solution by Gardner and the two-dimensional one are equivalent to the thickness to height ratio of the order of one-third or less. For the case where the thickness to height ratio is larger than one-third, the temperature distribution in the fin becomes two-dimensional. The efficiency of the thick fin becomes smaller with an increase in the half-thickness to height ratio a and b , because the contraction effect of the heat flow causes a large temperature drop through the fin-root.

(4) The temperature distribution and efficiency of a radiation plate-fin with constant thickness are analysed. The efficiency of the thick fin becomes smaller with an increase in the half-

thickness to height ratio a and b . The efficiency is incorrect because of the non-uniform temperature distribution across the fin-root.

The author is indebted to research members for assistance and advice in the work presented.

References

- (1) Y. Okamoto : *Bulletin of JSME*, Vol. 7, No. 28 (1964), p.751.
- (2) H.H. Keller and E.V. Somers: *Trans. ASME*, Ser. C, Vol.81, No.2 (1959), p.151.
- (3) K.A. Gardiner: *Trans. ASME*, Vol.67, No.8 (1945), p.621.
- (4) Y. Okamoto: *Proc. 13th Japan Nat. Congr. Appl. Mech.* 1963 (1965), p.225.

536.25:536.33

Thermal Performances of Radiative and Convective Plate-Fins with Mutual Irradiation*

By Yoshizo OKAMOTO**

In using radiative and convective fins on heat exchangers, space radiators or nuclear fuel elements for heat release at high temperature, there are cases when the surface of the fins emits radiative and convective energy with mutual irradiation.

Most of analyses studied thus far have been carried out for an isolated fin and have neglected the mutual irradiation.

In this paper, thermal performances of radiative and convective plate-fins are systematically analysed on the basis of a derived steady-state integro-differential equation. This has been solved for a wide range of values of the normalized parameters. Lastly, the relation between the cases when there is mutual irradiation and when there is not is formulated.

1. Introduction

For many purposes, the finned surface is being used to increase the heat transfer, especially for high temperature work. Heat exchangers, space-vehicle radiators, and fuel-elements of gas-cooled reactors require the use of finned surface for high-flux heat-transfer. In this case, the finned surface emits the radiative and convective energy to other surfaces of an enclosure and the gas. And the fin itself is accompanied by the internal heat generation.

Most of analyses studied thus far have been carried out for an isolated fin and have neglected

the mutual irradiation between the surfaces. Thermal performance of a radiation fin which emits the heat to a vacuum space of mean space-temperature as in the case of space-vehicle radiators is given by Chambers⁽⁶⁾.

Sparrow⁽⁵⁾ and Eckert⁽⁴⁾ referred to the analysis of net-radiation fins which can exchange heat with each other as well as the external environment. Heat-balance equation is complicated, and it is possible to arrive at an integro-differential equation.

In this paper, this problem is taken up to further the analysis of radiative and convective plate-fins. The mathematical representation of complex process of radiant interaction, convection, internal heat generation, and conduction requires an integro-differential equation. The heat-transfer

* Received 14th November, 1964.

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mechanism is classified into the following;

1. Convective plate-fins
2. Radiative plate-fins to the gas
3. Convective and radiative plate-fins to gas
4. Radiative plate-fins to heat-sink surface
5. Radiative plate fins to heat-sink surface and gas

On the basis of the derived equation, the temperature distribution and efficiency are systematically analysed for a wide-range of variables of normalized parameter. Lastly, the relation between the cases when there is mutual irradiation and when there is not is formulated. An approximate equation of the efficiency in each one of the cases is introduced.

$$\frac{d^2T(x)}{dx^2} + \frac{q_i}{\lambda_m} = \frac{\alpha}{\lambda_m y_b} \{T(x) - T_g\} + \frac{\sigma e}{10^8 \lambda_m y_b} \left[\int_0^{x_f} \{T(x)^4 - T(x')^4\} \frac{d\phi(x, x')}{dx'} dx' \right. \\ \left. + \int_0^{x_f} \{T(x)^4 - T(\bar{x})^4\} \frac{d\phi(x, \bar{x})}{d\bar{x}} d\bar{x} + \phi_s(x) \{T(x)^4 - T_r^4\} \right. \\ \left. + \phi_c(x) \{T(x)^4 - T_c^4\} + \phi_g(x) \{T(x)^4 - T_g^4\} \right] \dots\dots\dots (1)$$

Equation has to satisfy the boundary condition

$$\left. \begin{array}{l} x=0 : T(0) = T_r \\ x=x_f : \frac{dT(x_f)}{dx} = 0 \end{array} \right\} \dots\dots\dots (2)$$

where

$x, x', \bar{x}$: distance from fin-root

$T(x)$: temperature of fin

T_g : temperature of gas

T_r : fin-root temperature

T_c : heat-sink temperature

q_i : internally generated heat per unit volume per unit time

e : emissivity of fin surface

σ : Stefan-Boltzmann's constant

α : surface heat-transfer coefficient

λ_m : thermal conductivity of fin

$\phi(x, x')$: overall configuration factor from the fin to its own fin

$\phi(x, \bar{x})$: overall configuration factor from the fin to the opposite-side fin

$\phi_s(x)$: overall configuration factor from the fin to the fin-root surface

$\phi_c(x)$: overall configuration factor from the fin

2. Basic equation

As shown in Fig. 1, plate-fins are simplified to a specified fin division containing the fin-root surface S , two fins $F, \bar{F}$, heat-sink surface C , and the gas $G^{(1)(2)}$.

Various quantities of the specified fin division are shown in Table 1, where

$2y_b$: thickness of fin

x_f : height of fin

p' : pitch of fin

l' : height of channel

From the heat balance on the infinitesimal segment (dF) at the distance from its root, the steady-state conduction equation is represented by

Fig. 1 Schematic diagram of specified fin

Table 1 Various quantities of the plate fin

Surface	Sign	Area	Emissivity	Number
Fin	$F(dF)$ (dF')	$x_f (\Delta x)$	e	$j = x_f / \Delta x$
Opposite side fin	$\bar{F}(d\bar{F})$	$x_f (\Delta x)$	e	$j = x_f / \Delta x$
Fin root	S	p'	e	1
Heat sink	C	p'	e_c	1
Reflector	R	$2(l' - x_f)$	0	2
Gas	G	—	e_g	—

Total $2j+4$

to the heat-sink surface

$\phi_g(x)$: overall configuration factor from the fin to the gas

Eqs. (1) and (2) are transformed into dimensionless forms

$$\left. \begin{aligned} X &= \frac{x}{x_f}, \quad X' = \frac{x'}{x_f}, \quad \bar{X} = \frac{\bar{x}}{x_f} : && \text{dimensionless length} \\ \theta(X) &= T(x)/T_r && : \text{dimensionless temperature} \\ \theta_g &= T_g/T_r && : \text{dimensionless gas temperature} \\ \theta_c &= T_c/T_r && : \text{dimensionless heat-sink temperature} \\ m &= \frac{\alpha x_f^2}{\lambda_m y_b} && : \text{dimensionless convective number} \\ l &= \frac{\sigma \epsilon x_f^2 T_r^3}{10^8 \lambda_m y_b} && : \text{dimensionless radiative number} \\ n &= \frac{q_i x_f^2}{\lambda_m T_r} && : \text{dimensionless number of internal heat generation} \\ \Phi(X, X'), \Phi(X, \bar{X}), \Phi_s(X), \Phi_c(X), \Phi_g(X) &: && \text{overall configuration factor expressed by} \\ &&& \text{dimensionless length } X, X' \text{ and } \bar{X} \end{aligned} \right\} \dots\dots\dots (3)$$

The dimensionless equation is represented by

$$\begin{aligned} \frac{d^2 \theta(X)}{dX^2} &= m \{ \theta(X) - \theta_g \} + l \left[\int_0^1 \{ \theta^4(X) - \theta^4(X') \} \frac{d\Phi(X, X')}{dX'} dX' \right. \\ &\quad + \int_0^1 \{ \theta^4(X) - \theta^4(\bar{X}) \} \frac{d\Phi(X, \bar{X})}{d\bar{X}} d\bar{X} + \Phi_s(X) \{ \theta^4(X) - 1 \} \\ &\quad \left. + \Phi_c(X) \{ \theta^4(X) - \theta_c^4 \} + \Phi_g(X) \{ \theta^4(X) - \theta_g^4 \} \right] - n \dots\dots\dots (4) \end{aligned}$$

The dimensionless boundary condition is represented by

$$X=0 : \theta(0)=1, \quad X=1 : \frac{d\theta(1)}{dX}=0 \dots\dots\dots (5)$$

Solving Eqs. (4) and (5), the temperature distribution is introduced. The fin efficiencies, heat-transfer ratio, and heat-transfer rate are represented by the definition. If

$$K = m(1 - \theta_g) + l \left[(1 - \theta_g^4) \int_0^1 \Phi_g(X) dX + (1 - \theta_c^4) \int_0^1 \Phi_c(X) dX \right] \dots\dots\dots (6)$$

Convection efficiency :

$$\eta_c = \frac{m}{K} \int_0^1 \{ \theta^4(X) - \theta_g \} dX \dots\dots\dots (7)$$

Radiation efficiency to fin-root surface :

$$\eta_{rs} = -\frac{l}{K} \int_0^1 \{ \theta^4(X) - 1 \} \Phi_s(X) dX \dots\dots\dots (8)$$

Radiation efficiency to heat-sink surface :

$$\eta_{rc} = \frac{l}{K} \int_0^1 \{ \theta^4(X) - \theta_c^4 \} \Phi_c(X) dX \dots\dots\dots (9)$$

Radiation efficiency to gas :

$$\eta_{rg} = \frac{l}{K} \int_0^1 \{ \theta^4(X) - \theta_g^4 \} \Phi_g(X) dX \dots\dots\dots (10)$$

Overall radiation efficiency :

$$\eta_r = -\eta_{rs} + \eta_{rc} + \eta_{rg} \dots\dots\dots (11)$$

Conduction efficiency :

$$\eta = \eta_r + \eta_c \dots\dots\dots (12)$$

Convection heat transfer ratio :

$$\epsilon_c = \frac{\eta_c}{\eta_c + \eta_{rc} + \eta_{rg}} \dots\dots\dots (13)$$

Radiation heat-transfer ratio to fin-root surface :

$$\epsilon_{rs} = \eta_{rs} / (\eta_c + \eta_{rg} + \eta_{rc}) \dots\dots\dots (14)$$

Radiation heat-transfer ratio to heat-sink surface :

$$\epsilon_{rc} = \eta_{rc} / (\eta_c + \eta_{rg} + \eta_{rc}) \dots\dots\dots (15)$$

Radiation heat-transfer ratio to gas :

$$\epsilon_{rg} = \eta_{rg} / (\eta_c + \eta_{rg} + \eta_{rc}) \dots\dots\dots (16)$$

Total radiation heat transfer ratio :

$$\epsilon_r = \eta_r / (\eta_c + \eta_{rg} + \eta_{rc}) \dots\dots\dots (17)$$

Conduction heat-transfer ratio :

$$\epsilon_{cond} = \eta / (\eta_c + \eta_{rg} + \eta_{rc}) \dots\dots\dots (18)$$

Heat transfer rate :

$$Q = \frac{\lambda_m y_b T_g K \eta}{x_f}$$

η shows the rate of the conduction heat in the fin.

3. Angle factor

In a finned enclosure shown in Fig. 1, an angle factor viewing the surface is expressed by D'Lambert's law. Angle factors among the surfaces dF , dF' , $d\bar{F}$, S , C , R , and G are introduced.

If $p = p'/x_f$, $L = l'/x_f$, then

(a) Infinitesimal surface dF

Angle factor viewing the infinitesimal surface

of opposite side fin $\frac{dF_1(X, \bar{X})}{d\bar{X}}$:

$$\frac{dF_1(X, \bar{X})}{d\bar{X}} = \frac{1}{2} \frac{p^2}{[(X - \bar{X})^2 + p^2]^{3/2}} \dots\dots\dots (19)$$

Angle factor viewing the infinitesimal surface of its own fin $F(X, X')$:

$$F(X, X') = 0$$

Angle factor viewing the fin-root surface $F_2(X)$:

$$F_2(X) = \frac{1}{2} \left(1 - \frac{X}{\sqrt{X^2 + p^2}} \right) \quad \dots\dots\dots (20)$$

Angle factor viewing the heat-sink surface $F_3(X)$:

$$F_3(X) = \frac{1}{2} \left(1 - \frac{L-X}{\sqrt{(L-X)^2 + p^2}} \right) \quad \dots\dots\dots (21)$$

Angle factor viewing the reflector surface $F_4(X)$:

$$F_4(X) = \frac{1}{2} \left(\frac{L-X}{\sqrt{(L-X)^2 + p^2}} - \frac{1-X}{\sqrt{(1-X)^2 + p^2}} \right) \quad \dots\dots\dots (22)$$

Thus, there is obtained,

$$\int_0^1 \frac{dF_1(X, \bar{X})}{d\bar{X}} d\bar{X} + F_1(X) + F_2(X) + F_3(X) = 1$$

(b) Fin-root surface S

Angle factor viewing the infinitesimal surface of the fin $\frac{dF_5(X)}{dX}$:

$$\frac{dF_5(X)}{dX} = \frac{1}{2p} \left(1 - \frac{X}{\sqrt{X^2 + p^2}} \right) \quad \dots\dots\dots (23)$$

Angle factor viewing the heat-sink surface F_6 :

$$F_6 = \frac{1}{p} (\sqrt{L^2 + p^2} - L) \quad \dots\dots\dots (24)$$

Angle factor viewing the reflector F_7 :

$$F_7 = \frac{1}{p} (\sqrt{1 + p^2} - 1 + L - \sqrt{L^2 + p^2}) \quad \dots\dots\dots (25)$$

Thus, there is obtained,

$$2 \int_0^1 \frac{dF_5(X)}{dX} dX + F_6 + F_7 = 1$$

(c) Heat-sink surface

Angle factor viewing the infinitesimal surface of the fin $\frac{dF_8(X)}{dX}$:

$$\frac{dF_8(X)}{dX} = \frac{1}{2p} \left(1 - \frac{L-X}{\sqrt{(L-X)^2 + p^2}} \right) \quad \dots\dots\dots (26)$$

Angle factor viewing the fin-root surface F_9 :

$$F_9 = \frac{1}{p} (\sqrt{L^2 + p^2} - L) \quad \dots\dots\dots (27)$$

Angle factor viewing the reflector surface F_{10} :

$$F_{10} = \frac{1}{p} \{ L - 1 + p - \sqrt{(L-1)^2 + p^2} \} \quad \dots\dots\dots (28)$$

Thus, there is obtained,

$$2 \int_0^1 \frac{dF_8(X)}{dX} dX + F_9 + F_{10} = 1$$

(d) Reflector surface R

Angle factor viewing the infinitesimal surface of the fin $\frac{dF_{11}(X)}{dX}$:

$$\frac{dF_{11}(X)}{dX} = \frac{1}{2(L-1)} \left\{ \frac{L-X}{\sqrt{(L-X)^2 + p^2}} - \frac{1-X}{\sqrt{(1-X)^2 + p^2}} \right\} \quad \dots\dots\dots (29)$$

Angle factor viewing the fin-root surface F_{12} :

$$F_{12} = \frac{1}{2(L-1)} (\sqrt{1 + p^2} - \sqrt{L^2 + p^2} - 1 + L) \quad \dots\dots\dots (30)$$

Angle factor viewing the heat-sink surface F_{13} :

$$F_{13} = \frac{1}{2(L-1)} \{ L - 1 + p - \sqrt{(L-1)^2 + p^2} \} \quad \dots\dots\dots (31)$$

Angle factor viewing the reflector surface F_{14} :

$$F_{14} = \frac{1}{L-1} \{ \sqrt{(L-1)^2 + p^2} - p \} \quad \dots\dots\dots (32)$$

Thus, there is obtained,

$$\int_0^1 \frac{dF_{11}(X)}{dX} dX + F_{12} + F_{13} + F_{14} = 0$$

4. Determination of the energy rate $U_{ij}^{(1)}$ and the overall configuration factor ϕ_{ij}

The overall configuration factor ϕ_{ij} is solved by the angle factors $\frac{dF_1(X, \bar{X})}{d\bar{X}}$, $F_2(X)$, ..., $F_{14}(X)$.

A numerical iterative method for determining the configuration factor is used.

For the case when the finned enclosure contains 1, 2, 3, ..., n surfaces and the gas, the proportion of the energy rate from i surface to j surface directly and indirectly through the reflector and the gas is given by

$$U_{ij}^{(1)} = (1 - e_g) \left(F_{ij} + \frac{F_{ij} F_{Rj}}{\frac{1}{1 - e_g} - F_{RR}} \right) \quad \dots\dots\dots (33)$$

The absorbing rate in the gas is given by

$$U_{iG}^{(1)} = e_g \left(1 + \frac{F_{iR}}{\frac{1}{1 - e_g} - F_{RR}} \right) \quad \dots\dots\dots (34)$$

where

$$\sum U_{ij}^{(1)} + U_{iG}^{(1)} = 1$$

The proportion of the reaching and absorbing rate is given by the use of Eqs. (33) and (34).

(a) When the unit energy rate is emitted from the dF surface, the proportion reaching into the dF' surface on its own fin $\frac{dU_1^{(1)}(X, X')}{dX'}$:

$$\frac{dU_1^{(1)}(X, X')}{dX'} = (1 - e_g) \frac{F_4(X) dF_{11}(X')/dX'}{\frac{1}{1 - e_g} - F_{14}} \quad \dots\dots\dots (35)$$

The proportion reaching into the dF̄ surface on opposite-side fin $\frac{dU_2^{(1)}(X, \bar{X})}{d\bar{X}}$:

$$\frac{dU_2^{(1)}(X, \bar{X})}{d\bar{X}} = (1-e_g) \left\{ \frac{dF_1(X, \bar{X})}{dX} + \frac{F_4(X)dF_{11}(\bar{X})/d\bar{X}}{\frac{1}{1-e_g} - F_{14}} \right\} \dots\dots\dots (36)$$

The proportion reaching into the fin-root surface $U_3^{(1)}(X)$:

$$U_3^{(1)}(X) = (1-e_g) \left\{ F_2(X) + \frac{F_4(X)F_{12}}{\frac{1}{1-e_g} - F_{14}} \right\} \dots\dots\dots (37)$$

The proportion reaching into the heat-sink surface $U_4^{(1)}(X)$:

$$U_4^{(1)}(X) = (1-e_g) \left\{ F_3(X) + \frac{F_4(X)F_{13}}{\frac{1}{1-e_g} - F_{14}} \right\} \dots\dots\dots (38)$$

The proportion absorbed by the gas $U_5^{(1)}(X)$:

$$U_5^{(1)}(X) = e_g \left(1 + \frac{F_4(X)}{\frac{1}{1-e_g} - F_{14}} \right) \dots\dots\dots (39)$$

(b) When the unit energy rate is emitted from the fin-root surface, the proportion reaching into the dF surface on the fin $\frac{dU_6^{(1)}(X)}{dX}$:

$$\frac{dU_6^{(1)}(X)}{dX} = (1-e_g) \left\{ dF_5(X)/dX + \frac{F_7dF_{11}(X)/dX}{\frac{1}{1-e_g} - F_{14}} \right\} \dots\dots\dots (40)$$

The proportion reaching into the fin-root surface $U_7^{(1)}$:

$$U_7^{(1)} = (1-e_g) \frac{F_7F_{12}}{\frac{1}{1-e_g} - F_{14}} \dots\dots\dots (41)$$

The proportion reaching into the heat-sink surface $U_8^{(1)}$:

$$U_8^{(1)} = (1-e_g) \left\{ F_6 + \frac{F_7F_{13}}{\frac{1}{1-e_g} - F_{14}} \right\} \dots\dots\dots (42)$$

The proportion absorbed by the gas $U_9^{(1)}$:

$$U_9^{(1)} = e_g \left(1 + \frac{F_7}{\frac{1}{1-e_g} - F_{14}} \right) \dots\dots\dots (43)$$

(c) When the unit energy rate is emitted from the heat-sink surface, the proportion reaching into the dF surface on the fin $\frac{dU_{10}^{(1)}(X)}{dX}$:

$$\frac{dU_{10}^{(1)}(X)}{dX} = (1-e_g) \left\{ \frac{dF_8(X)}{dX} + \frac{F_{10}dF_{11}(X)/dX}{\frac{1}{1-e_g} - F_{14}} \right\} \dots\dots\dots (44)$$

The proportion reaching into the fin-root surface $U_{11}^{(1)}$:

$$U_{11}^{(1)} = (1-e_g) \left(F_9 + \frac{F_{10}F_{12}}{\frac{1}{1-e_g} - F_{14}} \right) \dots\dots\dots (45)$$

The proportion reaching into the heat-sink surface $U_{12}^{(1)}$:

$$U_{12}^{(1)} = (1-e_g) \left(\frac{F_{10}F_{13}}{\frac{1}{1-e_g} - F_{14}} \right) \dots\dots\dots (46)$$

The proportion absorbed by the gas $U_{13}^{(1)}$:

$$U_{13}^{(1)} = e_g \left(1 + \frac{F_{10}}{\frac{1}{1-e_g} - F_{14}} \right) \dots\dots\dots (47)$$

The proportion of the energy rate transferred to 1, 2, 3..., i..., n is represented by $dU_1^{(1)}(X, \bar{X})/d\bar{X}$, ...,

$U_{13}^{(1)}$. And the surfaces reflect and absorb the energy. Thus, the secondary transferred energy rate is represented by $dU^{(2)}(X, \bar{X})/d\bar{X}$, ..., $U_{13}^{(2)}$. In general, the proportion of k -th energy rate $dU^{(k)}(X, \bar{X})/d\bar{X}$, ..., $U_{13}^{(k)}$ is represented by the proportion of $k-1$ -th energy rate $dU^{(k-1)}(X, \bar{X})/d\bar{X}$, ..., $U_{13}^{(k-1)}$.

$$\begin{aligned} & \begin{pmatrix} dU_1^{(k)}(X, X')/dX' \\ dU_2^{(k)}(X, \bar{X})/d\bar{X} \\ U_3^{(k)}(X) \\ U_4^{(k)}(X) \\ U_5^{(k)}(X) \end{pmatrix} = (1-e) \int_0^1 dU_1^{(k-1)}(X, X'')/dX'' \begin{pmatrix} dU_1^{(1)}(X'', X')/dX' \\ dU_2^{(1)}(X'', \bar{X})/d\bar{X} \\ U_3^{(1)}(X'') \\ U_4^{(1)}(X'') \\ U_5^{(1)}(X'') \end{pmatrix} dX'' \\ & + (1-e) \int_0^1 dU_1^{(k-1)}(X, \bar{X}')/d\bar{X}' \begin{pmatrix} dU_2^{(1)}(\bar{X}', X')/dX' \\ dU_1^{(1)}(\bar{X}', \bar{X})/d\bar{X}' \\ U_3^{(1)}(\bar{X}') \\ U_4^{(1)}(\bar{X}') \\ U_5^{(1)}(\bar{X}') \end{pmatrix} \\ & + (1-e)U_3^{(k-1)}(X) \begin{pmatrix} dU_6^{(1)}(X')/dX' \\ dU_6^{(1)}(\bar{X})/d\bar{X} \\ U_7^{(1)} \\ U_8^{(1)} \\ U_9^{(1)} \end{pmatrix} + (1-e_c)U_4^{(k-1)}(X) \begin{pmatrix} dU_{10}^{(1)}(X')/dX' \\ dU_{10}^{(1)}(\bar{X})/d\bar{X} \\ U_{11}^{(1)} \\ U_{12}^{(1)} \\ U_{13}^{(1)} \end{pmatrix} \dots\dots\dots(48) \end{aligned}$$

The overall configuration factor of the finned enclosure is represented by

$$\left. \begin{aligned} \frac{d\Phi(X, X')}{dX'} &= e \sum_{k=1}^k \frac{dU_1^{(k)}(X, X')}{dX'}, & \frac{d\Phi(X, \bar{X})}{d\bar{X}} &= e \sum_{k=1}^k \frac{dU_2^{(k)}(X, \bar{X})}{d\bar{X}}, \\ \Phi_s(X) &= e \sum_{k=1}^k U_3^{(k)}(X), & \Phi_c(X) &= e_c \sum_{k=1}^k U_4^{(k)}(X), & \Phi_g(X) &= \sum_{k=1}^k U_5^{(k)}(X) \end{aligned} \right\} \dots\dots\dots(49)$$

Substituting the configuration factors into Eqs. (4) and (5), the temperature can be calculated for various values of dimensionless parameters.

5. Heat-transfer mechanism of plate-fins

The heat-transfer mechanism of the plate-fins is classified into the following 5 cases:

- (1) The net convective fins with mutual irradiation ($e_g = e_c = \theta_c = 0$)
- (2) The gas radiative fins with mutual irradiation ($m = e_c = \theta_c = 0$)
- (3) The convective and radiative fins to the gas and with mutual irradiation ($e_c = \theta_c = 0$)
- (4) The radiative fins to the heat-sink and with mutual irradiation ($m = e_g = \theta_g = 0$)
- (5) The radiative fins to the heat-sink and the gas and with mutual irradiation ($m = 0$)

6. The net convective plate-fins with mutual irradiation (Case 1)

Fig. 2 shows a finned division of convective fins with mutual irradiation. An enclosure of the finned division contains the finned surface $F, \bar{F}$, the fin-root surface S , and the reflector R . The enclosure is filled with transparent gas. For the case when convection energy is transferred from the surface to the gas, the temperature and efficiency are introduced. The convective plate-fin is being

used to increase the heat-transfer surface of fuel-elements of gas-cooled reactors and heat exchanger, especially at high temperature. In this case, Eq. (4) is simplified to

$$\begin{aligned} \frac{d^2\theta(X)}{dX^2} &= l \int_0^1 \{\theta^4(X) - \theta^4(X')\} \frac{d\Phi(X, X')}{dX'} dX' \\ &+ l \int_0^1 \{\theta^4(X) - \theta^4(\bar{X})\} \frac{d\Phi(X, \bar{X})}{d\bar{X}} d\bar{X} \\ &+ l\Phi_s(X) \{\theta^4(X) - 1\} + m \{\theta(X) - \theta_g\} \dots\dots\dots(50) \end{aligned}$$

Solving Eqs. (50) and (5), the temperature distribution $\theta(X)$ is introduced. The fin efficiency Φ_c , Φ_{rs} , heat-transfer ratio ε_{rs} , ε_{cond} , and heat-transfer rate are given by Eqs. (7), (8), (13), (14) and (18), when K is

$$K = m(1 - \theta_g) \dots\dots\dots(51)$$

The efficiency η_c, η_{rs} and heat-transfer rate

Fig. 2 Specified fin space

are obtained for various values of l, m, p, θ_g , and L .

Figs. 3 and 4 show distribution of the overall configuration factor $\frac{d\phi(X, X')}{dX'}$, $\frac{d\phi(X, \bar{X})}{d\bar{X}}$, $\phi_s(X)$ at $p=0.2$, $e=0.8$, and $L=2.0$.

The mountainous curve of $\frac{d\phi(X, X')}{dX'}$ becomes maximum at $X=X'$. And $\frac{d\phi(X, X')}{dX'}$ near the fin-tip ($X \rightarrow 1$) becomes large because of reflection of the radiation ray. On the other hand, the curve of $\frac{d\phi(X, \bar{X})}{d\bar{X}}$ is same mountainous curve as $\frac{d\phi(X, X')}{dX'}$, and becomes maximum at $X=\bar{X}$. $\frac{d\phi(X, \bar{X})}{d\bar{X}}$ is 8 times large than $\frac{d\phi(X, X')}{dX'}$.

Fig. 5 shows the relation between the fin-pitch

ratio p and the overall configuration factors $\phi_F'(p)$, $\phi_F(p)$ and $\phi_s(p)$. As shown in this Figure, the configuration factor coming back to its own fin $\phi_F' = \int_0^1 \int_0^1 \frac{d\phi(X, X')}{dX'} dX' dX$ has a constant value of 0.15, and is independent of the pitch ratio. And the configuration factor going into the opposite side of the fin $\phi_F = \int_0^1 \int_0^1 \frac{d\phi(X, \bar{X})}{d\bar{X}} d\bar{X} dX$ becomes suddenly smaller with increasing value of the pitch ratio p . The configuration factor $\phi_s = \int_0^1 \phi_s(X) dX$ becomes suddenly larger with increasing value of p .

When the emissivity of the fin surface e is smaller, the curves of $\frac{d\phi(X, X')}{dX'}$, $\frac{d\phi(X, \bar{X})}{d\bar{X}}$, $\phi_s(X)$

Fig. 3 Distribution of overall configuration factor $X' \sim \frac{d\phi(X, X')}{dX'}$

Fig. 5 Distribution of total configuration factor

Fig. 4 Distribution of total configuration factor $\bar{X} \sim \frac{d\phi(X, \bar{X})}{d\bar{X}}$, $\phi_s(X)$

Fig. 6 Temperature distribution of convective plate-fins

become slightly flat, and are independent of the channel height ratio L . These configuration factors are introduced into Eq. (2).

Fig. 6 shows the temperature distribution with the dimensionless radiative number as a parameter, when $m=1$, $p=0.2$, $e=0.8$, $\theta_g=0.5$, and $L=2.0$. The curve of the convective fin without mutual irradiation is given by the dotted line ($l=0$).

Gardner⁽⁸⁾ introduced the rigorous solution of

Fig. 7 Fin efficiency of convective plate-fins η_c

Fig. 8 Fin efficiency of convective plate-fins

Fig. 9 Heat-transfer ratio of convective plate-fins

the net convective fin represented by $\theta(X)=\theta_g + (1-\theta_g) \frac{\cosh \sqrt{m}(1-X)}{\sqrt{m}}$. The greater l is, the more flat the temperature becomes, because the mutual radiation interference becomes active. When the radiation number l becomes infinite, the temperature distribution becomes flat [$\theta(X)=1$].

Fig. 7 shows the relation between the efficiency η_c and the dimensionless radiative number l with the dimensionless convective number and p as a parameter, when $e=0.8$, $\theta_g=0.5$, and $L=2.0$.

The continuous lines show the efficiency for the value of $m=1$. And dotted lines show the efficiency for the value of $m=2$.

When l is zero, the efficiency is that of a convective fin and is represented by $\eta_{c0} = \tanh \sqrt{m} / \sqrt{m}$ ⁽⁸⁾. The greater l is, the greater the efficiency becomes. When l becomes infinite, the efficiency approaches the constant line ($\eta_c=1$). In this Figure, the efficiency becomes greater for a increasing value of the pitch ratio p , because of the increase in $\phi_s(X)$ and the decrease of $\frac{d\phi(X, \bar{X})}{d\bar{X}}$. The efficiency is to be larger with an increasing value of l and decreasing values of m .

Fig. 8 shows the relation between the convective efficiency and the dimensionless convective number with l as a parameter.

The dotted lines show the efficiency of a convective plate-fin without mutual irradiation $\eta_{c0} = \tanh \sqrt{m} / \sqrt{m}$ ⁽⁸⁾. The greater the l is, the greater the efficiency

becomes. And the efficiency approaches the constant line ($\eta_c=1$). The efficiency is slightly increasing for an increasing value of θ_g . But η_c does not depend upon the channel height L .

Fig. 9 shows the relation between the dimensionless radiative number l and the heat-transfer ratio ϵ_{rs} and ϵ_{cond} with fin-pitch ratio p as a parameter. Total heat-transfer rate Q_t is the sum of the conduction rate through the fin and the radiation rate directly transferred from the fin-root to the fin, and is represented by

$$Q_t = Q_{rs} + Q_{cond} \quad (52)$$

where

Q_t : total heat-transfer rate transferred to the

gas

Q_{cond} : conduction heat rate through the fin

Q_{rs} : radiation heat rate directly transferred from the fin-root to the fin surface

Eq. (52) is represented by the heat-transfer ratio.

$$\epsilon_{\text{cond}} = -\epsilon_{rs} + \epsilon_c$$

where

$$\epsilon_{rs} > \epsilon_c = 1$$

Thus, there is obtained

$$1 = \epsilon_{\text{cond}} + \epsilon_{rs}$$

As shown in Fig. 9, the heat-transfer ratio ϵ_{rs} becomes larger with an increasing value of the fin-pitch ratio p , because of the increase in $\Phi_s(X)$.

For example, in the case when $p=1.0$, $m=1.0$ and $l=8$, the heat-transfer ratios are to be $\epsilon_{\text{cond}}=0.14$, and $\epsilon_{rs}=0.86$. These show that 14% of the total heat-transfer rate is due to conduction, and 86% is due to radiation from the fin root.

When p is in the order of 1.0 or more and l is sufficiently greater than m , the radiation from the fin-root is larger than the conduction through the fin. The approximate solution for the efficiency is

$$\eta_c = \frac{\tanh \sqrt{m}}{\sqrt{m}} + 0.04 \left(1 - \frac{\tanh \sqrt{m}}{\sqrt{m}} \right) \times (1 - 0.01e)(1 + \sqrt{p+0.2})l \quad (53)$$

This equation is useful to calculate the convective efficiency for various values of m , l , p , e and L . The accuracy of this equation is within 3%, when $0 < m < 8$, $0 < l < 10$, $0 < p < 2.0$, and $0 < e < 1$.

7. The gas-radiative plate-fins with mutual irradiation ($m = e_c = \theta_c = 0$)

When the plate-fin emits radiation energy to

Fig. 10 Temperature distribution of radiative plate-fins to gas

the gas with mutual irradiation, the temperature and efficiency are introduced.

The radiative fins to the gas are used to increase the heat transfer of chemical reactors and combustors. Eq. (4) is simplified to

$$\begin{aligned} \frac{d^2 \theta(X)}{dX^2} = & l \int_0^1 \{ \theta^4(X) - \theta^4(X') \} \frac{d\Phi(X, X')}{dX'} dX' \\ & + l \int_0^1 \{ \theta^4(X) - \theta^4(\bar{X}) \} \frac{d\Phi(X, \bar{X})}{d\bar{X}} d\bar{X} \\ & + l \Phi_s(X) \{ \theta^4(X) - 1 \} + l \Phi_g(X) \{ \theta^4(X) - \theta_g^4 \} \end{aligned} \quad (54)$$

Solving Eqs. (54) and (5), the temperature is introduced. The efficiencies and heat-transfer ratio are given by Eqs. (8), (10), (14) and (18), when K is

$$K = l(1 - \theta_g^4) \int_0^1 \Phi_g(X) dX \quad (55)$$

The efficiencies η_{rg} , η and heat-transfer ratio are obtained for various values of l , p , θ_g , θ_s , and L .

Fig. 10 shows the temperature distribution with dimensionless radiative number as a parameter, when $p=1.0$, $e=0.8$, $e_g=0.4$, $\theta_g=0.5$, and $L=2.0$. The dotted line is the temperature distribution of a plate-fin⁽¹⁾ solved by

$$\frac{d^2 \theta(X)}{dX^2} = l \{ \theta^4(X) - \theta_g^4 \} \quad (56)$$

The temperature along the fin becomes flat with an increasing value of l . The temperature of plate-fin is always larger than the temperature of a plate-fin, because radiative energy with mutual irradiation is uniformly distributed over the fin surface.

Fig. 11 shows the relation between the efficiency and the dimensionless radiative number l with fin-pitch ratio as a parameter.

The radiative efficiency to the gas η_{rg} is represented by a straight line, and the conduction efficiency η is represented by a dotted line. The

Fig. 11 Fin efficiency of radiative plate-fins to gas

efficiency of a single sheet of a gas-radiative plate-fin η_{rg} is represented by a one-dotted line. The radiative efficiency to the gas η_{rg} becomes larger and the conduction efficiency η becomes smaller with an increasing value of p , because the configuration factor $\phi_s(X)$ becomes larger with an increase in p . On the other hand, the efficiencies η and η_{rg} decrease for an increasing value of l . When the dimensionless radiative number l becomes larger, the radiation heat from the fin root is to increase and the conduction heat through the fin is to decrease.

Fig. 12 shows the relation between the emissivity e and the efficiencies η and η_{rg} are to decrease with an increase in θ_g and e_g .

Fig. 13 shows the relation between the efficiencies η, η_{rg} and the gas emissivity e_g with l as a parameter.

The efficiencies η and η_{rg} are to decrease with an increasing value of e_g .

An approximate equation of the efficiency to the gas is introduced.

$$\eta_{rg} = 1 - \frac{\sqrt{l}}{1 + 0.925\sqrt{l}} (1 - 0.01\theta_g) \times (1 - 0.12p - 0.15p^{0.1}) \times (0.63 + 0.37e_g)(1 - 0.24e) \dots (57)$$

This equation is useful to calculate the efficiency for various values of l, p, e, e_g, θ_g , and L . An accuracy of the equation is in the order of 5%, when $0 < l < 10, 0 < p < 2, 0 < e, e_g < 1, 0 < \theta_g < 1$.

8. The convective and radiative plate-fins to the gas with mutual irradiation ($e_c = \theta_c = 0$)

For the case when the plate-fins emit radiative and convective energy to the gas, the temperature and efficiency are introduced. The solution of a

Fig. 12 Fin efficiency of radiative plate-fins to gas $\eta, \eta_{rg} \sim e$

radiative and convective plate-fin is already given by Okamoto⁽²⁾. The radiative and convective plate-fins to the gas are being used to increase the heat-transfer of boiler superheaters and fuel-elements of gas-cooled reactors. The basic equation is represented by

$$\frac{d^2\theta(X)}{dX^2} = l \int_0^1 \{\theta^4(X') - \theta^4(X'')\} \frac{d\phi(X, X')}{dX'} dX' + l \int_0^1 \{\theta^4(X) - \theta^4(X'')\} \frac{d\phi(X, X'')}{dX''} dX'' + l\phi_s(X) \{\theta^4(X) - 1\} + l\phi_g(X) \{\theta^4(X) - \theta_g^4\} + m\{\theta(X) - \theta_g\} \dots (58)$$

Solving Eqs. (58) and (5), the temperature $\theta(X)$ is introduced for various of $l, m, e, e_g, \theta_g, p$, and L .

The efficiencies $\eta_c, \eta_{rg}, \eta_{rs}$ and η , heat-transfer ratio $\varepsilon_c, \varepsilon_{rg}, \varepsilon_{rs}$, and ε_{cond} are given by Eqs. (7), (8), (10), (12), (13), (14), (16), (17), and (18), when K is

$$K = m(1 - \theta_g) + l(1 - \theta_g^4) \int_0^1 \phi_g(X) dX \dots (59)$$

Fig. 14 shows the temperature distribution with dimensionless radiative number l as a parameter. When l is zero, the temperature of a net convective plate-fin⁽²⁾ is represented by

$$\frac{\theta(X) - \theta_g}{1 - \theta_g} = \frac{\cosh \sqrt{m}(1 - X)}{\cosh \sqrt{m}} \dots (60)$$

On the other hand, the author solved the conductive equation of a radiative and convective fin to the gas represented by

$$\frac{d^2\theta(X)}{dX^2} = l\{\theta^4(X) - \theta_g^4\} + m\{\theta(X) - \theta_g\} \dots (61)$$

Solving Eqs. (61) and (5), the temperature of a single sheet of the radiative and convective plate-fin is represented by dotted lines with l as

Fig. 13 Efficiency of radiative plate-fins to gas $\eta, \eta_{rg} \sim e_g$

a parameter. The temperature of the fins is always larger than the temperature of a single sheet of the fin at the same value of l .

Figs. 15 and 16 show the relation between the dimensionless radiative number l and the efficien-

cies η_{rg} and η_{rc} with pitch ratio p as a parameter. When l is to be zero, the plate-fins become a single sheet of a convective fin.

When the dimensionless convective number m is to be constant, the sum of the convective

$m=1.0, p=0.2, e=0.7, e_g=0.8, \theta_g=0.5, L=2$

Fig.14 Temperature distribution of radiative and convective plate-fins to gas

$m=1.0, e=0.7, e_g=0.8, \theta_g=0.5, L=2.0$

Fig.15 Fin efficiency of radiative and convective plate-fins to gas $l \sim \eta_c, \eta_{rg}$

$m=4.0, e=0.7, e_g=0.8, \theta_g=0.5, L=4.0$

Fig.16 Fin efficiency of radiative and convective plate-fins to gas

$m=1, e=0.8, \theta_g=0.5, L=2.0$

Fig.17 Heat-transfer rate of radiative and convective plate-fins to gas $l \sim \epsilon_{rc}, \epsilon_{rg}, \epsilon_{cond}$

Fig.18 Schematic diagram of specified fin

efficiency η_c plus the gas-radiative efficiency η_{rg} becomes smaller with an increasing value of l and a decreasing value of p . When p is to be zero, integration terms of Eq. (58) can be neglected. And the efficiency at $p=0$ is equal to the efficiency of a single sheet of the plate-fins $\eta_{rg} + \eta_{rc}$ is always larger than the efficiency of a single sheet of the fin.

Fig. 17 shows the relation between the heat-transfer ratios $\varepsilon_c, \varepsilon_{rg}, \varepsilon_{rs}$ and ε_{cond} with p as a parameter. When l becomes larger and p becomes smaller, ε_c become smaller, and ε_{rg} become larger.

9. The radiative plate-fins to the heat-sink with mutual irradiation ($m=e_g=\theta_g=0$)

For the case when the plate-fins emit radiative energy to the heat-sink surface, the temperature and efficiency are solved. The plate-fins are used

to dissipate heat from radiator surfaces of space-vehicles to cosmic space.

Chamber⁽⁶⁾ and Bartas⁽⁷⁾ analysed a single sheet of a fin not referring to the radiation from the fin root and the mutual irradiation. Sparrow⁽⁵⁾ dealt with the case of black-body fins with mutual irradiation. In this report, this problem is taken up for further analysis of the gray-body fins. Fig. 18 shows the specified division of the fins.

The fin surface $F, \bar{F}$ and the fin-root surface transfer heat to the heat-sink surface. The basic equation is represented by

$$\begin{aligned} \frac{d^2\theta(X)}{dX^2} = & l \int_0^1 \{\theta^4(X) - \theta^4(X')\} \frac{d\phi(X, X')}{dX'} dX' \\ & + l \int_0^1 \{\theta^4(X) - \theta^4(\bar{X})\} \frac{d\phi(X, \bar{X})}{d\bar{X}} d\bar{X} \\ & + l\phi_s(X) \{\theta^4(X) - 1\} + l\phi_c(X) \{\theta^4(X) - \theta_c^4\} \end{aligned} \quad \dots\dots\dots (62)$$

Solving Eqs. (62) and (5), the temperature is introduced. The efficiencies $\eta_{rc}, \eta_{rs}, \eta$, heat-transfer

$p=0.2, e=0.8, \varepsilon_c=1.0, \theta_c=0, L=5.0$

Fig.19 Temperature distribution of radiative plate-fins to heat-sink $X \sim \theta(X)$

$p=0.5, e=0.8, \varepsilon_c=1.0, \theta_c=0, L=5.0$

Fig.20 Temperature distribution of radiative plate-fins to heat-sink $X \sim \theta(X)$

$e=0.8, \varepsilon_c=1.0, \theta_c=0, L=5.0$

Fig.21 Fin efficiency of radiative plate-fins to heat-sink

$p=0.2, \varepsilon_c=1.0, \theta_c=0, L=10$

Fig.22 Fin efficiency of radiative plate-fins to heat-sink $l \sim \eta_{rc}, \eta_{rs}, \eta$

ratios $\varepsilon_{rc}, \varepsilon_{rs}, \varepsilon_{cond}$, and heat-transfer rate Q are represented by Eqs. (8), (9), (12), (15), (17) and (18), when K is

$$K = l(1 - \theta_c^4) \int_0^1 \phi_c(X) dX \dots\dots\dots (63)$$

Fig.23 Fin efficiency of radiative plate-fins to heat-sink $l \sim \eta_{rc}, \eta_{rs}, \eta$

Fig.24 Temperature distribution of radiative plate-fins to gas and heat-sink

(a) Fin efficiency of radiative plate-fins to heat-sink and gas

(b) Heat-transfer ratio

Fig. 25

to the tip becomes.

Fig. 21 shows the relation between l and the efficiency with pitch ratio as a parameter. The solid lines show the radiative efficiency to the heat sink η_{rc} and dotted lines show the conduction efficiency η . Bartas⁽⁷⁾ solved the efficiency of a single sheet of the plate-fin represented by the equation

$$\frac{d^2\theta(X)}{dX^2} = l\theta^4(X) \dots\dots\dots (64)$$

The larger p is, the larger the efficiency η_{rs} becomes and the smaller η becomes. Because the configuration factors $\Phi_c(X)$ and $\Phi_s(X)$ become larger with an increasing value of p .

Fig. 22 shows the relation between the efficiencies η_{rc} , η_{rs} , η and dimensionless radiative number l with emissivity e as a parameter. The efficiencies η_{rc} and η of the black-body surface are largest, and η_{rc} and η become smaller with a decreasing value of e .

$$p=0.2, e_g=0.4, e_c=1.0, \theta_g=0.8, \theta_c=0.6, L=2.0$$

(a) Fin efficiency of radiative plate-fins to heat-sink and gas

(b) Heat-transfer ratio

Fig. 26

Fig. 23 shows the relation between the efficiency and l with channel-height ratio L as a parameter. The efficiencies η_{rc} and η become greater with an increasing value of L . When L is larger than 10, the efficiency approaches asymptotically the value at $L=\infty$.

An approximate equation of the radiative plate-fins to the heat-sink is introduced, when $e_c=1, \theta_c=0$.

$$\begin{aligned} \eta_{rc} = & 1 - 0.876\sqrt{l}(1 + 1.43\sqrt{p}) \\ & \times \{1 + 0.6(1 - 0.1\sqrt{L})^{0.8}\} \\ & \times (1 - 0.1\sqrt{l})(1 - 0.187e) \dots\dots\dots (65) \end{aligned}$$

The equation is useful to calculate the efficiency for various values of l, p, e , and L . An accuracy of the equation is in the order of 6%, when $0 < l < 10, 0 < p < 3, 1 < L < 20, 0 < e < 1$.

10. The radiative plate-fins to the gas and the heat-sink

For the case when the plate-fins emit radiative energy to the gas and heat-sink, the temperature and efficiency are solved. In this case, the radiative plate-fins are used to increase the heat-transfer of combustors and reactors. The basic equation is represented by

$$\begin{aligned} \frac{d^2\theta(X)}{dX^2} = & l \int_0^1 \{\theta^4(X) - \theta^4(X')\} \\ & \times \frac{d\Phi(X, X')}{dX'} dX' \\ & + l \int_0^1 \{\theta^4(X) - \theta^4(\bar{X})\} \frac{d\Phi(X, \bar{X})}{d\bar{X}} d\bar{X} \\ & + l\Phi_s(X) \{\theta^4(X) - 1\} + l\Phi_g(X) \\ & \times \{\theta^4(X) - \theta_g^4\} + l\Phi_c(X) \\ & \times \{\theta^4(X) - \theta_c^4\} \dots\dots\dots (66) \end{aligned}$$

Solving Eqs. (66) and (5), efficiencies η_{rg} , η_{rc} , η_{rs} , η , and heat-transfer ratios ϵ_{rg} , ϵ_{rc} , ϵ_{rs} , ϵ_{cond} are represented by Eqs. (8), (9), (10), (12), (14), (15), (16) and (18), when K is

$$\begin{aligned} K = & l \left\{ (1 - \theta_c^4) \int_0^1 \Phi_c(X) dX \right. \\ & \left. + (1 - \theta_g^4) \int_0^1 \Phi_g(X) dX \right\} \dots\dots\dots (67) \end{aligned}$$

These are obtained for various values of $l, e, e_g, e_c, \theta_g, \theta_c, p$ and L .

Fig. 24 shows the temperature distribution with dimensionless radiative number l as a parameter. The greater l is, the more steep

Fig. 27

(a) Fin efficiency of radiative plate-fins to heat-sink and gas

(b) Heat-transfer ratio

Fig. 28

the temperature gradient is.

Fig. 25(a) shows the relation between the efficiency η_{rg} , η_{rc} and l as a parameter. When l increases, η_{rg} , η_{rc} comes to decrease gradually. η_{rg} is to decrease and η_{rc} is to increase with an

increasing value of p , because the total configuration factor $\Phi_s(X)$, $\Phi_c(X)$ becomes larger and $\Phi_g(X)$ becomes smaller with an increasing value of p .

Fig. 25(b) shows the relation between the heat-transfer ratios ϵ_{rg} , ϵ_{rc} , ϵ_{cond} and l . When l is to increase, ϵ_{rg} is to decrease. After reaching a minimum value, ϵ_{rg} gradually increases. ϵ_{cond} becomes smaller with an increasing value of l .

Fig. 26(a) shows the relation between η_{rg} , η_{rc} and l with emissivity e as a parameter. η_{rg} becomes smaller with an increasing value of e .

Fig. 26(b) shows the heat-transfer ratios ϵ_{rg} , ϵ_{rc} and ϵ_{cond} with e as a parameter. ϵ_{rg} and ϵ_{cond} become smaller with a decreasing value of e .

Figs. 27(a), 27(b), 28(a) and 28(b) show the efficiency and heat-transfer ratio with gas-emissivity e_g and channel height ratio L as parameters. The efficiencies η_{rg} , $\eta_{rg} + \eta_{rc}$ and heat-transfer ratios ϵ_{rg} and ϵ_{cond} become smaller with an increasing value of e_g and with a decreasing value of L .

11. Conclusion

(1) The heat-transfer mechanism of plate fins is simplified to a problem of a specified finned division. From the heat balance of the fin, an integro-differential conduction equation is introduced.

(2) Angle factors of gray surfaces in the fin

are calculated. In using a numerical conversion method expressed as a matrix form, overall configuration factors of the fin are calculated.

(3) The equation is solved by the use of a digital computer.

The temperature, heat-transfer ratio, and heat-transfer rate are obtained for various values of dimensionless parameters.

(4) The heat-transfer mechanism of the plate-fins is classified into five cases. In each case, the performance of fin is discussed.

Approximate equation is introduced. In general, it is concluded that the temperature distribution of the plate-fins becomes flat because of the mutual

irradiation and so the efficiency is larger than that of a single sheet of a plate-fin.

References

- (1) Y. Okamoto: *Bulletin of JSME*, Vol.7, No.28(1964), p.751.
- (2) Y. Okamoto: *Proc. 13th Japan Nat. Congr. Appl. Mech.* 1963, (1965), p.225.
- (3) K.A. Gardner: *Trans. ASME*, Vol.67, No.8 (1945), p.621.
- (4) E.R.G. Eckert, T.F. Irvine and E.M. Sparrow: *ARL-TN-60-160* (1960), p.15.
- (5) E.M. Sparrow, G.B. Miller, and V.K. Jonsson: *Jour. Aeron., Sci.* Vol.29, No.11 (1962), p.1291.
- (6) R.L. Chambers and E.V. Somers: *Trans. ASME, Ser. C*, Vol.81, No.4 (1959), p.327.
- (7) J.G. Bartas and W.H. Seller: *Trans. ASME, Ser. C*, Vol.82, No.1 (1960), p.73.