by the static tension test because the elongated necks are evenly distributed under the impact pressure.

(3) The yield strength under high velocity deformation is not affected by cold work hardening.

(4) If the multilayer vessel which is loosely fitted to the liner is loaded by impact pressure, the vessel will be deformed as the total work of deformation of multilayers is equivalent to the work of deformation of the single fold shell loaded by the same shock energy.

(5) If the multilayer vessels which are tightly fitted with liners are loaded by the impact pressure, the shock wave will be directly transmitted to the outer layer so that the periodically fluctuated stress with high frequency will occur on the outer

layer as Nishimura⁽⁷⁾ has theoretically analysed. Study on the thick wall cylinder under impact pressure will be published later.

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On the Theoretical Analysis of Shore Hardness*

By Tadasu TSUKIZOE** and Terumasa HISAKADO***

The object of this study is to estimate the theoretical value of Shore hardness and to confirm theoretically whether the dynamic tests based on the height of rebound have a general relation to the static hardness. The value of Shore hardness is generally determined by the height of rebound of a falling indenter which will depend upon the elastic and restitution properties of the specimen.

If the equation showing the relation between the static load and the depth of indentation is applicable to the dynamic indentation, the energy which determines the height of rebound of hammer can be deduced theoretically on the basis of that equation. A comparison of the theoretical Shore hardness numbers and experimental ones of some specimens shows a good agreement between them.

1. Introduction

The hardness tests employed widely in factories and laboratories are generally static indentation ones, the values of which are determined by the measured diameter of permanent indentation and the load at the contact between a ball and a given specimen. On the contrary a Shore hardness test is a kind of the dynamic hardness one based on the height of rebound of a falling indenter. In spite of the wide utilization of Shore hardness test

it is so difficult to analyze theoretically the nature of contact between colliding surfaces that a theoretical analysis for that nature has not been made.

Some investigators tried to deduce the energy which determines the height of rebound of hammer in Shore hardness on the basis of the experimental relations between the dynamic load and the depth of indentation⁽¹⁾. Nevertheless the theoretical and quantitative analysis of Shore hardness has not been made yet.

In this paper in order to analyze the nature of contact between colliding surfaces it is assumed that the equations showing the relation between the static load and the depth of indentation are applicable to the dynamic indentation. On this assumption, the relation between the load caused by impact

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and the depth of the dynamic indentation over the purely elastic range may be shown by Hertz's equations. On the other hand if the mean flow pressure in the course of the plastic deformation of indentation increases as a result of work-hardening only, i.e. provided there are no forces of a quasi-viscous nature in the dynamic deformation of metals, the equation showing that relation can be deduced theoretically. Then the load caused by impact in Shore hardness test can be estimated from the energy of impact of hammer. Therefore the height of rebound of hammer can be deduced theoretically from the energy of elastic recovery of the surface of indentation. For this theoretical deduction it is necessary to estimate the mean flow pressure of the specimen.

Assuming that the mean flow pressure is equivalent to Meyer hardness number⁽²⁾, the relation between Shore hardness number and Brinell hardness number can be deduced theoretically too. These theoretical deductions are discussed in the light of experimental evidences.

2. Theory

2.1 The depth of indentation

If a flat seat is pressed by a spherical indenter under a load, it will at first deform elastically according to Hertz's equation:

$$a = \sqrt[3]{\frac{3}{8} \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right) WD} \quad \text{.....(1)}$$

where

a : radius of the circle of contact

W : load on the contact

D : diameter of the spherical indenter

E_1, E_2 : Young's moduli for the seat and the indenter

ν_1, ν_2 : Poisson's ratios for the seat and the indenter

Assuming that in the case of Shore hardness test there is no deformation at the tip of hammer because it is made of diamond, the depth of indentation λ' is found from Eq. (1) to be

$$\lambda' = \frac{a^2}{D} = \sqrt[3]{\frac{9}{64} \left(\frac{1-\nu_1^2}{E_1} \right)^2 \frac{W^2}{D}} = k \sqrt[3]{\frac{W^2}{D}} \quad \text{.....(2)}$$

where

λ' : depth of indentation in the course of elastic deformation

$$k = \sqrt[3]{\frac{9}{64} \left(\frac{1-\nu_1^2}{E_1} \right)^2}$$

As the load W is increased, the mean pressure over the area of contact increases until it reaches such a value that at a critical point within the metal the elastic limit is exceeded. This occurs in the region where the shear stresses are maximum,

and this region is situated at a point about $0.5a$ below the center of the circle of contact. And the onset of plastic deformation occurs around this point when the mean pressure reaches a value of 1.1 times the elastic limit.

If the load is increased further, a stage is soon reached at which the whole of the metal around the region of contact is flowing plastically. After this stage of full plasticity the mean pressure, which we may call the mean flow pressure p_m , is found to be almost independent of the load, provided there is no work-hardening. In contrast, when the mean flow pressure p_m increases as a result of work-hardening in the course of indentation, p_m is given by the following equation:

$$p_m = \frac{4}{\pi} \frac{K^{2/n}}{D^{2(1-2/n)}} W^{(1-2/n)} = C W^{(1-2/n)} \quad \text{.....(3)}$$

where

n : Meyer index

K : coefficient dependent on the metal

$$C = \frac{4}{\pi} \frac{K^{2/n}}{D^{2(1-2/n)}}$$

Hence, the depth of indentation λ is given by

$$\lambda = \frac{W}{\pi D p_m} = \frac{W^{2/n}}{\pi D C} \quad \text{.....(4)}$$

Fig. 1 Relation between load and depth of indentation

Assuming that Eq. (2) is still valid beyond the onset of plastic deformation, the relation between the load and the depth of indentation is shown in Fig. 1(a). The portion OW_e' represents the increase in the depth of indentation with load over the elastic range where the deformation follows Eq. (2). Beyond the load W_e' , where full plasticity occurs, the deformation follows the curve $W_e'W_p$ [Eq. (4)]. At the end of the indentation the load reaches the maximum value W_p . Finally a release of elastic stresses in the surface of metal takes place, as the result of which rebound occurs and the depth of permanent indentation λ_r is left. Fig. 1(b) shows the variation of the depth of indentation.

In the following we shall suppose that the relation between the load and the depth of indentation is still valid in the dynamic indentation, and we may analyze Shore hardness theoretically.

2.2 The elastic deformation of the flat surface

In the case of contact between a flat seat and a spherical indenter the depth of indentation in the course of the elastic deformation of the contact zone is obtained from Eq. (2). In practice, however, not only the indentation shown in Fig. 1(b) but also the elastic deformation of the flat surface around the indentation will be produced, i.e. some region of the flat surface around the indentation (OAD shown in Fig. 2) will be deformed elastically ($O'A'D'$). Then we will call this deformation "the elastic deformation of the flat surface", the value of which is given by Hertz's equation as follows:*

$$\delta' = \sqrt[3]{\frac{9}{64} \left(\frac{1-\nu_1^2}{E_1} \right)^2 \frac{W^2}{D}} = k \sqrt[3]{\frac{W^2}{D}} \quad \text{..... (5)}$$

On the other hand, in the course of full plasticity in the contact zone, the elastic deformation of the flat surface must also be produced.

At the end of indentation process (DA_pC_o' in Fig. 2), the amount of the elastic deformation of the flat surface is given by δ_p (Fig. 2) and the load is given by W_p (Fig. 1). Then, as the result of removing the load, a permanent indentation with the depth λ_r is left on the surface of specimen.

In Fig. 3 the spherical surface AC_oB of diameter D gives the form of contact surface at the end of indentation process and the spherical surface AC_pB of diameter D_1 represents the permanent indentation when the load is removed. Then, suppose we

* In the case of contact between a flat seat and a spherical indenter with the diameter D , the displacement of the indenter is given by Hertz's equation:

$$l = \sqrt[3]{\frac{9}{8} \left(\frac{1-\nu_1^2}{E_1} \right)^2 \frac{W^2}{D}}$$

As the depth of the indentation on the flat seat can be calculated by Eq. (2), the value of difference between l and λ' ($\delta' = l - \lambda'$) will give the amount of the elastic deformation of the flat surface.

reapply the indenter of diameter D with the load W_p . The indentation should now be in exactly the same condition as it was when it was formerly formed (AC_oB). The whole of this process (removal and reapplication of the load) follows Hertz's equation as follows.

In the case of contact between a spherical seat with the diameter D_1 and a spherical indenter with the diameter D (Fig. 3), the amount of the elastic deformation of the flat surface is given by**

$$\begin{aligned} \delta &= \sqrt[3]{\frac{9}{64} \left(\frac{1-\nu_1^2}{E_1} \right)^2 \frac{W^2}{D} \left(1 - \frac{D}{D_1} \right)^{1/3}} \\ &= k \sqrt[3]{\frac{W^2}{D} \left(1 - \frac{D}{D_1} \right)^{1/3}} \quad \text{..... (6)} \end{aligned}$$

In the above case of contact, the radius of the circle of contact a can be written as

$$a = \sqrt[3]{\frac{3}{8} \left(\frac{1-\nu_1^2}{E_1} \right) W \left(\frac{D_1 D}{D_1 - D} \right)^{1/3}}$$

Hence, the depth of indentation λ_o is given by

$$\lambda_o = \frac{a^2}{D} = k \sqrt[3]{\frac{W^2}{D} \left(\frac{D_1}{D_1 - D} \right)^{2/3}} \quad \text{..... (7)}$$

Fig. 2 Shape and depth of indentation in process of deformation

** In the case of contact between a spherical seat with the diameter D_1 and a spherical indenter with the diameter D , the displacement of the indenter is given by Hertz's equation:

$$l = \sqrt[3]{\frac{9}{8} \left(\frac{1-\nu_1^2}{E_1} \right)^2 \frac{W^2}{D} \left(1 - \frac{D}{D_1} \right)^{1/3}}$$

The depth of indentation on the spherical seat λ under the load (λ is equivalent to λ_e in Fig. 3) can be written as

$$\lambda = \lambda_o \left(1 - \frac{D}{D_1} \right)$$

Substituting Eq. (7) in this equation

$$\lambda = \sqrt[3]{\frac{9}{64} \left(\frac{1-\nu_1^2}{E_1} \right)^2 \frac{W^2}{D} \left(1 - \frac{D}{D_1} \right)^{1/3}}$$

Hence, the value of the difference between l and λ will give the amount of the elastic deformation of the surface.

the value of Young's modulus or the mean flow pressure of the specimen becomes, the further the maximum load caused by impact increases. It is also evident that softer metals usually have a smaller effect of Young's modulus.

2.4 Theoretical Shore hardness number

In Fig. 3 the permanent indentation AC_pB has a radius of curvature larger than that of the indenter since the occurrence of rebound implies that there has been elastic recovery of the surface of the indentation. We may calculate the extent to which the indentation has shallowed as a result of this effect and relate it to the energy of rebound. In the case of contact between a spherical seat with the diameter D_1 and a spherical indenter with the diameter D , the displacement of the indenter is

$$l = \sqrt[3]{\frac{9}{8} \left(\frac{1-\nu_1^2}{E_1} \right)^2 \frac{W^2}{D} \left(1 - \frac{D}{D_1} \right)^{1/3}} \\ = 2k \sqrt[3]{\frac{W^2}{D} \left(1 - \frac{D}{D_1} \right)^{1/3}} \quad \dots\dots\dots(14)$$

Hence, the energy of rebound is estimated by

$$A = \int_0^{W_p} W dl = \frac{4}{5} k \frac{W_p^{5/3}}{D^{1/3}} \left(1 - \frac{D}{D_1} \right)^{1/3} \\ = G(h + \lambda_0 + \delta_p) \quad \dots\dots\dots(15)$$

where

h : height of rebound

Neglecting the negligible small terms λ_0 and δ_p in comparison with h in this equation, the height of rebound h is given by

$$h = \frac{4}{5} \frac{\pi^{1/2} k^{3/2} C^{1/2}}{G} W_p^{(2n-1)/n} \quad \dots\dots\dots(16)$$

If the height of rebound is h , the Shore hardness is⁽⁸⁾

$$H_S = \frac{10\,000}{65} \frac{h}{h_0}$$

Hence,

Fig. 5 Relation between the maximum load caused by impact and the mean flow pressure for highly work-hardened metals

$$H_S = \frac{10\,000}{65} \frac{4}{5} \frac{\pi^{1/2} k^{3/2} C^{1/2}}{G h_0} W_p^{(2n-1)/n} \quad \dots\dots\dots(17)$$

3. Experimental justification of the theory

3.1 Comparison of theoretical and experimental Shore hardness

In order to estimate the theoretical value of Shore hardness it is necessary to find the maximum load W_p caused by impact as a function of the mean flow pressure p_m of a specimen, which may be measured in general by the static indentation tests⁽⁴⁾.

Assuming that the mean flow pressure p_m in the course of the impact has the same variation as that in the static indentation, it is clear that in Shore hardness p_m is equivalent to Meyer hardness number, which is dependent on the diameter of the indenter and the load as given by Eq. (3)⁽²⁾. Then it is not difficult to deduce theoretically the relation between Shore hardness and Brinell hardness employed widely.

The mean flow pressure p_m in Shore hardness test is obtained from Eq. (3), and that in Brinell hardness test is given by

$$\frac{W_B}{\frac{\pi}{4} d_B^2} = \frac{4}{\pi} \frac{K^{2/n}}{D_B^{2(1-2/n)}} W_B^{(1-2/n)} \quad \dots\dots\dots(18)$$

where

W_B : load in Brinell hardness test

D_B : diameter of the ball in Brinell hardness test

d_B : chordal diameter of the indentation in Brinell hardness test

K : coefficient dependent on metals

Eliminating K between Eq. (3) and Eq. (18), the relation between C and d_B is shown by the following equation:

$$d_B = \frac{W_B^{1/n}}{\left(\frac{\pi}{4} C \right)^{1/2}} \left(\frac{D_B}{D} \right)^{(1-2/n)} \quad \dots\dots\dots(19)$$

On the one hand from this equation we can determine Brinell hardness number in terms of C , and on the other hand from Eq. (17) we can calculate Shore hardness number in terms of C . Consequently the relation between Shore hardness and Brinell hardness is deduced by using a parameter C . The theoretical curve for highly work-hardened metals ($n=2$, $C=p_m$) is shown in Fig. 6. The dotted line indicates the experimental curve provided in JIS (Japanese Industrial Standard), and some experimental results⁽⁵⁾ are also plotted. In this figure Vickers hardness numbers for carbon steel are converted into Brinell hardness numbers according to the table provided in SAE and ASTM. It is

evident from Fig. 6 that a good agreement between the theoretical and the experimental values is found over a fairly wide range of Shore hardness number. However the theoretical (full line) and experimental curve (dotted line) intersect each other. The reason why the theoretical values of Shore hardness are a little smaller than the experimental ones for softer metals may be that the dynamic flow pressure is higher than the static because of forces of a quasi-viscous nature involved in the dynamic deformation of metals. Then the above-mentioned

consideration seems to be reasonable from the fact that an abnormally high Brinell hardness number is obtained by rapid loading⁽⁵⁾.

On the contrary the reason why the theoretical values of Shore hardness are a little larger than the experimental ones for harder metals may be that the dynamic flow pressure is lower than the static because of the decrease of the amount of plastic deformation and of the increase of the amount of elastic recovery from indentation.

It is therefore clear from the above analysis

that the theoretical values of Shore hardness agree considerably well with the experimental ones within some range and may be estimated more exactly if the dynamic flow pressure of specimens is determined reasonably. The above consideration seems to be reasonable for the collision of a hard hammer on metals such as Shore hardness test. Under this consideration the relation of Shore hardness number to Brinell hardness number can be deduced theoretically and the effect of the elastic and the plastic properties of the specimen in Shore hardness test may be made clear.

Fig. 6 Hardness numbers conversion curve

Fig. 7 Relation between Shore hardness numbers and the depth of permanent indentation for highly work-hardened metals

3.2 Shore hardness and the depth of permanent indentation

If the elastic recovery of the surface of indentation in the direction of AB in Fig. 3 is negligibly small, the shape of permanent indentation becomes spherical (AC_pB). Hence the relation of Shore hardness number H_S to the depth of the permanent indentation λ_r produced in its test can be deduced theoretically as follows:

$$\lambda_r = \lambda_o \frac{D}{D_1} = \frac{W_p^{2/n}}{\pi DC} (1 - \pi^{3/2} k^{3/2} DC^{3/2} W_p^{(n-3)/n}) \dots \dots \dots (20)$$

This enables us to estimate the relation between H_S and λ_r . The theoretical curve for highly work-hardened metals ($n=2$, $C=p_m$) is shown in Fig. 7, and some experimental results are also plotted. Each experimental result measured with D-type Shore hardness tester represents respectively the mean value for twenty experiments. The depth of permanent indentation is measured on the profile curve recorded with a stylus instrument. The error in the measured values of depth seems to be due to the difficulty of measuring it at the deepest

point of spherical indentation. However it is evident from Fig.7 that a comparison of the theoretical results and the experimental ones of some specimens show a good agreement between them.

4. Conclusion

Assuming that the relations of the static load to the depth of indentation are applicable to the dynamic indentation, the maximum load caused by impact for some metals can be estimated theoretically in Shore hardness test: i.e. the hard metals have a high load caused by impact, and the metals with a small value of Young's modulus have a low load. Then the theoretical value of Shore hardness and the depth of permanent indentation produced in the test can be determined from Hertz's equations which give the relation between load and displacement in the case of contact between a spherical seat and a spherical indenter.

These theoretical deductions are discussed in the experimental evidences, and it is seen that the agreement between experiment and theory is fairly good for the relation between Shore hardness and

Brinell hardness as well as in the relation between Shore hardness and the depth of permanent indentation.

We can conclude that the dynamic tests based on the height of rebound may have a general constant relation to the static hardness values because the values of Shore hardness are almost proportional to the dynamic flow pressure.

Also, applying the theory of Shore hardness, we can confirm theoretically the influences on Shore hardness of the elastic and restitution properties of the specimen, of the weight of hammer, of the initial height of fall and of the shape of the tip of hammer.

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Discussion

K. IIZUKA : (1) I think it should be shown how the omission of minor terms in deducing Eq. (13) is valid for Shore hardness test.

(2) As Brinell test yields deeper indentations and therefore larger representative strain, I doubt the validity of equation $p_m \doteq H_B$.

(3) How are the H_B and H_V scales correlated in Fig.6? Do you obtain a similar theoretical relation also for C-type Shore testers?

T. YOSHIZAWA : Please tell us about the following items: (4) The similarity of the flow pressure and Brinell hardness or Meyer hardness,

(5) the relation between the values of flow pressure and the magnitude of the radius of diamond point,

(6) the effect of material of balls (steel or tungsten carbide) on Brinell hardness values,

(7) the theoretical equation of conversion between Shore hardness values and Brinell hardness values, and

(8) the origin of the radius of diamond hammer ($D=3.4$ mm).

Authors' closure

(1) The omission of minor terms in deducing Eq. (13) produces the error ΔW_p of the maximum load caused by impact W_p , but the value of the

error ΔW_p is in the range of 0.03 kg to 0.18 kg for the range of p_m of 100 to 600 kg/mm² in the case of highly work-hardened metals with large Young's modulus ($E_1=2.1 \times 10^6$ kg/cm², $\nu_1=0.30$, $n=2.0$ and $C=p_m$). Therefore it is seen that the value of the error ΔH_S of Shore hardness number is in the range of 0.0144 to 0.304.

On the other hand in the case of highly work-hardened metals with small Young's modulus ($E_1=0.8 \times 10^6$ kg/cm², $\nu_1=0.31$, $n=2.0$ and $C=p_m$) ΔW_p is 0.25 kg for p_m of 300 kg/mm² (the highest value for the metals with small Young's modulus). Therefore it is seen that ΔH_S is 0.65.

As the allowable deviation ΔH_S of the measured value from the nominal Shore hardness number is 1.0 as provided in JIS, it is clear that the omission of minor terms in deducing Eq. (13) is reasonable.

(2) As there is no work-hardening in the course of indentation on highly work-hardened metals, it is clear from Eq. (3) that the mean flow pressure p_m is independent of both the load and the diameter of the spherical indenter. Hence p_m is equivalent to Meyer hardness number and approximately equal to Brinell hardness number for highly work-hardened metals.

(3) In Fig.6 Vickers hardness numbers for carbon steel are converted into Brinell hardness

numbers according to the table provided in SAE and ASTM.

The theoretical Shore hardness numbers of highly work-hardened steel ($E_1=2.1 \times 10^6$ kg/cm², $\nu_1=0.30$, $n=2.0$ and $C=p_m$) for C-type Shore hardness testers are shown in Append-Table 1.

It is evident from this table that a good agreement between the theoretical values for C-type and for D-type Shore hardness tester is found over a wide range. Hence our theoretical analysis of Shore hardness is also considered to be reasonable from this table. But to check the validity of the theory further it may be necessary to calculate the theoretical values by the use of the measured values (the values of D , G and h_0) of C-type Shore hardness tester.

(4) In our theory it is assumed that the mean flow pressure is equivalent to Meyer hardness number. Shore hardness number in terms of C can be calculated from Eq. (17) and Brinell hardness number in terms of C can also be determined from Eq. (19). Hence the relation between Shore hardness and Brinell hardness is deduced by using a parameter C .

(5) The relation between the mean flow pressure and the diameter of the spherical indenter is given by Eq. (3).

(6) The experimental values of Brinell hardness were measured by using a steel ball. If a tungsten carbide ball is used, the experimental

Append-Table 1

H_B		100	200	300	400	500	600
Type							
C-type	$D=3.0$ mm	11.3	26.1	41.6	57.6	73.2	88.3
	$D=3.4$ mm	12.5	28.3	44.5	62.6	79.2	95.2
D-type	$D=3.4$ mm	12.5	27.3	43.8	60.6	75.7	92.7

Remarks: Assumptions in the calculation of the theoretical values for C-type Shore hardness tester are as follows:

- (1) The diameter of the spherical surface of the hammer D is in the range 3.0 to 3.4 mm.
- (2) The weight of the hammer G has a standard value of 2.36 g.
- (3) The initial height of fall h_0 has a standard value of 254 mm.
- (4) The height of rebound per unit hardness number has a standard value of 1.651 mm.
- (5) p_m is equivalent to H_B .

values of Brinell hardness will become larger for $H_B > 450$.

(7) The approximate relation between W_p and p_m for steel deduced from Fig. 5 is to be: $W_p = 6.04 p_m^{0.408}$. Inserting this equation into Eq. (17), we obtain the theoretical equation between Shore hardness number and Brinell hardness number for highly work-hardened metals ($n=2.0$, $C=p_m$) as follows:

$$H_S = 7.53 \times 10^{-2} H_B^{1.112},$$

where it is assumed that p_m is equivalent to H_B .

(8) The diameter of the spherical surface of the diamond hammer D is a measured value of the D-type Shore hardness tester used in experiment.