced an equation of the bending moment⁽¹⁾. But the amounts of the bending moment and the axial force, which cause the deflection and the dimensional change of sheared blanks, have not yet been known. Accordingly the Maeda equation is not verified by experimental results. The authors in this experiment detected the vertical and horizontal component forces acting on the cutting edges of a parallel-straight-edged tool and calculated the bending moment by the above-mentioned equation. At the same time the bending moment and the axial force were calculated on the bending deformation of the material. As for the accuracy of the dimensions of sheared blanks, the authors discussed how the spring back of the bending deformation influenced the dimensions of sheared blanks and confirmed it by the blanking test with a circular tool.

2. Deflection of sheared blanks

From the point of view that the shearing phenomenon is comparatively simple, the shearing

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