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Adhesion Characteristics and Self-Excited Vibrations of Driving Axle System of Individual Drive Rectifier A. C. Locomotives*

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The ideal quality of the locomotive is that it produces a great tractive effort with less adhesive weight and with little vibrating stress in driving axle system, so that it has a superior adhesive characteristics and mechanical strength. In order to attain the ideal, the adhesive characteristics must be improved, and self-excited vibration of the driving axle system during slipping must be suppressed.

Research has been performed in these directions. The fundamental equation for slip motion of AC locomotives was derived, and the conditions for suppressing self-excited vibration were sought. These data were studied by means of analog computers and truck testing equipment, and actual tests on service lines were conducted. As a result, success was attained in suppressing self-excited vibration.

Studies were subsequently conducted of methods of improving the adhesive characteristics while simultaneously suppressing self-excited vibration. As a result, it was established that this objective could be attained.

1. Introduction

An ideal locomotive must be light in weight and have a great tractive effort. At the same time, the stress generated in the driving axle system must be as small as possible, and there must be a superior mechanical strength.

In order to obtain lightweight locomotives with a great tractive effort, it is necessary to improve the adhesion characteristics and to utilize as much as possible of the adhesive force occurring between the driving wheels and the rails as tractive effort. Ideally, all of this adhesive force ought to be utilized.

However, there are wide fluctuations in the adhesive force even though the rail conditions remain uniform. When all of the adhesive force has been utilized as tractive effort at a moment when there is a great adhesive force, in the following moment there may be only a very small adhesive force, and the tractive effort at the periphery of the driving wheels may exceed the adhesive force. As a result, the driving wheels will be accelerated and slipping will occur.

Upon the occurrence of slipping, there will be a sudden reduction in the adhesion coefficient

together with the increasing slip speed. Not only will this cause a further increase in the slip speed, but self-excited vibrations will also occur in the driving axle system. This will result in the occurrence of great mechanical stress, finally leading to accidents such as cracking or breakage. Such accidents are even more likely to occur because of the mechanical strength, which has been lowered relatively on account of the improved adhesion characteristics.

At the present time, nearly all A. C. locomotives are of the rectifier type. Rectifier locomotives generally have adhesion characteristics superior to those of other types of locomotives, and this is one of the chief advantages inherent in A. C. electrification. At the same time, there have been numerous accidents which are believed to be attributable to self-excited vibrations.

Seki et al. previously reported their findings concerning the adhesion characteristics of rectifier type A. C. locomotives⁽¹⁾. The present writers have been engaged in research about how to prevent the occurrence of self-excited vibrations and how to improve further the adhesion characteristics.

In this report, the present writers describe their findings concerning the individual drive system, which is the most widely used system at

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the present time.

2. Fundamental equations

The driving system of an individual drive rectifier A. C. locomotive is illustrated in Fig. 1. In the system shown in the figure, the A. C. source voltage E_a is converted by the rectifier MR into D. C. voltage and is supplied to the main motor M. The driving torque generated in the main motor armature is transmitted to the gear which meshes with the pinion directly coupled to the armature. It is thence transmitted to the driving wheels via a quill spring or a rubber spring.

The greatest tractive effort is required for starting in this driving system. There are two possible reasons for the occurrence of slipping in the driving wheels. First, when there is a rise in the terminal voltage of the main motor when the notch has been advanced, the driving torque will increase, and the tractive effort at the periphery of the driving wheels will be greater than the adhesive force. Second, slipping may occur when there is a sudden decrease in the adhesive force and when the latter has become smaller than the tractive effort at the periphery of the driving wheels.

Here let us consider the second case. Suppose that slipping has occurred on the driving wheels when there has been a decrease of $\Delta\mu$ in the adhesion coefficient. In this case, the relation between the slip speed and the adhesion coefficient will be as shown in Fig. 2. There is a considerable dispersion of the values, and the average values show a nonlinear distribution. Nevertheless, for the sake of simplicity, let us assume that we may take the average values and approximate

Fig. 1 Driving system of rectifier A. C. locomotive

them to a linear distribution in the area where there is a low slip speed. When this has been done, after the adhesion has been suddenly reduced by $\Delta\mu$ from μ_0 and slipping has occurred, the adhesion coefficient during slipping may be approximated by means of the following equation:

$$\mu_s = (\mu_0 - \Delta\mu) - Ar\dot{\phi}_2 \quad (\dot{\phi}_2 \geq 0) \quad \dots\dots\dots (1)$$

Here,

$\dot{\phi}_2$: Slip angular velocity of driving wheels rad/sec

ϕ_2 : Slip angle of driving wheels rad

A : Constant sec/m

r : Driving wheel radius m

If the above equation is expressed as a Laplace transformed equation, we shall obtain the following:

$$\mu_s = (\mu_0 - \Delta\mu) - Arp\phi_2 \quad \dots\dots\dots (2)$$

We shall obtain the following equation for the slip motion of the driving axle system, if we express the slip angle of the gear as ϕ_1 (rad), the polar moment of inertia of the armature and of the gear and pinion converted into terms of the moment of inertia of the gear axle as Θ_1 (kg m^2), the polar moment of inertia of the driving wheel and the driving wheel axis as Θ_2 (kg m^2), the weight on the driving wheels (the adhesion weight) as W (N), the driving torque converted into terms of the gear axle as τ (Nm), and the complex spring constant of the quill spring or rubber spring as $k_0(1 + T_v p)$:

$$\begin{bmatrix} \Theta_1 p^2 + k_0(1 + T_v p) & -k_0(1 + T_v p) \\ -k_0(1 + T_v p) & \Theta_2 p^2 - AW r^2 p + k_0(1 + T_v p) \end{bmatrix} \times \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} \tau \\ (\Delta\mu - \mu_0) W r \end{bmatrix} \quad \dots\dots\dots (3)$$

Fig. 2 Relation between slip speed and adhesion coefficient during slip

If the main motor current is I_M (A), the D.C. inductance of the main armature circuit is L (H), the equivalent resultant resistance of the main circuit is R (Ω), the equivalent output voltage of the rectifier is E_R (V), and the counter electromotive force of the main motor is E_M (V), then the following equation will apply:

$$(Lp + R)I_M = E_R - E_M \quad (4)$$

If the gear ratio is s , then the revolving speed of the main motor will be $s\varphi_1$. If the ratio between this and the counter electromotive force of the main motor E_M , i.e., the unit revolution flux, is $\zeta\phi$ (Vs), then the following equation will apply when the train speed at starting is nearly zero:

$$E_M = s\zeta\phi\varphi_1 \quad (5)$$

The torque τ generated by the main motor, converted into terms of the gear, will be:

$$\tau = s\eta\zeta\phi I_M \quad (6)$$

Here, η is the efficiency of torque transmission.

When the total equivalent resistance of the main circuit R is divided up into the internal resistance of the main motor R_M and the internal resistance of the power source R_1 , and when the D.C. inductance of the main circuit L is divided up into the internal inductance of the main motor L_M and the inductance of the power source L_1 , then the main motor terminal voltage E_1 will be:

$$\begin{aligned} E_1 &= E_R - I_M R_1 - L_1 p I_M \\ &= E_M + I_M R_M + L_M p I_M \end{aligned} \quad (7)$$

Let us suppose that slipping has occurred in the driving wheels and the revolving speed of the main motor has increased. If E_M has increased for this reason according to Eq. (5), and if I_M has decreased according to Eq. (4), then E_1 will increase according to Eq. (7).

Most of the ignitron A.C. locomotives in Japan have control devices to maintain this terminal voltage at a uniform value⁽²⁾⁽³⁾. In cases when these control devices are provided, the current will be reduced to an extent where the increase in E_1 in Eq. (7) will be canceled out. The driving torque determined by Eq. (6) will be reduced, and the slip speed will be diminished.

If $F(p)$ is taken as the transfer function for this control device, then the following will be

$$\begin{bmatrix} \Theta_1 p^2 + k_0(1 + pT_v) & -k_0(1 + pT_v) & -s\eta\zeta\phi \\ -k_0(1 + pT_v) & \Theta_2 p^2 - AWr^2 p + k_0(1 + pT_v) & 0 \\ s\zeta\phi p & 0 & R + Lp \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ I_M \end{bmatrix} = \begin{bmatrix} 0 \\ (\Delta\mu - \mu_0)Wr \\ E_R \end{bmatrix} \quad (9)$$

One can obtain the following characteristic equation from the foregoing:

$$A_0 + A_1 p + A_2 p^2 + A_3 p^3 + A_4 p^4 = 0 \quad (10)$$

Here,

$$A_0 = \eta(s\zeta\phi)^2 k_0 - AWr^2 k_0 R \quad (11)$$

Fig. 3 Block diagram of driving system of individual drive rectifier A.C. locomotive during slipping

obtained:

$$F(p) = \left\{ \int_0^\infty E_R e^{-pt} dt \right\} \left\{ \int_0^\infty (E_s - E_1) e^{-pt} dt \right\}^{-1} \quad (8)$$

Here, E_s is the reference voltage.

The foregoing are the fundamental equations for this system. A block diagram plotted on the basis of these equations is shown in Fig. 3.

In this block diagram the unit revolution flux $\zeta\phi$ is a function of the main motor current I_M . It will fluctuate when there are fluctuations of I_M . However, slip phenomena present problems in the large-current region during starting, where $\zeta\phi$ is regarded as being practically constant. Therefore, in the ensuing analysis, $\zeta\phi$ is assumed to be constant.

3. Stability criteria of self-excited vibrations

The vibrations in the driving axle system generated through the quill springs or the rubber springs are generally in the vicinity of 10 c/sec or greater, while the vibrations originating in the automatic voltage regulator are about 2 c/sec or less. For this reason, it is believed that the automatic voltage regulator exerts almost no direct influence on the vibrations of the driving axle system. Since there are also many locomotives which do not have automatic voltage regulators, our study here will pertain to locomotives lacking such automatic voltage regulators.

In cases when there are no automatic voltage regulators, the following equation can be derived from the preceding fundamental equations:

$$A_1 = (\Theta_1 + \Theta_2 - AWr^2 T_v) k_0 R - AWr^2 k_0 L + \eta (s\zeta\phi)^2 (k_0 T_v - AWr^2) \dots\dots\dots (12)$$

$$A_2 = (k_0 \Theta_1 T_v + k_0 \Theta_2 T_v - AWr^2 \Theta_1) R + (\Theta_1 + \Theta_2 - AWr^2 T_v) k_0 L + \eta (s\zeta\phi)^2 \Theta_2 \dots\dots\dots (13)$$

$$A_3 = \Theta_1 \Theta_2 R + (k_0 \Theta_1 T_v + k_0 \Theta_2 T_v - AWr^2 \Theta_1) L \dots\dots\dots (14)$$

$$A_4 = \Theta_1 \Theta_2 L \dots\dots\dots (15)$$

Since this system is stable on the basis of Hurwitz's discriminant, it is clear that self-excited vibrations will not occur if the system is composed so as to satisfy the following conditions:

$$A_0 > 0 \quad A_1 > 0 \quad A_2 > 0 \quad A_3 > 0 \quad A_4 > 0 \dots\dots\dots (16)$$

and

$$A_1 A_2 A_3 - A_0 A_3^2 - A_1^2 A_4 > 0 \dots\dots\dots (17)$$

In cases when the main motor is suspended on the bogie frame, quill springs are used to transmit the power in order to eliminate shock from the driving wheels. In such cases, however, the spring constants can generally be expressed in terms of the constant part only, i. e., in terms of k_0 only, and A_1 , A_2 , and A_3 will be simplified in the following manner⁽⁴⁾:

$$A_1 = k_0 (\Theta_1 + \Theta_2) R - AWr^2 k_0 L - AWr^2 \eta (s\zeta\phi)^2 \dots\dots\dots (18)$$

$$A_2 = -AWr^2 \Theta_1 R + k_0 (\Theta_1 + \Theta_2) L + \eta (s\zeta\phi)^2 \Theta_2 \dots\dots\dots (19)$$

$$A_3 = \Theta_1 \Theta_2 R - AWr^2 \Theta_1 L \dots\dots\dots (20)$$

In studying these data, let us focus attention upon R , L , and k_0 , the values of which can be varied relatively easily. On the basis of Eq. (11), we obtain:

$$R < \frac{\eta (s\zeta\phi)^2}{AWr^2} \dots\dots\dots (21)$$

On the basis of Eqs. (18), (19), and (20), we obtain:

$$L < \frac{\Theta_1 + \Theta_2}{AWr^2} R - \frac{\eta (s\zeta\phi)^2}{k_0} \dots\dots\dots (22)$$

$$L < \frac{\Theta_2}{AWr^2} R \dots\dots\dots (23)$$

$$L > \frac{AWr^2 \Theta_1 R - \eta (s\zeta\phi)^2 \Theta_2}{k_0 (\Theta_1 + \Theta_2)} \dots\dots\dots (24)$$

However, in cases when Eq. (23) is valid, Eq. (24) ordinarily is also valid. For this reason, the following expression may be made concerning L :

$$\frac{\Theta_2}{AWr^2} R > L > \frac{AWr^2 \Theta_1 R - \eta (s\zeta\phi)^2 \Theta_2}{k_0 (\Theta_1 + \Theta_2)} \dots\dots\dots (25)$$

In this case, let us suppose that

$$R \leq \frac{\eta (s\zeta\phi)^2 \Theta_2}{AWr^2 \Theta_1} \dots\dots\dots (26)$$

[Since ordinarily $\Theta_2 < \Theta_1$, these conditions are stronger than the conditions in Eq. (21)]. In such a case, the right side of Eq. (25) will have a value less than zero. However, since no negative inductance is present, Eq. (25) will be as follows:

$$L < \frac{\Theta_2}{AWr^2} R \dots\dots\dots (27)$$

If the electric time constant of the main circuit is T_E , then it will have the following value:

$$T_E = \frac{L}{R} < \frac{\Theta_2}{AWr^2} \dots\dots\dots (28)$$

Consequently, one must first determine a value for R which will satisfy Eq. (21) or Eq. (26).

Then one determines a value for L which will satisfy Eq. (25) or Eq. (27). When this has been done, the coefficients in the characteristic equation will all have positive values. Now if one determines a value for k_0 which will satisfy Eq. (17) after R and L have both been determined in this manner, a situation will be created in which self-excited vibrations will not occur. In other words, with k_0 as an unknown, one substitutes the values of A_0 , A_1 , $\dots$, A_4 in Eq. (17) in the following manner:

$$\begin{aligned} A_1 A_2 A_3 - A_0 A_3^2 - A_1^2 A_4 &= a_1 (a_2 c_3 - a_1 c_4) k_0^2 \\ &+ (a_1 c_2 c_3 - a_2 c_1 c_3 - a_0 c_3^2 + 2 a_1 c_1 c_4) k_0 \\ &- c_1 (c_2 c_3 + c_1 c_4) > 0 \dots\dots\dots (29) \end{aligned}$$

Here,

$$\left. \begin{aligned} a_0 &= \eta (s\zeta\phi)^2 - AWr^2 R = A_0 \\ a_1 &= (\Theta_1 + \Theta_2) R - AWr^2 L \\ a_2 &= (\Theta_1 + \Theta_2) L \\ c_1 &= \eta (s\zeta\phi)^2 AWr^2 \\ c_2 &= \eta (s\zeta\phi)^2 \Theta_2 - AWr^2 \Theta_1 R \\ c_3 &= \Theta_1 \Theta_2 R - AWr^2 \Theta_1 L = A_3 \\ c_4 &= \Theta_1 \Theta_2 L = A_4 \end{aligned} \right\} \dots\dots\dots (30)$$

If the range of k_0 capable of satisfying these conditions is not within the range where the results can be realized with positive real numbers, the values of R and L must be varied within the aforementioned conditions (ordinarily, they need only be made smaller) so that k_0 will come within a range where the results can be realized with positive real numbers.

Next, let us perform the calculations, referring to the constants in the Type ED 71 ignitron A. C. locomotive, one of Japan's representative locomotives. The constants for these locomotives differ slightly depending upon the year of manufacture. In the ED 716 locomotives manufactured in 1960, the main motors are suspended on the bogie frames, and approximately the following values apply:

$$\begin{aligned} \Theta_1 &= 614 \text{ kg m}^2 & \Theta_2 &= 218 \text{ kg m}^2 \\ r &= 0.56 \text{ m} & W &= 1.56 \times 10^5 \text{ N} \\ s &= 5.47 & \eta &= 0.9 \\ L &= 0.005 \text{ H} & \zeta\phi &= 5.5 \text{ V sec/rad} \end{aligned}$$

Since the main motors are suspended on the bogie frames, the power is transmitted by quill springs, the spring constant of which is:

$$k_0 = 4.65 \times 10^5 \text{ N m/rad}$$

Since the main circuit is arranged as shown in Fig. 4, the equivalent resistance of the main circuit R will, it is thought, be equal to 0.08Ω when slipping occurs in only one axle, and to 0.2Ω when all the axles are slipping at the same time.

The dynamic friction reduction coefficient A is supposed to be 0.027 sec/m on the basis of Fig. 2.

If we substitute the constants given above in Eqs. (11), (15), and (18)~(20), the following will be obtained when R is 0.08Ω .

$$A_0 = 290 \times 10^6 \text{ kg}^2 \text{ m}^4 / \text{sec}^3$$

$$A_1 = 295 \times 10^5 \text{ kg}^2 \text{ m}^4 / \text{sec}^2$$

$$A_2 = 203 \times 10^4 \text{ kg}^2 \text{ m}^4 / \text{sec}$$

$$A_3 = 6.64 \times 10^3 \text{ kg}^2 \text{ m}^4$$

$$A_4 = 6.7 \times 10^2 \text{ kg}^2 \text{ m}^4$$

Although all of the coefficients of the characteristic equation will have positive values, the provisions of Eq. (17) will not be fulfilled because

$$A_1 A_2 A_3 - A_0 A_3^2 - A_1^2 A_4 \\ = -20 \times 10^{16} \text{ kg}^6 \text{ m}^{12} / \text{sec}^3$$

When the calculations are performed for $R = 0.2\Omega$, it is found likewise that the coefficients of the characteristic equation assume positive values but that the provisions of Eq. (17) will not be fulfilled.

Consequently, it may be supposed that self-excited vibrations will occur in this locomotive during slipping.

Next, the constants enumerated above are inserted into the block diagram in Fig. 3, and analysis is performed by an analog computer for a locomotive having no automatic voltage regulator. Results such as those shown in Fig. 5 are obtained, and it is found that self-excited vibrations

R_r : Equivalent internal resistance of power source
 R_L : Internal resistance of smoothing inductance DCL
 R_f : Internal resistance of series field winding SF
 R_m : Armature resistance of main motor M

Fig. 4 Main circuit of Type ED 71 locomotive

of approximately 8 c/sec are generated.

The oscillogram shown in Fig. 6 was taken during actual slipping in a locomotive using no automatic voltage regulator. It is evident that self-excited vibrations of about 8 c/sec are occurring in each of the axles. These findings coincide in general with the results of analysis by the analog computer.

Next, calculations were undertaken in order to find out how to prevent the occurrence of self-excited vibrations in this type of locomotive. If, for instance, only the values of R and L were changed and the other constants are left unchanged, it was learned that it would be sufficient to adopt a value of R equivalent to about 0.04Ω or less and a value of L/R equivalent to about 0.025 sec or less.

In the ED 7145 locomotives produced in 1962, the newly developed semi-suspended motor mounting system was adopted. In these locomotives, the power is transmitted through rubber springs, the spring constant of which is as follows:

$$k_0 = 2.3 \times 10^6 \text{ N m/rad}$$

The value of T_v differs according to the measuring frequency. However, in the vicinity of

Fig. 5 Analytical results of ED 716 locomotives when automatic voltage regulator was not used

Fig. 6 Oscillogram taken during slipping of ED 716 locomotive when automatic voltage regulator was not used (Since slipping did not cease, readhesion was obtained by lowering the notch)

the vibration frequency of 18 c/sec, at which vibrations are thought to occur in the driving axle system, its value will be:

$$T_v \approx 1 \times 10^{-3} \text{ sec}$$

The other constants are approximately the same as those of the ED 716 locomotives. Therefore, when these constants are substituted in Eqs. (11)~(15), the following values will be obtained when R is 0.08 Ω .

$$A_0 = 1440 \times 10^6 \text{ kg}^2 \text{ m}^4 \Omega / \text{sec}^3$$

$$A_1 = 1450 \times 10^5 \text{ kg}^2 \text{ m}^4 \Omega / \text{sec}^2$$

$$A_2 = 980 \times 10^4 \text{ kg}^2 \text{ m}^4 \Omega / \text{sec}$$

$$A_3 = 15.2 \times 10^3 \text{ kg}^2 \text{ m}^4 \Omega$$

$$A_4 = 670 \text{ kg}^2 \text{ m}^4 \Omega \text{ sec}$$

In this case, all of the coefficients in the characteristic equation will have positive values, and the following results will also be obtained:

Fig. 7 Analytical results of ED 7145 locomotive when automatic voltage regulator was not used

Fig. 8 Oscillogram taken during slipping of ED 7145 locomotive when automatic voltage regulator was not used

$$A_1 A_2 A_3 - A_0 A_3^2 - A_1^2 A_4$$

$$= 2.13 \times 10^{19} \text{ kg}^6 \text{ m}^{12} \Omega^3 / \text{sec}^3 > 0$$

Thus the provisions of Eq. (17) will be fulfilled, and self-excited vibrations will not occur.

The constants given above were inserted into the block diagram in Fig. 3, and analysis was performed by an analog computer. The results are shown in Fig. 7. Test results obtained with the actual locomotives are shown in Fig. 8. In neither case is the occurrence of self-excited vibrations observable.

4. Adhesion characteristics

When slipping has occurred in the driving wheels, the most important requirement for improving the adhesion characteristics is to suppress the slip speed to the minimum and also to bring about readhesion as rapidly as possible.

If the composition satisfies the aforementioned conditions for suppressing the self-excited vibrations, self-excited vibrations will not occur. When studying the adhesion characteristics in such a case, we may doubtless assume that the gear and the driving wheel work together as an integral whole. If in Eq. (3) we assume that

$$\varphi_1 = \varphi_2 \dots \dots \dots (31)$$

we obtain the following equation:

$$\theta p n_s - A W r^2 n_s = \tau + \Delta \mu W r - \mu_0 W r \dots \dots \dots (32)$$

Here,

$$\theta = \theta_1 + \theta_2 \dots \dots \dots (33)$$

$$n_s = p \varphi_1 = p \varphi_2 \dots \dots \dots (34)$$

A block diagram such as one shown in Fig. 3 will be obtained.

Here, if v_s is the slip speed converted into terms of the speed at the periphery of the driving wheel, taking into consideration the fact that $\tau - \mu_0 W r \approx 0$, we shall obtain the following results in cases when there is no automatic voltage regulator:

$$v_s(p) = r n_s(p) = \frac{(Lp + R) \Delta \mu W r^2}{L \theta p^2 + (R \theta - A W r^2 L) p + \eta (s \zeta \phi)^2 - A W r^2 R} \dots \dots \dots (35)$$

From this equation, it is clear that the slip speed will grow infinitely unless

Fig. 9 Block diagram assuming driving wheel and gear form an integral whole

$$R < \eta(s\zeta\phi)^2 / AWr^2 \dots\dots\dots (36)$$

It is also clear that increasing vibration will result unless the following is valid:

$$T_E = L/R < \theta / AWr^2 \dots\dots\dots (37)$$

The vibrations occurring in such cases are called armature circuit vibrations.

Among the two conditions mentioned above, the former is the same as Eq. (21); consequently, the conditions are less rigid than in Eq. (26).

In the latter case also, the conditions are less strict than in Eq. (27). Thus, both of these conditions will unfailingly apply if Eqs. (26) and (27) are both valid.

Since Seki and his colleagues have already reported on their studies concerning Eq. (35), let us here conduct studies with an analog computer, referring to the constants of the aforementioned Type ED 71 locomotives, concerning the results occurring when the equivalent resistance of the main circuit R and the inductance L are varied.

First, let us assume the inductance of the main circuit L to have a constant value of 0.005 H. Let us vary the equivalent resistance R from 0.2 Ω to 0.02 Ω . In this case, the temporary changes in the value of the slip speed v_s , as well as the relations between the slip speed and the tractive

force at the periphery of the driving wheel will be as shown in Fig. 10. As R is gradually reduced, the maximum slip speed will become proportionally smaller, and the slip speed will become vibratory. As a result, it is clear that readhesion will become easier. In addition, the converged value of the slip speed will also become smaller. Applying Eq. (35), we can obtain the following converged value:

$$v_s(0) = \frac{R\Delta\mu W r^2}{\eta(s\zeta\phi)^2 - AWr^2 R} \text{ m/sec} \dots\dots\dots (38)$$

In this case, the block diagram mentioned above will be valid only in the region in which v_s has a positive value. It will not apply when the value of v_s is smaller than zero, and the region in this diagram where v_s has a negative value is meaningless. Nevertheless, it is shown by means of broken lines for the sake of reference.

Next, in Fig. 11 are shown the results when the value of the equivalent resistance of the main circuit R was maintained at a constant value of 0.08 Ω and the inductance of the main circuit L was varied.

It is clear from this that, as the value of L is gradually increased, the slip speed will become increasingly vibratory and there will be an increasing possibility of readhesion; however, the maximum slip speed will also increase simulta-

Fig. 10 Results when equivalent resistance of main circuit R was varied ($L=0.005$ H, without automatic voltage regulator, $\Delta\mu=0.1$)

Fig. 11 Results when inductance of main circuit L was varied ($R=0.08$ Ω , without automatic voltage regulator, $\Delta\mu=0.1$)

neously. Naturally enough, the converged value of the slip speed will remain unchanging.

In addition, if the value of L is made too large, Eq. (27) will no longer be valid, and self-excited vibrations will occur when quill springs are used.

Next, let us consider cases in which automatic voltage regulators are used. Generally speaking, automatic voltage regulators tend to have complex transfer functions. However, for the sake of simplicity, let us assume that in this case the transfer function can be approximated in the primary system, i. e., that it can be approximated as

$$E(p) = \frac{A_f}{1+1.2p} \quad (39)$$

In this case, let us analyze the patterns of variation in the slip speed when the gain A_f was changed. The results shown in Fig. 12 are obtained. In this analysis, R_M was assumed to be 0.01Ω , and L_M was assumed to be 0. From these results, it is clear that as A_f is gradually increased, the maximum slip speed and the converging point both decrease proportionally. In addition, the slip speed becomes increasingly vibratory, and readhesion becomes easier. The automatic

Fig. 12 Results when using an automatic voltage regulator with a transfer function of $A_f/(1+1.2p)$

voltage regulators installed in the Type ED 71 locomotives generally correspond to the case when A_f is 14.

In cases when the automatic voltage regulator can be approximated in the integral system, i. e., when approximation can be performed as

$$F(p) = \frac{1}{Tp} \quad (40)$$

let us analyze the patterns of variation in the slip speed when T was changed. The results of this analysis are shown in Fig. 13. It is clear that as T is gradually decreased, the maximum slip speed will decrease proportionally. In this case also, R_M was assumed to be 0.01Ω , and L_M was assumed to be 0.

Circuits with practically the identical composition as in these analyses were prepared, and tests were conducted. As a result, although considerable dispersion was observed, the same general tendencies as the results of the above-mentioned analyses with analog computers were observed⁽⁵⁾.

The automatic voltage regulator does not have any direct effects upon the self-excited vibrations of the driving axle system through the quill springs or the rubber springs. However, it is effective since it suppresses the slip speed to a minimum

Fig. 13 Results when using an automatic voltage regulator with a transfer function of $1/Tp$

Fig. 14 Self-excited vibration pattern when there is an automatic voltage regulator (ED 716 locomotive used). It was assumed that $v_s < 0$ and $\mu_s < 0$ when performing analysis with analog computer. $\Delta\mu$ was assumed to be 0.1)

value, produces readhesion, and causes the vibrations to cease.

This pattern was tested in an actual locomotive. The results of the actual tests are shown in Fig. 14 together with the results of their analysis by means of an analog computer⁽⁶⁾.

Taking the above-mentioned results into consideration, let us now consider what measures can be taken to improve the adhesion characteristics and to suppress the occurrence of self-excited vibrations in the driving axle system. Generally speaking, the following methods may be followed.

(1) The conditions for suppressing self-excited vibrations can be satisfied chiefly by reducing sufficiently the equivalent resistance of the main circuit R . At the same time, the slip speed is minimized in order to make possible readhesion.

(2) In addition to fulfilling the conditions for suppressing the self-excited vibrations by one means or another, it is further necessary to provide an automatic voltage regulator or the like for the purpose of controlling the main motor terminal voltage. In this way, the slip speed can be suppressed to a minimum, and readhesion can be brought about.

The first objective of minimizing the equivalent resistance of the main circuit R can be attained in various ways. First, we can adopt the saturable reactor which has recently been developed and which is now being put into actual use in Type

ED 75 locomotives. Second, we can also apply the low-tension no-arc tap-changing continuous control system using silicon controlled rectifiers⁽⁷⁾.

As mentioned above, the automatic voltage regulator mentioned in second place was put into use for the first time in the world in the Type ED 71 locomotives. As is well known, these automatic voltage regulators have been in actual service with an excellent record.

5. Conclusion

In the foregoing the writers have discussed their findings concerning the adhesion characteristics of individual drive rectifier A. C. locomotives and concerning the self-excited vibrations of their driving axle systems. First of all, the fundamental equations for the slip motion of these locomotives were derived, and the conditions for preventing the occurrence of self-excited vibrations were found. Next, these data were analyzed by means of analog computers, and the analytical results were compared with the results of tests conducted with actual locomotives. It was ascertained that the theories were generally correct. Next, studies were performed on methods of improving the adhesion characteristics, and methods of suppressing the occurrence of self-excited vibrations and of improving the adhesion characteristics were discussed.

In conclusion, the writers express their profound gratitude to the personnel of the Japanese National Railways, of the Hitachi Research Laboratory, and of the Mito Works, Hitachi, Ltd., who so kindly guided and assisted them in this research.

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