of reverse flow from the impeller as well as its absolute value of axial and rotational components of velocity is increased progressively.

(2) Maintaining approximately the similar profiles, both of the axial and rotational components of velocity decay with the distance from the pump flange.

(3) The maximum value of the radial component of velocity remains within the value of order 1/100 times that of the axial or rotational component.

(4) Considerable amounts of the energy developed by the pump are carried back upstream from impeller by the reverse flow and almost all of them is dissipated in its flow process.

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Swirling Flow in Suction Pipe of Centrifugal Pumps*

(2nd Report, Distribution of Shearing Stress)

By Mitsukiyo MURAKAMI** and Naomichi HEYA***

For the swirling flow created in the suction pipe of centrifugal pumps at partial discharges, the authors evaluated the distribution of shearing stresses by using measured velocity and pressure and compared it with strain velocity.

The stress tensor at any point within the pipe is verified to be in proportion with the rate of strain velocity. Although the absolute value of the radial component of velocity exceeds hardly one hundredth that of the axial or rotational component, it contributes considerably to the stress tensor and can not be neglected.

Eddy kinetic viscosity of the flow near the pipe wall ranges from 20 to 70 times the kinetic viscosity due to molecular causes.

1. Introduction

A flow accompanied by swirling component is seen in many cases of industrial practice, for example, vortex tubes, cyclones, and so on. Especially, the study of vortex tubes^{(1)~(4)}, in which a strong swirling flow is created, has recently become the subject of many investigations. As

the basic investigation of swirling flow, Meldau⁽⁵⁾ and Yen⁽⁶⁾ have made the measurements on skin friction and intensity of turbulence of the swirling flow in pipes. For all these studies, the details of swirling flow are yet insufficiently understood and further investigations will be required.

In this paper, the authors have evaluated the distribution of shearing stresses by using the measured velocities and pressures for the swirling flow⁽⁷⁾ which is created in the suction pipe of centrifugal pumps. Although the experimental procedure was not so accurate as would be required

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for quantitative investigation of shearing stresses, it is believed that the results are good enough to obtain an information relevant to the character of distribution of shearing stress or decay of swirling velocity.

The results of this investigation will be also supplemental to the study of swirling flows developed in various fields of industrial practice.

2. Equation to calculate shearing stresses

Taking cylindrical co-ordinate as shown in Fig. 1, the equations to determine the shearing stresses are represented as follows⁽⁸⁾:

$$\tau_{\theta}/\gamma = v_r v_{\theta}/g + (1/gr^2) \int_0^r r^2 \{ \partial(v_{\theta} v_z) / \partial z \} dr \quad \dots (1)$$

$$\tau_z/\gamma = v_r v_z/g + (1/r) \int_0^r r [\partial \{ (v_z^2/g) + (p/\gamma) \} / \partial z] dr \quad \dots (2)$$

where r , θ , and z denote the radial, rotational, and axial co-ordinates, respectively, v_r , v_{θ} , and v_z the velocity components in the respective directions, p the static pressure at radius r , γ the specific weight of liquid, g the acceleration of gravity, and τ_{θ} and τ_z the shearing stresses in directions of θ and z , respectively.

According to our experiments, the maximum value of $|v_r|$ is seen to exceed hardly one hundredth that of $|v_{\theta}|$, or of $|v_z|$ and the radial flow component can be neglected for the measurement of flow velocity. But the component itself contributes remarkably to the shearing stress between water particles and the neglect of it will bring about a serious error as described in Section 3.3.

As v_r is zero at pipe wall, the components of shearing stresses on the wall are given from Eqs. (1) and (2) as:

$$(\tau_{\theta}/\gamma)_0 = (1/gR^2) \int_0^R r^2 \{ \partial(v_{\theta} v_z) / \partial z \} dr \quad \dots (1)'$$

$$(\tau_z/\gamma)_0 = (1/R) \int_0^R r [\partial \{ (v_z^2/g) + (p/\gamma) \} / \partial z] dr \quad \dots (2)'$$

The static pressure p at radius r is determined

Fig. 1 Cylindrical co-ordinate

approximately from the equation

$$p = p_w - (\gamma/g) \int_R^r (v_{\theta}^2/r) dr \quad \dots (3)$$

where the kinetic energy contribution of the radial velocity component and the viscous action are neglected. The above calculations may be carried out by a graphical method, in use of experimental values.

The values of the shearing stresses $(\tau_{\theta})_0$ and $(\tau_z)_0$ on pipe wall are approximately in relation of $(\tau_z)_0 \doteq (\tau_{\theta})_0 (v_z)_0 / (v_{\theta})_0 \quad \dots (4)$ where $(v_z)_0$ and $(v_{\theta})_0$ represent the velocity components of water close to the wall.

For the determination of the value of τ_z , two successive graphical calculations are needed using Eqs. (2)' and (3), while for that of τ_{θ} only one process using Eq. (1)' will suffice. It is probable that the accuracy of τ_z will be less than that of τ_{θ} , since graphical method may always bring about some errors. Thus, to determine the value of τ_z near the wall, where the relationship of Eq. (4) is approximately correct, more accurate result is expected from use of Eq. (4) than from the graphical method.

3. Distribution of shearing stress

3.1 Distribution of shearing stress along pipe

The distribution of flow velocity at successive sections of a long straight suction pipe for $Q/Q_n = 0.18$ is shown in Fig. 2, where Q and Q_n denote a certain discharge and the rated discharge of pump, respectively. The pump employed in the test was a diagonal flow type of 150 mm inlet diameter and specific-speed 745 (in m, m/sec, and rpm units). The positive sign for axial flow refers to the flow toward the upstream from pump flange, namely, "reverse flow", and that for the swirling flow in direction of impeller rotation. Symbols S_0 , S_1 , S_2 , ..., S_7 denote the measured sections.

It is seen that the swirling velocity decays with distance from pump flange, exhibiting a closer similarity to profile of the forced vortex motion. The curves of axial component are also deformed similarly to the swirling velocity profile. Nearly similar profile is obtained for all of the other flow rates lower than 86 percent of the rated discharge. At discharges higher than this, no reverse flow is seen and all of the liquid particles in suction pipe move toward the impeller and show a nearly uniform profile of velocity.

The decay of $v_{\theta \max}$ with distance from pump flange is shown in terms of $v_{\theta \max}/V_{\theta \max}$ in Fig. 3, where $v_{\theta \max}$ and $V_{\theta \max}$ denote the maximum swirling velocity at any section and the initial section

Fig. 2 Velocity distributions of flow at several sections

Fig. 3 Variation of maximum v_θ with axial distance

(a)

(b)

(c)

Fig. 4 Variation of wall friction with axial distance and flow rate

S_0 , respectively. In Figs. 4 (a), (b), and (c), the variation of wall friction along the pipe axis is plotted in terms of non-dimensionalized form $(\tau)_0/\rho u_{10}^2$, $(\tau_\theta)_0/\rho u_{10}^2$, and $(\tau_z)_0/\rho u_{10}^2$, where $(\tau)_0 = \sqrt{(\tau_\theta)_0^2 + (\tau_z)_0^2}$, and u_{10} is the peripheral velocity of impeller at outer edge of impeller eye.

From the experimental results given in Fig. 5, it is seen that the value of velocity ratio $(v_z)_0/(v_\theta)_0$ near the wall remains constant along the pipe line and is also independent of flow discharge. Thus, the value of $(\tau_z)_0/(\tau_\theta)_0$ also remains constant along the pipe wall by means of Eq. (4). As is seen in Fig. 4, an intensive shearing stress is caused on the wall at the starting section of reverse flow, and firstly it decreases rapidly, then gradually with distance from pump flange. More severe gradient of the curve is observed for higher pump discharge. The shearing stress of a flat plate calculated from Blasius formula by assuming a uniform velocity field V_z is also plotted as a broken line in Fig. 4 (b), where V_z is the same velocity as the axial velocity of reverse flow just outside the boundary layer of section S_0 . The stress $(\tau_z)_0$ in the swirling flow shows much higher value than that experienced in a simple parallel flow. Results of calculation for the coefficient of wall friction expressed in terms of $C_f = \tau/(\rho v_{\max}^2/2)$, $C_{f(\theta)} = \tau_\theta/(\rho v_{\theta \max}^2/2)$, and $C_{f(z)} = \tau_z/(\rho v_{z \max}^2/2)$ are shown in Fig. 6. The values remain constant for every

Fig. 5 Flow angle near pipe wall

Fig. 6 Variation of frictional coefficient along pipe line

section and any different pump discharge, representing a rough character of the surface and $C_{f\theta} = 0.046$, $C_{f(\theta)} = 0.013$, and $C_{f(z)} = 0.044$.

3.2 Transverse distribution of shearing stress

In Figs. 7 (a)~(c), transverse distributions of τ_θ and τ_z at several sections are shown for different pump discharges. In the figure, the right- and left sides of the symbol $\odot$ refer to the regions of reverse and forward flow, respectively. Positive sign of the stresses refer to positive directions of θ and z .

3.2.1 Distribution of τ_θ Within the region near the wall, the swirling velocity of liquid is slowed down by the wall friction and hence a negative value of τ_θ is caused. At zero discharge, it is seen that the absolute value of negative τ_θ in a section near the pump reduces from the wall to the pipe center and reaches zero at about $r/R = 0.87$. And then it increases positively toward inner portion and reaches the positive maximum at about $r/R = 0.75$ and again decreases toward the pipe axis. At a distant section from the pipe flange, a modified profile of τ_θ is observed and at the section where the reverse flow mostly disappears, a negative value of τ_θ is caused over the whole section. This distribution of τ_θ is closely related with the presence of radial component of the flow. At zero discharge, all of the flow in suction pipe is purely circulating between the region of impeller and suction pipe. In this case, none of the flow from the reservoir of water supply participates. Hence, the picture of decay of reverse flow along the pipe line is directly characterized by the radial component. At sections S_1 and S_2 , an intensive swirling reverse flow with a slight inward radial component is maintained in outer portion of pipe. The angular momentum of liquid in outer zone is transported to the central portion of pipe by inward flow and there results a higher swirling velocity than that expected in a typical forced vortex motion. In this region, the liquid element in outer layer with a slower velocity exerts a retarding force to the liquid in inner layer. At low discharges other than zero similar profiles of τ_θ are obtained at different sections. By increasing the discharge, the percentage of liquid elements coming without swirling component from a suction well is increased in the pipe and a severe decay of the swirling motion occurs in the central region and a positive τ_θ is always caused.

3.2.2 Distribution of τ_z The axial flow near the wall is slowed down by the wall friction and intensive negative stress τ_z is caused on the wall. The value falls abruptly to zero within a short

Fig. 7 Variation of stress distribution with axial distance

distance from the wall, then it increases to a positive value and again decreases toward the pipe axis. The profile of τ_z varies similarly with that of τ_θ . The similar characteristics is observed at different sections and at different discharges except zero. At zero discharge, τ_z has a negative value in the central region of pipe near the pump flange. This peculiarity has a closer relationship with the radial component of velocity. At sections S_1 and S_2 , an outward radial flow is developed in the central region of pipe as shown in Fig. 2. Due to this radial flow, a mass of fast moving liquid is transported to the outer region and an increase of forward velocity is brought about there. Consequently, the liquid in outer zone will exert an acceleration to the stratum of inner portion.

3.3 Contribution of radial flow to shearing stress

Contribution of the radial flow to shearing

stresses is pictured in Figs. 8 (a), (b), and (c), in which the distribution of strain velocity is also shown (for zero discharge). A high degree of contribution of the radial flow to τ_θ is seen but not to τ_z . The curves of shearing stress determined with regard to the radial flow exhibits a

Fig. 8 Distributions of shearing stress and velocity

similar profile to that of strain velocity. Thus, it has been verified that the radial flow contributes considerably to the stress tensor and its neglect should not be allowed, though the absolute value of the radial component of velocity exceeds hardly one hundredth that of the axial or rotational component.

3.4 Eddy kinetic viscosity

To estimate the degree of turbulence on swirling flow, authors made a calculation of eddy kinetic viscosity by the following equations

$$\frac{\varepsilon_\theta}{v_{\theta*}R} = \frac{\tau_\theta/\rho v_{\theta*}^2}{\partial(v_\theta/v_{\theta*})/\partial(r/R) - (v_\theta/v_{\theta*})/(r/R)} \quad \dots (5)$$

$$\frac{\varepsilon_z}{v_{z*}R} = \frac{\tau_z/\rho v_{z*}^2}{\partial(v_z/v_{z*})/\partial(r/R)} \quad \dots (6)$$

where ε_θ and ε_z denote the coefficients of eddy viscosity in θ - and z -direction, respectively, and ρ the density of liquid and the friction velocities are introduced as $v_{\theta*} = \sqrt{(\tau_\theta)_0/\rho}$ and $v_{z*} = \sqrt{(\tau_z)_0/\rho}$. The results are shown in Figs. 9 (a) and (b), where

the value of eddy kinetic viscosity for the flow without rotational component is also plotted. A remarkable discrepancy in two curves, especially in the central region, is seen but the complicated details of the behavior are indistinct, because the experimental procedure was not so accurate as would be required for quantitative investigation of them. However, the value of eddy kinetic viscosity near the wall seems to range from 20 to 70 times the kinetic viscosity μ/ρ due to molecular causes.

4. Conclusion

For the swirling flow created in a suction pipe of a centrifugal pump with high specific-speed, the authors evaluated the distribution of shearing stresses by using measured velocities and pressures, and compared it with strain velocity. The results may be summarized as follows: (1) Extremely high value of wall friction is experienced at the starting section of the reverse flow and with an increased distance from pump flange, firstly it decreases rapidly, then decreases gradually, representing a steeper gradient for higher pump discharge. (2) Along a radius of a pipe section, changes of the sign of τ_θ and τ_z are

Fig. 9 Eddy kinetic viscosity across pipe section

observed when a strong swirling flow is created but the radial distances at which the sign of τ_θ and τ_z changes do not fall into one point. These are noticeable characteristics of the swirling flow. (3) Although the absolute value of a radial component of flow velocity hardly exceeds one hundredth that of the axial or rotational component, it contributes considerably to the stress tensor and can not be neglected except for the region near the axis and wall of pipe. (4) For the swirling flow, the value of eddy kinetic viscosity near the wall is almost the same order as that for the non-swirling flow but its distributions across pipe section show a different profile. The value of eddy kinetic viscosity ranges from 20 to 70 times the kinetic viscosity of liquid in the neighbourhood of

wall.

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Swirling Flow in Suction Pipe of Centrifugal Pumps*

(3rd Report, Effect of Bend in Pipe)

By Mitsukiyo MURAKAMI** and Naomichi HEYA***

The swirling flow generated in an elbow suction pipe of a centrifugal pump at partial discharges was studied and the results were compared with that generated in a straight suction pipe.

Distribution of velocity may be considered to be nearly in symmetry about the pipe axis but the wall pressures measured at four taps in a section of the pipe spaced 90 deg. circumferentially show a considerable difference. The averaged value of the pressure is elevated by the centrifugal action of the swirling motion and the increase of pressure is affected considerably by the piping installation.

The swirling flow due to a back flow from the impeller can not penetrate a 90 deg.-elbow but some of it passes through a 45 deg.-elbow, resulting in a markedly complex flow picture upstream of the elbow.

1. Introduction

A swirling flow developed in a long straight suction pipe installed in a high specific-speed pump was studied in the foregoing report⁽¹⁾. The swirling motion shifts the flow from the concentric condition. In many cases of engineering practice, an elbow suction pipe is in wide use. When a suction pipe is bent near the pump flange, a

secondary flow due to the bend will change the unsymmetric conditions and a complicated nature of flow will be brought about. In the present paper, by using elbows in different piping installations, the velocity and wall pressure of flow were measured and the results were compared with those obtained with a straight suction pipe.

2. Experimental apparatus and procedure

The pump used in the test and the experimental apparatus are almost the same as those in the foregoing study⁽¹⁾, the details of them being

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