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A Structural Analysis of Semi-Monocoque Car Bodies*

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The purpose of this paper is to develop a practical method of stress and deformation analysis of body structures such as sedans and station wagons subjected to bending or twisting loads.

Many of these are reinforced shell structures, the so-called "semi-monocoque" type of construction, but they contain relatively enormous structural "cut-outs" (such as door and window openings).

In conducting research, an attempt is made to analyze the car body as a structure consisting of rigid frame parts and reinforced shells both components being rigidly jointed. The matrix method of structural analysis is used and the calculations are carried out on the digital computer IBM 7074.

In order to check the validity of the present method, experiments are carried out on two- and three-dimensional models. Comparison between the calculated results and the tests indicates that the agreement of stress and deflection is fairly good.

The purpose of this paper is to develop a practical method of stress and deformation analysis of body structures such as sedans and station wagons.

A method that has been widely employed in order to make the structural analysis at all possible is one using a simplified (idealized) structure consisting of rigid frames. However, many of the recent car bodies are a reinforced shell structure, the so-called "semi-monocoque" type of construction. We consider, therefore, that the idealized structure should contain shells as well as rigid frames. From this point of view, it seems that the analytical method which has been widely employed in aeronautical engineering is applicable to the present analysis. But, the car body contains such enormous door and window openings that it can hardly be dealt with in the same way as aircraft. An attempt is made to analyze the car body as a structure consisting of rigid frame parts and reinforced shells.

In order to check the validity of this method, experiments were carried out on models—a two-dimensional model of simplified automobile side structures subjected to bending loads and three-dimensional models subjected to twisting loads.

Conventions for Cartesian co-ordinate systems in the case of space rigid frame

Except for the inclined members $\overline{4\ 3}$, $\overline{10\ 9}$ and $\overline{5\ 6}$, $\overline{11\ 12}$, we select a Cartesian co-ordinate system ox, oy, oz as shown in Fig. 1 (b), (c) (note that this co-ordinate system is also applied to the members $\overline{3\ 9}$, $\overline{4\ 10}$, $\overline{5\ 11}$ and $\overline{6\ 12}$). A Cartesian co-ordinate system which is turned clockwise to angle α with respect to the y -axis is used for the inclined members $\overline{4\ 3}$, $\overline{10\ 9}$ and, similarly, one which is turned counterclockwise to angle β with respect to the y -axis is used for the inclined members $\overline{5\ 6}$, $\overline{11\ 12}$.

Sign conventions for bending and twisting moments and shear flows

As a general rule, the bending moments are considered positive if they produce a tensile stress inside of the closed rigid frame or the space rigid frame. Members not being defined by this rule are considered positive if they produce a curvature as shown in Fig. 1. For twisting moments and shear flows, see Figs. 2 and 3, respectively.

1. Theoretical analysis

Analysis is first made on extremely simplified models—a two-dimensional model of the simplified

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side structure subjected to a vertical or horizontal load and a three-dimensional rectangular parallelepiped model covered with plates over both upper and lower surfaces subjected to twisting loads (See Fig. 9).

Analysis is next carried out on the cabin models of the 4-door sedan and the station wagon subjected to twisting loads. We may obtain various simplified models of car bodies by attaching thereto the proper front and rear compartments in series as shown in Fig. 4.

Fig. 1 Sign conventions for bending moments

Fig. 2 Sign conventions for twisting moments

Fig. 3 Sign conventions for shear flows

Actual structure
Fig. 4 Station wagon

1.1 Assumption and method of analysis

As previously mentioned, an attempt is made to analyze the car body as a structure consisting of both rigid frame parts and reinforced shells. Illustrating both components in the structure shown in Fig. 5, the reinforced shell refers to the part which consists of the quarter panel and the reinforcements $\overline{78}$, $\overline{85}$, $\overline{56}$ and $\overline{67}$, while the rigid frame refers to the other part composed of members $\overline{12}$, $\overline{23}$, $\overline{34}$, $\overline{45}$, $\overline{17}$ and $\overline{38}$.

Let us, therefore, make the following assumptions on each of the components.

Assumptions about rigid frames are:

(a) Frame can be described by its elastic line.

(b) Rigid frames are built-up by uniform component beams.

(c) Deformations due to shear and normal forces are neglected.

(d) Joints are infinitely stiff.

(e) Two principal axes and an elastic line of cross section are parallel to Cartesian co-ordinate systems which have been defined properly.

Assumptions about reinforced shells⁽¹⁾⁽²⁾ are:

(a) Sheet is so thin that variations of stress across the thickness can be neglected.

(b) Reinforced shell is plane.

(c) All sheet material is assigned a purely shear carrying role.

(d) Reinforcements are pinned together at the ends and shear stresses are uniformly distributed along the edges for rectangular sheets. The direct stresses are, therefore, assumed to vary linearly between nodal points. But, we neglect the deformation due to the direct strains in reinforcements, because a car body contains enormous structural "cut-outs" and thus the overall dis-

placements are due mainly to bending (and twisting) deformations of each member as in the case of (space) rigid frames.

(e) Reinforcements carry only direct stresses.

(f) None of the panels will buckle.

(g) Reinforcements have infinite flexural rigidity in bending parallel to their plane, while in bending perpendicular to their plane, their flexural rigidity is finite without any restriction by the sheet. The sheets offer no resistance to torques.

The assumptions can also be made that this theory is restricted to linearly elastic bodies and that the system has no redundant supports. Under these assumptions, the matrix method⁽³⁾ of structural analysis based on the principle of complementary virtual work is used and the calculations are carried out on the digital computer IBM 7074.

1.2 Model of simplified side structure

We can obtain the basic system by cutting the redundant members as shown in Fig. 6. The structure is six times redundant.

The analysis in this paragraph is as shown in Appendix A.

Next, the theoretical calculations and the experiments on the test model as indicated in Paragraph 2.1 bring about the results which follow:

(a) Deflections Table 1 shows the results of deflection test and the theoretical results when $V=100$ kg or $H=100$ kg, where the theoretical ones based on the theory of rigid frames ignore the plate.

The results show that the deflections obtained by the present method are relatively good; however,

Table 1 Comparison between measured and calculated deflections at point of application (Fig.24)

	Experimental result	Theoretical result based on theory of rigid frames	Difference	Theoretical result based on present method	Difference
Deflection (mm) due to $V=100$ kg sec	2.05	2.67	30%	1.93	6%
Deflection (mm) due to $H=100$ kg sec	2.35	3.54	51%	2.06	12%

Fig. 5 Model of the simplified side structure

Fig. 6 Basic system

if we do not take the plate into consideration, the theoretical results are fairly different from the experimental ones.

(b) Stress distributions The stress distributions obtained in the theoretical calculation and in the experiment under a vertical and a horizontal load are shown in Figs. 7 and 8, respectively.

Comparing the theoretical results with the experimental ones, the agreement of the stress distribution is fairly good.

1.3 Rectangular parallelepiped model

In order to analyze a rectangular parallelepiped rigid frame covered with plates over both upper and lower surfaces subjected to twisting loads (Fig. 9), consider first the space rigid frame without panel. The analysis of the former is easily made from that of the latter.

1.3.1 Rectangular parallelepiped rigid frame

We must select the basic system of the rec-

Fig. 7 Measured and calculated bending stress distribution in test model (Fig. 24) due to $V=100$ kg

Fig. 8 Measured and calculated bending stress distribution in test model (Fig. 24) due to $H=100$ kg

Fig. 9 Rectangular parallelepiped rigid frame covered with plates over upper and lower surfaces ($1' 2' 3' 4'$ is a plane of symmetry)

tangular parallelepiped rigid frame. This is easily done by cutting the redundant members as shown in Fig. 10. Though it is a 30 times redundant structure, the system is effectively 15 times redundant due to symmetry of structure and antisymmetry of loading; we have only to select two shearing forces and one twisting moment at the intersections ($1', 3'$ and $4'$) of the plane of symmetry with cross-members, whereas at the cut ends 4 and $4''$, as in general cases, we must select six redundancies—one normal force, two shearing forces and three moments.

Since the structure with panels is analyzed later, we prefer the basic system, as shown in Fig. 11, consisting of the U-shaped frames connected by three-dimensional pinned joints*. We can select 15 redundancies as shown in Fig. 12. This basic system is, of course, unstable under general loads, but stable and statically determinable under twisting loads P (See Fig. 13).

The analysis in this paragraph is shown in Appendix B.

1.3.2 Rectangular parallelepiped rigid frame

covered with plates over upper and lower surface Consider the structure subjected to twisting

Fig. 10 Rectangular parallelepiped rigid frame Basic system (1)

Fig. 11 Rectangular parallelepiped rigid frame Basic system (2)

* In the space frames, we should define three kinds of pinned joints as follows:

One-dimensional pinned joints—Pinned joints dealt with in the plane frames, e.g., those of scissors.

Two-dimensional pinned joints—Those which act as pinned joints with respect to two axes, e.g., universal joints of shaft.

Three-dimensional pinned joints—Those which have no resistance to moments in any directions, e.g., ball joints.

loads as shown in Fig. 9. We can obtain the basic system by changing the basic system of the rectangular parallelepiped rigid frame (Fig. 11) into the system shown in Fig. 14 as follows:

(a) Changing the stiff joints of the U-shaped frames which will be covered with the plates into one-dimensional pinned joints that act as pinned joints which can only move parallel to the plates.

(b) Covering each prescribed surface with the plate and removing X_9 and X_{15} shown in Fig. 12. Thus we are able to obtain pinned joints at the points where the redundancies are removed.

Fig. 12 15 redundancies for the basic system (Fig. 11)

Fig. 13 Bending and twisting moments in the basic system (Fig. 11) due to twisting loads P

Fig. 14 Rectangular parallelepiped rigid frame covered with plates over upper and lower surfaces (Basic system)

Using this basic system, we can analyze the structure with panels over both upper and lower surfaces.

The analysis in this paragraph is shown in Appendix C.

Let us now show the results of the theoretical analysis and of the experiments on the test model as indicated in Paragraph 2.2.

(a) Deflection Table 2 shows the experimental and the theoretical results when $P=100$ kg. The result shows that the deflection obtained by the present method is fairly good.

(b) Stress distributions The stress distributions obtained in the experiment and in the theoretical calculation are shown in Fig. 15. Comparing the theoretical results with the experimental ones, the agreement of the stress distribution is also fairly good.

1.4 4-door sedan type

Considering the cabin model of the idealized 4-door sedan [Fig. 3 (b)], dealing first with a space

Fig. 15 Measured and calculated bending and twisting stress distribution in test model (Fig. 25)

Fig. 16 Space rigid frame of 4-door sedan type (Basic system)

Table 2 Comparison between measured and calculated deflections (Fig. 25)

	Experimental result	Theoretical result based on theory of rigid frames	Difference	Theoretical result based on present method	Difference
Deflection (mm) due to $P=100$ kg sec	22.6	31.4	39%	20.9	8%

rigid frame without panel, next one with panels. This will be the "semi-monocoque" type of construction, whose analysis is easily made from that of the former as mentioned in the previous paragraph 1.3.

1.4.1 Space rigid frame of 4-door sedan type

In order to analyze the space rigid frame obtained when all panels are stripped from the structure as shown in Fig. 3 (b), select its basic system as shown in Fig. 16. We can select 27 redundancies as shown in Fig. 17.

1.4.2 Semi-monocoque body of 4-door sedan type

Consider the semi-monocoque body of the 4-door sedan type with twisting loads shown in Fig. 3 (b). We can get the basic system by changing the basic system of the space rigid frame into the system as follows (Fig. 18):

Fig. 17 27 redundancies for basic system (Fig. 16)

- Two-dimensional pinned joint
- Three-dimensional pinned joint

Fig. 18 Cabin model of the idealized 4-door sedan (Basic system)

(a) Changing the stiff joints of the U-shaped frames which will be covered with the sheets into one-dimensional pinned joints.

(b) Covering each prescribed surface with the plate and removing X_3 , X_6 , X_{12} and X_{18} shown in Fig. 17.

1.5 Station wagon type

Let us consider the model of the idealized station wagon shown in Fig. 3 (c).

In this case, we will begin with the analysis of the space rigid frame without panel.

1.5.1 Space rigid frame of station wagon type

In order to analyze the space rigid frame of the station wagon type which is deprived of all panels from the structure shown in Fig. 3 (c), we may select the basic system shown in Fig. 19. Fig. 20 shows the 30 redundancies.

Fig. 19 Space rigid frame of station wagon type (Basic system)

Fig. 20 30 redundancies for basic system (Fig. 19)

1.5.2 Semi-monocoque body of station wagon type

Consider the semi-monocoque body of the station wagon type subjected to twisting loads. We can get the basic system by changing the basic system of the space rigid frame into the system as follows (Fig. 21):

(a) Changing the stiff joints of U-shaped frames which will be covered with the sheets into one-dimensional pinned joints.

(b) Covering each prescribed surface with the plate and removing $X_1, X_3, X_4, X_5, X_{12}, X_{18}$ and X_{28} shown in Fig. 20.

(c) Selecting a self-equilibrating stress system of $X_{31}=1$ illustrated in Fig. 22.

The theoretical results and the experimental ones on the test model as indicated in Paragraph 2.3 are shown to provide their correlations as follows:

(a) Deflections Table 3 shows the result of deflection test and the theoretical result when $P=100$ kg. It was found that the deflection ob-

Table 3 Comparison between measured and calculated deflections (Fig. 26)

	Experimental result	Theoretical result based on theory of rigid frames	Difference	Theoretical result based on present method	Difference
Deflection (mm) due to $P=100$ kgsec	3.64	19.75	5.5 times exp. result	3.82	5%

Fig. 21 Basic system for the structure shown in Fig. 3 (c)

Fig. 22 Self-equilibrating stress system $X_{31}=1$

tained by the present method was very good, but, if the plates in considering such a system are neglected, the theoretical result based on the theory of rigid frames is extremely different from the experimental one.

(b) Stress distributions The stress distributions obtained in the experiment and in the theoretical calculation are shown in Fig. 23. Comparing the theoretical results with the experimental ones, the agreement of the stress distribution is also very good.

2. Experiment with models

In order to check the validity of the method thus far developed by making a number of exploratory assumptions, the experiments are carried out on two- and three-dimensional models.

2.1 Model of simplified side structure

Fig. 24 shows the two-dimensional model of simplified side structure analyzed in Paragraph 1.2. The dimensions of this model are: length, 2 m; height, 1 m; thickness of sheet, 1 mm. All

Fig. 23 Measured and calculated bending stress distributions in test model shown in Fig. 26

Fig. 24 Test model (1)

frames are made of steel pipes: outside diameter, 49 mm; inside diameter, 47 mm. External loading conditions (See Fig. 5): bending load, $H=0$ kg, $V=100$ kg; shearing load, $H=100$ kg, $V=0$ kg.

Dial gages are employed for the measurement of deflections and electric-resistance wire gages for stresses. The results are shown in Table 1, Figs. 7 and 8.

2.2 Rectangular parallelpiped model covered with plates over upper and lower surfaces

The test model as shown in Fig. 25 is used for the structure dealt with in Paragraph 1.3.2. The dimensions of this model are: length, 2 m; width, 1 m; height, 1 m; thickness of sheets, 1 mm. All frames are made of steel pipes: outside diameter, 47.6 mm; inside diameter, 45.2 mm.

In the experiment, the three corners of the under-structure are freely supported and a vertical load $P=100$ kg is applied to the remaining corner. The results are shown in Table 2 and Fig. 15.

2.3 Semi-monocoque body of station wagon type

Fig. 26 shows the cabin model of the station wagon type tested. The dimensions of this model are: length, 2 m; width, 1 m; height, 1 m; thickness of sheets, 1 mm. All frames are made of steel pipes: outside diameter, 50.8 mm; inside diameter, 47.6 mm.

The external loading condition is the same as in the case in Paragraph 2.2. The results are shown in Table 3 and Fig. 23.

The shear stresses due to twisting moment in each member and the shear stresses in each

panel are too small to measure by means of wire gages.

3. Conclusions

First of all, extremely simplified structures are analyzed and then complicated objects are studied step by step.

Next, in order to check the validity of the present method, the theoretical and the experimental results on the test models as mentioned in Section 2 are compared. The agreements of stress distribution and deformation are both fairly good.

The present structures contain openings too enormous to be analyzed as semi-monocoque type of construction as in the case of aircraft. An attempt, therefore, is made to analyze the car body as a structure consisting of both rigid frames and reinforced shells.

The differences between the theoretical value when shear stiffness of plate tends to infinity ($Gt \rightarrow \infty$) and the value obtained in the present method are very small in each case. This fact means that the total shearing strain energy of the panels is very small. Thus, we believe that the agreement of the theoretical results with the experimental ones is obtained not because the shearing strain energy is taken into account in the present analysis, but because the plates impose certain local constraints on the rigid frame without panels. Fig. 27 illustrates the function of plates in the structures dealt with here; i.e., the plates serve as a diagonal in a truss does—to restrict the shear deformation of a rectangular rigid frame which should occur unless the plate were attached, and to constrain the deformation in bending parallel to its plane. From the mechanical point of view, the approach to the present problem is, therefore, considered to be analogous to one of the frame structure problems under such boundary conditions.

4. Nomenclature

- A : cross-sectional area
- E : Young's modulus
- G : shear modulus
- b : rectangular transformation matrix for internal forces
- b' : transpose of matrix b

Fig. 27 Figure to illustrate the function of plate.

Fig. 25 Test model (2)

Fig. 26 Test model (3)

- $\mathbf{b}^{-1}$: reciprocal or inverse matrix of $\mathbf{b}$
 $\mathbf{f}_e$: flexibility matrix of individual element
 $\mathbf{f}$: flexibility matrix of all elements
 I : moment of inertia of cross section
 I_x, I_y, I_z : moment of inertia of cross section parallel to a Cartesian co-ordinate system ox, oy, oz
 J : torsion constant
 l : length of structure
 M_x, M_y, M_z : moments parallel to a Cartesian co-ordinate system ox, oy, oz , (for sign conventions, see Fig.1 and Fig.2)
 $M_{y2,3}$: bending moment M_y at the end 2 in the member $\overline{2\ 3}$
 $\mathbf{0}$: zero matrix
 $\mathbf{q}$: column matrix of shear flows
 q : shear flow
 $\mathbf{R}$: column matrix of forces
 P, H, V : force
 $\mathbf{r}$: column matrix of displacements
 $\mathbf{S}$: column matrix of internal forces
 t : thickness
 T : twisting moment
 $\mathbf{X}$: column matrix of redundancies
 (m, n) : rectangular matrix with m rows and n columns

Subscripts

- f : floor
 r : roof

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Appendix A

The structure is subjected to a vertical and a horizontal load (Fig.5)

$$\mathbf{R} = \{VH\} \dots\dots\dots (\text{A}\cdot 1)$$

We denote by $\mathbf{X}$ the column matrix of the six (unknown) redundancies,

$$\mathbf{X} = \{X_1 X_2 \dots X_6\} \dots\dots\dots (\text{A}\cdot 2)$$

The column matrix $\mathbf{S}$ of the stresses in the actual structure can always be written in the simple form,

$$\mathbf{S} = \mathbf{b}_0 \mathbf{R} + \mathbf{b}_1 \mathbf{X} \dots\dots\dots (\text{A}\cdot 3)$$

Since the basic system is statically determinable, the two matrices $\mathbf{b}_0$ and $\mathbf{b}_1$ are found merely by statical reasoning, i.e., they denote the stress matrices corresponding to

$$\mathbf{R} = \{1\ 1\} \dots\dots\dots (\text{A}\cdot 4)$$

and

$$\mathbf{X} = \{1\ 1 \dots 1\} \dots\dots\dots (\text{A}\cdot 5)$$

respectively.

We write now the $\mathbf{S}$ matrix in the following form,

$$\mathbf{S} = \{M_{12} M_{21} M_{23} M_{32} M_{34} M_{43} M_{45} M_{54} M_{38} M_{83} M_{17} M_{71} q\} \dots\dots\dots (\text{A}\cdot 6)$$

Then, we can obtain the $\mathbf{b}_0$ and $\mathbf{b}_1$ matrices as follows:

$$\mathbf{b}_0 = \begin{array}{cccccccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & & & & & & & & & & & & & \\ & & & & & & & & h_2 & & & & & \\ & & & & & & & & & & & & & \\ & & & & & & & & & & & & & \\ & & & & & & & & & & & & & \\ & & & & & & & & & & & & & \end{array} \dots\dots (\text{A}\cdot 7)$$

$$\mathbf{b}_1 = \begin{array}{cccccccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & -h & -h & -h & -h & -h & -h & -h & & & & & & \\ & l_1 & l_1 & l_1 & l_1 & l_1 & l_1 & l_1 & & & & & & \\ & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & & & & & \\ & & & & & & & & h_2 & -h_2 & & & & \\ & & & & & & & & l_2 & l_2 & l_2 & & & \\ & & & & & & & & 1 & 1 & 1 & -1 & -1 & \end{array} \dots\dots (\text{A}\cdot 8)$$

The flexibility of the complete structure is

$$\mathbf{f} = \left[\begin{array}{cc} \frac{h}{3EI_{12}} & \frac{h}{6EI_{12}} \\ \frac{h}{6EI_{12}} & \frac{h}{3EI_{12}} \\ \frac{l_1}{3EI_{23}} & \frac{l_1}{6EI_{23}} \\ \frac{l_1}{6EI_{23}} & \frac{l_1}{3EI_{23}} \\ \frac{l_2}{3EI_{34}} & \frac{l_2}{6EI_{34}} \\ \frac{l_2}{6EI_{34}} & \frac{l_2}{3EI_{34}} \\ \frac{h_2}{3EI_{45}} & \frac{h_2}{6EI_{45}} \\ \frac{h_2}{6EI_{45}} & \frac{h_2}{3EI_{45}} \\ \frac{h_2}{3EI_{38}} & \frac{h_2}{6EI_{38}} \\ \frac{h_2}{6EI_{38}} & \frac{h_2}{3EI_{38}} \\ \frac{l_1}{3EI_{17}} & \frac{l_1}{6EI_{17}} \\ \frac{l_1}{6EI_{17}} & \frac{l_1}{3EI_{17}} \\ \frac{h_1 l_2}{Gt} \end{array} \right] \dots\dots\dots (\text{A}\cdot 9)$$

(All the elements remaining blank are zero.)

In the matrix method of structural analysis the stresses and deflections are given as follows:

$$\mathbf{S} = \mathbf{b}\mathbf{R} \quad \text{.....(A.10)}$$

where

$$\mathbf{b} = \mathbf{b}_0 - \mathbf{b}_1(\mathbf{b}_1' \mathbf{f} \mathbf{b}_1)^{-1} \mathbf{b}_1' \mathbf{f} \mathbf{b}_0$$

and

$$\mathbf{r} = \mathbf{b}_0' \mathbf{f} \mathbf{b} \mathbf{R} \quad \text{.....(A.11)}$$

Substituting the above $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{f}$ matrices in Eqs. (A.10) and (A.11), we can determine the stress distribution and the deformation.

Appendix B

Analyze the space rigid frame which is deprived of all panels from the structure shown in Fig. 9.

$$\mathbf{R} = \{P\} \quad \text{.....(B.1)}$$

$$\mathbf{X} = \{X_1 \ X_2 \ \dots \ X_{15}\} \quad \text{.....(B.2)}$$

We write now the $\mathbf{S}$ matrix in the following form,

$$\mathbf{S} = \{\mathbf{M}_x \ \mathbf{M}_y \ \mathbf{M}_z\} \quad \text{.....(B.3)}$$

where

$$\mathbf{M}_x = \{T_{12} \ T_{34} \ M_{x11'} \ M_{x22'} \ M_{x33'} \ M_{x44'} \ M_{x41} \ M_{x14} \ M_{x32} \ M_{x23}\} \quad \text{.....(B.4)}$$

$$\mathbf{M}_y = \{M_{y12} \ M_{y21} \ M_{y43} \ M_{y34} \ T_{11'} \ T_{22'} \ T_{33'} \ T_{44'} \ M_{y41} \ M_{y14} \ M_{y32} \ M_{y23}\} \quad \text{.....(B.5)}$$

$$\mathbf{M}_z = \{M_{z12} \ M_{z21} \ M_{z43} \ M_{z34} \ M_{z11'} \ M_{z22'} \ M_{z33'} \ M_{z44'} \ T_{14} \ T_{23}\} \quad \text{.....(B.6)}$$

Then we can obtain the two matrices $\mathbf{b}_0$ and $\mathbf{b}_1$ by means of statics as follows:

$$\mathbf{b}_0 = \{\mathbf{b}_{0x} \ \mathbf{b}_{0y} \ \mathbf{b}_{0z}\} \quad \text{.....(B.7)}$$

where

$$\mathbf{b}_{0x} = \{0 \ 10 \ 1\} \quad \text{.....(B.8)}$$

$$\mathbf{b}_{0y} = \{0 \ -1 \ 0 \ 0 \ 0 \ -1 \ 0 \ 0 \ 0 \ 0 \ 0\} \quad \text{.....(B.9)}$$

$$\mathbf{b}_{0z} = \{0 \ 10 \ 1\} \quad \text{.....(B.10)}$$

and

$$\mathbf{b}_1 = \{\mathbf{b}_{1x} \ \mathbf{b}_{1y} \ \mathbf{b}_{1z}\} \quad \text{.....(B.11)}$$

where

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1							1								
2													1		
3	1			h			1								
4							-1			1					
5				h						1					
6	1												-1		
7	1														
8	1			h											
9				h						1					
10										1					

.....(B.12)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1								1							
2	$\frac{2l}{b}$				$\frac{2hl}{b}$	$\frac{2l}{b}$		1							
3	$\frac{2l}{b}$				$\frac{2hl}{b}$	$\frac{2l}{b}$							$\frac{2l}{b}$	1	
4															
5				h											
6	$\frac{2l}{b}$			$\frac{2hl}{b}$	$\frac{2l}{b}$			1							
7	$\frac{2l}{b}$			$\frac{2hl}{b}$	$\frac{2l}{b}$										
8	$\frac{2l}{b}$			$\frac{2hl}{b}$	$\frac{2l}{b}$										
9															
10															
11															
12															

.....(B.13)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1									-1						
2				1					-1						
3				1										-1	
4														-1	
5		1							-1						
6				1					-1						
7											1				
8															
9															
10												-1			

.....(B.14)

Appendix C

The analysis of this structure can readily be made by altering part of the elements of the matrices about the rectangular parallelepiped rigid frame (Appendix B).

Since we remove X_9 and X_{15} , we denote by $\mathbf{X}$ the column matrix of the 13 (unknown) redundancies,

$$\mathbf{X} = \{X_1 \ \dots \ X_8 \ X_{10} \ \dots \ X_{14}\} \quad \text{.....(C.1)}$$

The column matrix $\mathbf{S}$ of the stresses required here is expressed in the following form,

$$\mathbf{S} = \{\mathbf{M}_x \ \mathbf{M}_y \ \mathbf{M}_z \mathbf{q}\} \quad \text{.....(C.2)}$$

where $\mathbf{M}_x$ and $\mathbf{M}_y$ are the same as Eqs. (B.4) and (B.5), respectively, but $\mathbf{M}_z$ is expressed in the simple form,

$$\mathbf{M}_z = \{T_{14} \ T_{23}\} \quad \text{.....(C.3)}$$

because we neglect the reinforcement deformability [assumption (g) about reinforced shells].

$$\mathbf{q} = \{q_r \ q_f\} \quad \text{.....(C.4)}$$

The $\mathbf{b}_0$ matrix is found easily as

$$\mathbf{b} = \{\mathbf{b}_{0x} \ \mathbf{b}_{0y} \ \mathbf{b}_{0z} \ \mathbf{b}_{0q}\} \quad \text{.....(C.5)}$$

where $\mathbf{b}_{0x}$ and $\mathbf{b}_{0y}$ are the same as Eqs. (B.8) and

(B.9), respectively. b_{0z} and b_{0q} are

$$b_{0z} = \{0 \ 0\} \dots\dots\dots (C.6)$$

$$b_{0q} = \{0 \ 0\} \dots\dots\dots (C.7)$$

Next, b_1 can be written in the following form,

$$b_1 = \{b_{1z} b_{1y} b_{1z} b_{1q}\} \dots\dots\dots (C.8)$$

where the b_{1z} and b_{1y} matrices are obtained by removing the columns corresponding to X_9 and X_{15} ; and b_{1z} is found by removing the same columns, retaining the rows corresponding to Eq. (C.3). That is,

$$b_{1z} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \dots\dots (C.9)$$

The transformation matrix for shear flows is

$$b_{1q} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{-2}{b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-2}{b} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots\dots (C.10)$$

The total flexibility follows

$$f = \begin{bmatrix} f_x & 0 & 0 & 0 \\ 0 & f_y & 0 & 0 \\ 0 & 0 & f_z & 0 \\ 0 & 0 & 0 & f_q \end{bmatrix} \dots\dots\dots (C.11)$$

where f_x , f_y , f_z and f_q correspond to Eqs. (B.4), (B.5), (C.3) and (C.4), respectively, e. g.,

$$f_q = \begin{bmatrix} \frac{bl}{2Gt_r} & 0 \\ 0 & \frac{bl}{2Gt_r} \end{bmatrix} \dots\dots\dots (C.12)$$

Similarly, we can analyze the idealized structures of the sedans and station wagons.

References

- (1) Samuel Levy: *Jour. Aeron. Sci.*, Vol.14 (1946-10).
- (2) Paul Kuhn: *Stresses in Aircraft and Shell Structures*, (1956), McGraw-Hill.
- (3) J. H. Argyris and S. Kelsey: *Energy Theorems and Structural Analysis*, (1960), Butterworths.

Discussion

M. YAMAMOTO: (1) When a steel sheet is welded onto a model pipe frame, remarkable deformations of frame and sheet result. Such irregularities in the sheet of your test models were rectified by the use of a torch, I guess. Although the irregularities of the sheet may be corrected by such method, the whole framework undergoes marked deformation, and this affects experimental results, as we experienced in testing car models in our laboratory some time ago. I notice that no definite mention is made of this effect in the present paper.

S. WATANABE: (2) I see that very good agreement has been obtained between theoretical and experimental results. However, an actual car body is considerably different from the model in that:

- 1) Frames are not built up by uniform component beams.
- 2) Sections are very complicated, consisting of thin sheets.
- 3) The members, particularly the upper frames, are not linear but rather they form three-dimensional curves.
- 4) The joints of each member are too weak to be assumed as rigid joints.
- 5) The roof and quarter panels acting as the shear field are arched in design. How should this method be applied in designing car bodies? I would like to know the data by which the distributing factor of loads in each member has been calculated qualitatively and quantitatively.

T. AKASAKA: (3) Effective shear modulus

of the sheet G is known to decrease if there is an initial deflection. Referring particularly to the roof and floor of this test structure, I should think that this effect must be considered in accordance with their weight and how these are fitted. What do you think?

(4) With reference to the test structure (Fig. 26), the ratio of the cross-sectional area of the reinforcement A_s and that of the panel A_p is expressed:

$$\frac{A_p}{A_s} = \frac{500 \times 1}{50 \times \pi \times 1.3} \doteq 2.5$$

In this case, the capacity of the panel to carry the axial force would be such as not to be negligible (or rather large) compared with the reinforcement. Therefore, the assumption of the shear flow being constant would not, in general, be justified. What is your view on this point?

(5) I well understand your answer to Question (4) as far as it means that the shear stress in the reinforced panel structure is carried for the most part by the panel rather than the flange. However, the problem here is whether or not a normal stress (σ_x, σ_y) in the panel can be disregarded against the shear stress (τ_{xy}). The validity of the assumption of the shear flow being constant depends on this point. What do you think on this matter in connection with your test model?

T. SEINO: (6) With reference to idealizing the joints of the semi-monocoque structure and the rigid frame, in the case where the problem as shown in Fig. 2 is to be solved by means of your method, this structure will have to be

idealized into either Fig. 3 (a) or (b):

Fig. 3 (a) Beam C is rigidly jointed with Reinforcement A.

Fig. 3 (b) Beam C is rigidly jointed with Reinforcement B.

Depending on the selection between the above two cases, a different solution will be obtained. What is your opinion? Also, which do you think should be adopted? (Dotted lines in Fig. 3 show approximate deflections.)

(7) 1) Will you please explain the reason for the latter half of your answer: "Otherwise, the one that works better in resisting external forces, or (a) in this instance, should be used".

2) The internal forces in the example as referred to in my previous question, if one takes it into consideration that all sheet materials are assigned a purely shear carrying role, would be expressed as in Fig. 4 [Figures for (a) alone are shown on account of limited space].

If the sheet is to carry the shear force alone, Member AB will have to carry the bending moment as shown in (2), and will be bent as the result. This goes contrary to the "Assumption about reinforced shells" (g).

If Member AB is not to be bent, it is because the sheet effects constraints. In this case, the sheet will have to carry the normal stress in plane also. Wouldn't this make it necessary to rewrite the assumption (c)? Also, the tensile rigidity of the sheet must necessarily be very great.

Meanwhile, it may be possible to assume that Member AB is infinite (or extremely large) in bending stiffness so as to be unbendable. However, would it be all right to consider pinned joints in idealizing such a structure whose reinforcements have large bending stiffness (assuming that Member AB and Member BB' have a bending stiffness of the same degree and are rigidly jointed to each other)? When the force alone is

applied to the joint of the reinforcements, assumptions and limitations for the shear field theory will suffice, but, when the moment is applied to the joint, should stricter assumptions and limitations be adopted?

K. YOSHIMINE: (8) Referring to the bending moment diagram of Fig. 23, View from B, I suspect that the bending stress in the left upper chord is excessively small as compared with the right side one.

Author's closure

(1) I was aware of that possibility and tried to minimize such effects by having as large a model as possible made with utmost care. Good results have been obtained not only from the models dealt with here, but also from other more complicated ones.

(2) That is true. A great number of difficult problems are involved in the analysis of an actual body. Therefore, instead of starting with an actual car, I dealt first with the simplified model, and intend to take up a model progressively closer to an actual car body. But I am not thinking of making direct use of the present analysis in designing car bodies. The next time, I hope to deal with a structure a little closer to an actual car body.

(3) I agree with you. It can be imagined that an initial deflection of the sheet would naturally result in a decreased shear modulus. I have not made any experiments on this point, and so cannot offer any definite opinion at present. As it is a very interesting subject, I would like to study it some time in the future.

(4) In answering your question on the sections, I would like to refer to the comparison, as shown in Append-Fig. 1, between the shear stress based on the shear field theory and that based on Navier-Bernoulli's assumptions i.e., engineering theory of bending (where, the flange is replaced with a square section equivalent to the pipe section in order to simplify the calculation).

The latter is about 12% larger in maximum value than the former but the shearing strain energy is less than 1% of the total strain energy.

Append-Fig. 4

In our test structure, the shear stress and strain energy of the sheet are both extremely small, the latter being less than 1% of the total strain energy of the structure. Consequently, I consider that there are no problems regarding the accuracy of the shear stress.

(5) You are right. 1. When the reinforcements are stiff and the panel is very thin, the normal stress (σ_x, σ_y) in the panel can be neglected, so $\tau = \text{const.}$ from the equation of equilibrium. 2. τ is considered constant from the equation of equilibrium in such cases as torsion tube where σ_x, σ_y are of the secondary order and τ_{xy} is of the first order. The test structure, which is of a discontinuous construction with a great deal of cut-out, is considered different from Case 2 cited above where a uniform shear flow resists the external torsional load as in the case of a torsion tube. Now, let us consider the large reinforced shell element of the roof and how it is subjected to load. Since there are only rigid frames (pillars) with no sheets in the neighborhood, the load is not introduced in the form of a uniform shear flow but as a horizontal concentrated load from the upper ends of pillars, which causes the shear deformation of the roof. From this reasoning, the test model is considered to correspond with Case 1. mentioned above. As referred to in the conclusion of this paper, such good agreement between theoretical and experimental results is obtained, not on account of shear strain energy in panels considered in the present analysis, but because of the condition that reinforcements have infinite flexural rigidity in bending parallel to the plane, this condition being necessarily introduced into analysis when shear field theory is applied. Therefore, the solution obtained in this method presents an appearance of that of the frame structure problem solved under boundary conditions imposing certain local constraints, which the plate, if attached, would effect on the rigid frame without panel. The assumption $\tau = \text{const.}$ is approximate as seen from Fig. 1, but the shear stress and shear strain energy in panels are of the second order when viewed in terms of the whole structure. As you pointed out initially, this assumption is based on the idealization of the structure whose frames are very stiff and sheets are very thin, so it naturally follows that the weaker the frame, the less constant the shear flow. This makes it necessary to check the accuracy of the structure from time to time, and the present analysis is

intended for this check.

(6) As you pointed out, deformations and internal forces in Fig. 3 are, of course, different depending on which case is used. As to which should be used, when a structure is connected in a manner obviously closer to one of the

Append-Fig. 5

two ways, that closer one should be used; otherwise, the one that works better in resisting external forces, or (a) in this instance, should be used.

(7) 1) The reason is that, when there are unknown factors, the severer one is ordinarily used.

2) Please take it into consideration that the present method is an engineering approximate theory based on a number of simplifying assumptions. The bending stiffness of the reinforcements is defined as infinite in the present method. When I find the internal forces, I consider that the external forces in the reinforcements consist of the force P and the moment $M_0 = Pl$ at point B (Fig. 5). There should be no problem about P . As regards M_0 , however, (a) and (b) in Fig. 3 are considered depending on whether M_0 acts on Member AB or Member BB', as you pointed out. In calculating, the couple, $(\text{force}) \times (\text{arm}) = \left(\frac{M_0}{h}\right)h$, is applied to both ends of Member AB in the case of (a), and the couple, $\left(\frac{M_0}{b}\right)b$ to both ends of Member BB' in the case of (b).

Substituting for the moment an equivalent couple is an approximate method being widely employed in theories of construction and elasticity. Also, even substituting pinned joints for rigid joints can be done as an engineering approach. For example, the shear field theory is also applied for analysing the building in some cases. In such cases, even the rigid joints of the frame are treated as pinned joints. In the present paper, whichever system may be selected, the difference in effects from any of the others may be ignored to the first order of magnitude, but I am interested in the analysis of cases where nodes are subjected to the moment and I hope to make a stricter analysis in the future.

(8) This matter has been proved in the experiments as well as in the analysis, and a different basic system has brought about the same result. So, I believe there is no error.