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X-Ray Investigation on Fatigue of Polycrystalline Metals*

(On the Relation between Crystal Orientation
and Deformation in fcc Crystal)

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When polycrystalline metals deform plastically, the behaviour of every grain would be affected by its orientation in regard to the direction of applied stress. The values of resolved shear stress that causes a slip would be different from grain to grain according to its orientation. Observing the diffraction plane dependence of the change in half-value breadth of X-ray diffraction line as has been reported previously by the authors, the influence of crystal orientation of grains during fatigue as well as after plastic deformation is investigated on copper. And the difference of behaviour of crystals between fatigue and plastic deformation is studied. It is found that the slip occurred in the most favourably oriented crystals in the case of fatigue, while in plastic deformation a slip was produced almost equally in all grains. These results support the fact that the Tresca's law is appropriate as the criterion of the fatigue failure under combined bending and torsional stresses. Further discussions are made on this point.

1. Introduction

A series of fatigue studies have been made by the authors, by adopting the value of half-value breadth of X-ray diffraction as the measure of fatigue damage that occurred in the material during fatigue stressing. In the studies^{(1)~(3)}, it was found that the magnitude of the change in half-value breadth of X-ray diffraction line during fatigue process was dependent on the diffraction plane⁽⁴⁾, and it has been reported that the finding is interpreted by taking account of the influence of the difference in the degree of slip in each grain according to its orientation⁽⁵⁾. Conversely,

these results give the possibility of getting an information concerning the effect of crystal orientation on the deformation from the difference of the change in half-value breadth of X-ray diffraction lines taken by X-rays of different wave lengths that are diffracted by separate atomic planes.

Metals used as strength members are generally of polycrystalline structure. The value of the resolved shear stress for slip is different in each crystal of polycrystalline aggregates, according to its orientation to the stress axis. Namely, the tendency for slip is different in each crystal according to its orientation. It is a problem of great interest how the crystal orientation influences on fatigue phenomena or plastic deformation.

Deformation and fracture of metals are studied by several ways of approach. Generally speaking, these studies may be divided into a few lines of

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studies, according to the magnitude of subdivided dimension of the material, for example

- (1) macroscopic plasticity
- (2) deformation of polycrystalline aggregates
- (3) slip in a single crystal
- (4) dislocation theory

Macroscopic studies of deformation or fracture are treated by introducing the law or criterion that governs the behaviour of materials. However, it is considered that macroscopic law or criterion must be supported by microscopic rule. In the field of mechanical engineering, studies have been made mainly from the standpoint (1), while in metal physics from (3) or (4). The subject (2) may play a role of combining these two fields of studies. Moreover, it may be subdivided in two parts:

(2a) deformation of polycrystals as a basis of macroscopic plasticity

(2b) roles of grain boundary, for instance, as a barrier to the motion of dislocations.

From the view point (2a), several theoretical attempts have been made^{(6)~(10)} to explain the work-hardening behaviour of polycrystalline aggregates from that of a single crystal of various orientations or the preferred orientation resulting from cold working, such as rolling.

Concerning fatigue, however, little investigations have been made from these points of view. The purpose of the present investigation is to study the role of crystal orientation in fatigue phenomena and also in plastic deformation, based on the above mentioned crystal orientation dependence of the change in half-value breadth of X-ray diffraction line. In this paper, as a basic study, copper was chosen among fcc crystals, because it has relatively few slip planes and slip directions, and also most of previous studies on plastic deformation of crystal aggregates are made on fcc crystal. The experimental results were discussed in relation to the fatigue strength under combined stresses.

2. Principle

2.1 Foreword

It is generally accepted that the tendency for slip in a grain can be represented by the orientation factor (Schmid factor) μ of slip planes. The resolved shear stress τ_s in slip direction on slip plane of a crystal in polycrystalline aggregates subjected to tension or torsion is expressed with the applied tensile stress σ or the torsional stress τ as

$$\tau_s = \mu_{\text{TEN}} \sigma, \quad \tau_s = \mu_{\text{TOR}} \tau \dots\dots\dots (1)$$

where μ_{TEN} and μ_{TOR} are the orientation factors in tension and in torsion, respectively. A slip plane that has a large value of τ_s would be easier to slip than the other planes. That is to say, the grain which contains a slip plane that has a large value of orientation factor would more tend to slip than the grains which have the planes of smaller orientation factor.

X-ray diffraction is highly sensitive to the change of minute variation of crystal planes that reflect X-ray. Let us consider that an X-ray pattern is taken from the surface of polycrystalline metals by a collimated beam of X-ray of a wave length. If the X-ray spot of irradiation on the surface includes many of slipped crystals, the X-ray pattern obtained would be broadened. In other words, the broadness of X-ray diffraction lines, taken from polycrystalline metallic materials that include plastic deformation of crystals by whatever cause, would depend on the geometry of diffraction. A diffraction of X-ray is characteristic to the wave length of X-ray and the indices of atomic planes, as is defined by the Bragg's equation. The direction of a crystal plane which diffracts X-ray is confirmed from the geometry of X-ray apparatus used for the diffraction. Namely, the diffraction line is obtained from those grains that have the crystal plane in the direction decided from the geometry of diffraction.

Consequently, in order to know the relation between crystal orientation and deformation, it is necessary to measure the broadening of the diffraction line from the grains which has a known value of orientation factor μ . In this connection the relation between the broadening of diffraction line, as a measure of degree of deformations, and the analytical value of the orientation factor is studied. For practical approach in this line of study, the following two methods are conceivable.

(1) To determine the orientation of a grain in polycrystalline aggregates from the Laue pattern and measure the diffuseness of the Laue spots. According to the character of the Laue spots, however, it is generally rather difficult to make a quantitative discussion on the broadness of the spots.

(2) To measure the broadening of the Debye profile, which comes from grains with a certain relationship of orientation according to the diffracting condition. Although the direction of the diffraction plane is confirmed by the geometry of diffraction, in this case, the orientation of the grain which contains the diffraction plane is variable around the normal of the diffracting plane. That is, according to the direction of stress axis.

in the grain which contains the diffraction plane, the value of orientation factor of an atomic plane changes. Thus, only the mean value of orientation factor μ of the diffracting grains can be discussed in relation to the change in half-value breadth of the Debye profile.

Although these two methods are considered to have individual merits, the latter method is used because of convenience of experimentation. For the purpose of studying the difference in the broadening of the diffraction line according to the orientation factor of the grains, it is necessary to choose an appropriate condition to get diffraction lines from grains with various values of μ . Two different methods are available for this purpose.

(2a) To get diffraction lines from planes with different Miller's indices (hkl), employing several sorts of characteristic X-ray of different wave lengths. In this case, experimentation has to be made so as to minimize the influence of geometry of diffraction, in order to extract the influence of difference in slip amount occurring in crystals of different orientations from the change in half-value breadth. For this purpose, usually diffractometer is employed⁽⁴⁾, where the atomic planes that reflect X-ray are kept parallel to the surface of materials for any combination of Miller's indices of atomic planes and wave length of X-ray. In this method, however, the influences other than orientation factor must be considered in the change in half-value breadth, for the Bragg angle θ is different in each reflection. That is, the broadening due to microstrain which is proportional to $\tan \theta$ and the broadening owing to the effective size of crystallite (subgrain) which is proportional to $1/\cos \theta$ ⁽¹¹⁾ can not be excluded. Then, it is necessary to separate their contributions by some other method.

(2b) To get diffraction lines of atomic planes of particular Miller's indices with different inclinations to the specimen surface. In this case, X-ray patterns are taken by collimated beam of a characteristic ray of various inclinations to the surface of specimen. In this process, however, an adequate correction has to be made for the geo-

metrical broadening of the diffraction line, for both irradiated area on the surface and reflecting depth in the specimen change with the inclination of X-ray beam to the specimen surface.

Method (2b) was adopted in this study. In order to exclude the effect of geometrical broadening, the change of the broadening of diffraction lines was presented by taking specific value, as will be mentioned afterwards.

2.2 Orientation factor (analytical consideration)

Now, it is necessary to get the analytical mean value of the orientation factor $\bar{\mu}$ of the diffracting crystals in practice. In the previous paper⁽⁵⁾, the authors reported on the graphical method for this purpose, adopting the stereographic projection of crystal. Let's consider an atomic plane (hkl) with an angle ψ to the specimen surface, as shown in Fig. 1. If the crystal orientation distributes uniformly in the polycrystalline aggregates, the axial directions of the diffracting crystals lie nearly uniformly on a circle on the stereographic projection, as shown in Fig. 2⁽¹²⁾. If we take the diffraction on (400) planes of copper, which is obtained with the characteristic X-rays of cobalt target, the circles of axial directions of crystals in the

Fig. 2 Possible directions of specimen axis when (hkl) plane are reflecting

Fig. 3 Possible directions of specimen axis when diffraction plane (400) is lying with inclination ψ to specimen surface

Fig. 1 Direction of diffraction plane

triangle of (011) standard stereographic projection becomes such as shown in Fig. 3, corresponding to several inclination angles ψ to the specimen surface.

On the other hand, the relation between the crystal orientation and the orientation factor μ can also be shown on the stereographic projection, in the form of μ -constant lines. Sachs⁽⁶⁾ calculated the distribution of m of fcc crystal (where $m=1/\mu$). The cases of tension and torsion are shown in Fig. 4 (a) and (b), respectively. The distribution of orientation factor corresponding to the possible directions of crystal axis can be obtained from taking the value of m along the curves in Fig. 3, for each value of ψ . Examples of such distribution for $\psi=20^\circ$ are shown in Fig. 5 in both cases of tension and torsion. The mean value $\bar{m}$ for the possible directions of crystal axis is obtained by averaging these distributions of m graphically. The results are shown in Table 1, where $\bar{\mu}$ is defined as $\bar{\mu}=1/\bar{m}$. In the table $\bar{\mu}$ is the mean value of orientation factor in all directions.

As is shown in the right column of Table 1, the specific value of

$$(\bar{\mu}/\bar{\mu})_{\text{Tension}} - (\bar{\mu}/\bar{\mu})_{\text{Torsion}} \quad \dots\dots\dots (2)$$

is different for each inclination angle ψ . That is to say, it is possible to know the effect of orien-

Table 1 Relation between inclination angle ψ and orientation factor μ , where (400) plane is diffracting

ψ	Tension		Torsion		$(\frac{\bar{\mu}}{\bar{\mu}})_{\text{Tension}} - (\frac{\bar{\mu}}{\bar{\mu}})_{\text{Torsion}}$
	$\bar{\mu}$	$\bar{\mu}/\bar{\mu}$	$\bar{\mu}$	$\bar{\mu}/\bar{\mu}$	
0°	0.46	1.04	0.69	0.89	0.15
10°	0.47	1.06	0.71	0.92	0.14
20°	0.46	1.03	0.78	1.01	0.02
30°	0.42	0.94	0.85	1.10	-0.16
40°	0.44	0.98	0.86	1.11	-0.13
	$\bar{\mu}=0.45$		$\bar{\mu}=0.77$

Fig. 4 Distribution of m on stereographic triangle⁽⁶⁾ (where $m=1/\mu$)

tation factor utilizing this difference.

2.3 Half-value breadth

As mentioned before, it is generally accepted that the change in the broadening of X-ray diffraction line is caused by subgrain formation or microstrain in the crystal⁽¹¹⁾. Namely, it represents the degree of disorder of crystal and is associated with the increase in dislocation density⁽¹³⁾.

It is said that the profile of the diffraction line from the specimen results from the superposition of the effect of the initial X-ray source, that of the geometry of diffraction apparatus and also of the minute distortion of the atomic planes of the material. Convolution integral⁽¹¹⁾ is generally used to get the true broadening β' corresponding to the deformation of crystal from experimental data, which include both the broadening B peculiar to the X-ray apparatus and the geometrical broadening β_ψ , too. It is also supported that the intensity distribution curve of the diffraction line is approximately expressed with the Cauchy distribution function. If the diffraction intensity curve is assumed to be of the Cauchy distribution type, the convolution integral is simplified as summation form of the breadth of the curve⁽¹¹⁾,

$$b_\psi = B_\psi + \beta' = B_0 + \beta_\psi + \beta' \quad \dots\dots\dots (3)$$

where, b_ψ and B_ψ are the half-value breadths of diffraction lines from inclined planes, after and before deformation, respectively, and B_0 corresponds to the case of $\psi=0$. B_0 and β_ψ are independent of the state of applied stress.

Now, to keep a correlation with the previous study⁽²⁾, β will be defined as

$$\beta = (b_\psi - B_\psi)/B_0 \quad \dots\dots\dots (4)$$

Let β_{Tension} and β_{Torsion} be the value of half-value breadths under tensile and torsional deformation,

Fig. 5 Distribution of m along possible direction of specimen axis

respectively, then from Eq. (4) we get

$$\beta_{\text{Ten}} - \beta_{\text{Tor}} = \frac{(b_{\phi})_{\text{Ten}} - (b_{\phi})_{\text{Tor}}}{B_0} \quad (5)$$

The value of the right side can be obtained experimentally. Consequently, the effect of the inclination β_{ϕ} can be excluded approximately by taking the difference in broadening between tension and torsion.

2.4 Summary

From the relation between the mean value of orientation factor [Eq. (2)] and the change in half-value breadth [Eq. (5)], it is possible to investigate the effect of crystal orientation on its deformation.

3. Experimental procedure

3.1 Specimens and test conditions

The chemical compositions of the material are listed in Table 2. The specimens were machined and then annealed at 500°C in vacuum for 1 hr. The surface of the specimen was finished with bright dip. The mean diameter of the grains was about 0.03 mm. The shapes and sizes of the specimen are shown in Fig. 6. The fatigue tests were carried out with both the Ono's type rotating

Table 2 Chemical compositions (%)

Cu	Sn	Pb	Ni	Fe	P	O ₂
99.896	0.018	0.008	0.003	0.001	0	0.043

bending fatigue testing machine and the Nishihara's type torsional fatigue testing machine. The S-N curve of bending fatigue test is shown in Fig. 7.

According to the experimental facts reported in the literature⁽²¹⁾⁽²⁴⁾, the magnitude of applied stress was chosen according to the Tresca's law in fatigue, while in plastic deformation it was determined by the von Mises' law. In the latter case, the equivalent stress σ^* and the equivalent strain ϵ^* are defined as⁽¹⁴⁾

$$\sigma^* = \sqrt{\sigma^2 + 3\tau^2}, \quad \epsilon^* = \sqrt{\epsilon^2 + \frac{1}{3}\gamma^2}$$

The condition of stress, however, is not essential here, because only the difference due to the inclination of diffraction plane is called to account.

3.2 X-ray measurements

The specimen was mounted axially on the specimen holder attached to the back reflection X-ray camera, and was rotated with a constant speed during the irradiation. Thus, the diffraction line was obtained on the film. In order to get the diffraction line from the plane with an inclination angle ϕ to the specimen surface, the specimen was tilted by the angle $\phi + \eta$ to the incident X-ray beams (where $\eta = 90 - \theta$; θ is the Bragg angle). The experimental conditions for X-ray are shown in Table 3. To reduce the sources of error, the films for tension and torsion were developed at the same time under the same condition.

From the film obtained, diffraction intensity curve was drawn with an automatic type microphotometer. As the half-value breadth, the integral breadth $b = S/(h_1 + h_2)$ was used, where S is the area surrounded by the diffraction curve, and h_1 and h_2 are the peak heights of $K\alpha_1$ and $K\alpha_2$ -lines, respectively.

4. Experimental results

The change in half-value breadth during bending fatigue process was measured as a basis. The result is shown in Fig. 8. The trend of the change was similar to that of low carbon steel⁽²⁾, namely, the half-value breadth increased rapidly at the initial stage and then gradually. Meanwhile, the

Table 3 Conditions of X-ray measurements

X-ray	CoK α
Diffraction plane	Cu (400)
Tube voltage	35 kV
Tube current	20 mA
Slit	1 mm double pin hole
Filter	Fe foil
Specimen to film	70 mm
Exposure time	60 min
Film	Fuji 100-type

changes in half-value breadth under plastic tension and torsion are shown in Fig. 9, where the stress-strain curve is also given.

On the basis of these results, the relations between μ and β were investigated for both cases of fatigue and plastic deformation, in order to get an information on the deformation of polycrystals. Fig. 10 shows the case of fatigue under stresses $\sigma=10 \text{ kg/mm}^2$ and $\tau=5 \text{ kg/mm}^2$ at the stress cycles $N=2 \times 10^5$. The ordinate is $(\beta_{\text{Ten}} - \beta_{\text{Tor}})$ cor-

Fig. 8 Change in half-value breadth during fatigue process

Fig. 9 Change in half-value breadth in plastic deformation

Fig. 10 Relation between change in half-value breadth and orientation factor

responding to Eq. (5), and the abscissa is the difference in the orientation factor given in Eq. (2). The experiment was performed on three stages of inclination angle $\psi=0^\circ, 20^\circ, 30^\circ$ (cf. Fig. 1). In the figure, standard errors of the half-value breadth in several measurements are also shown, where the scattering mainly arises from the measuring position on the film. Fig. 10 shows that in the case of fatigue the broadening β is larger in the grains with greater values of μ . In other words, it is likely that considerable amount of slips occurred in the preferentially oriented grains, while slips were less in the grains with small μ -values.

Fig. 11 shows the case just before fatigue fracture, and Fig. 12 is the case of lower stress levels $\sigma=7 \text{ kg/mm}^2$ and $\tau=3.5 \text{ kg/mm}^2$ at $N=3 \times 10^6$. In both figures an inclined linear relation is observed as well.

Now, Figs. 13 and 14 are the results of plastic deformation under the equivalent strain $\epsilon^*=0.5\%$ and $\epsilon^*=1.9\%$, respectively. In these cases the test results lie nearly on a horizontal line, which is in contrast with the above results on fatigue. From these data, it is suggested that in plastic deformation crystals are deformed nearly to the same degree, irrespective of their orientations. It is interesting that the orientation dependence

Fig. 11 Relation between change in half-value breadth and orientation factor

Fig. 12 Relation between change in half-value breadth and orientation factor

of crystal deformation is different in fatigue and in plastic deformation.

5. Discussion

5.1 Plastic deformation

Figs. 13 and 14 show that the change in half-value breadth is nearly constant irrespective of crystal orientation, indicating that the deformation is nearly uniform through all grains.

It was recognized from microscopic observation⁽¹⁵⁾ that when polycrystalline copper was deformed, the effect of grain boundary appeared above 0.1% strain and then near 1% strain multiple slips occurred. Hence, under the strains used in this investigation (0.5% and 1.9%) it might be expected that slip had occurred in all grains, but the distortion of the crystal was not so heavy. So, it is considered that the above result was obtained corresponding to the strain range adopted in this investigation.

In a strain range larger than that used in this study, ($\epsilon^* \approx 4\%$), it was found⁽¹⁶⁾ that the change in half-value breadth was dependent on crystal orientation, in contrast to the result of the present investigation. It was suggested that in the strain range the changes in half-value breadth could be interpreted more properly in terms of the orienta-

tion dependence of work-hardening rate ($d\sigma/d\epsilon$) of corresponding single crystal, than orientation factor μ . In other words, the results⁽¹⁶⁾ seemed to show that the orientation dependence of the work-hardening rate of crystals in polycrystalline aggregates was similar to that of single crystals⁽²⁵⁾.

5.2 Fatigue

From Figs. 10 to 12, it is considered that, during fatigue, many slips occurred in the most favourably oriented crystals. This is in accordance with the fact as shown by microscopic observation that heavy slip lines appeared in selected grains⁽¹⁷⁾.

Recent studies⁽⁸⁾⁽¹⁸⁾⁽¹⁹⁾ suggest the importance of the studies on the growing process of fatigue crack, including microcrack. The microcrack, as the origin of macroscopic crack, is considered to be nucleated in a grain during stress cycling.

Futhermore, it is often observed that the fatigue microcrack is produced along the slip plane. Then, it may be reasonable to consider that the fatigue strength is determined by whether or not the microcrack is nucleated and grows by slip in the most favourably oriented grain.

The situations are clarified by taking a model of crystal deformation into account. It will be assumed, according to Sachs⁽⁶⁾, that only one slip system is operative in a grain during deformation and the critical resolved shear stress is the same in all grains. Then, as a typical case two models are conceivable as are shown in Fig. 15 (a) and (b). In the case (b) slip occurs in the most preferentially oriented grain, while in (a) it appears in all grains, for the strain is restricted with one another.

It seems that there is some similarity between these models and the spatial models used in the probability theory⁽²⁰⁾. In the latter, the models (a) and (b) are called the bundle model and the weakest link model, respectively. Such correspondence might show some relation with the statistical

Fig. 13 Relation between change in half-value breadth and orientation factor

Fig. 14 Relation between change in half-value breadth and orientation factor

Fig. 15 Models of deformation of polycrystalline aggregates

distribution of fatigue life.

5.3 Correlation with the fatigue under combined stresses

As is shown in Fig. 16, it was reported by Nishihara and Kawamoto⁽²¹⁾ that the fatigue strength of fcc metals under combined bending and torsional stresses is in good agreement with the Tresca's law⁽²²⁾,

$$\sqrt{\sigma^2 + 4\tau^2} = \sigma_0 \quad \dots\dots\dots (6)$$

(where σ_0 is constant). The same tendency is also shown in the arrangement of fatigue data by Forrest⁽²³⁾.

On the other hand, as reported by Taylor et al.⁽²⁴⁾, the experimental result on yield stress of fcc metals under combined stresses is in accordance with the von Mises' law⁽²²⁾ (Fig. 16),

$$\sqrt{\sigma^2 + 3\tau^2} = \sigma_0 \quad \dots\dots\dots (7)$$

(In the plasticity theory, as is well known, the von Mises' law is interpreted as equivalent to the shear strain energy theory or the octahedral shear stress theory.)

Now, as has been pointed out by Sachs⁽⁶⁾, the yield condition can be interpreted from the slip in crystals in polycrystalline aggregates. That is, from Eq. (1) the ratio of torsional stress to tensile stress can be obtained, assuming that the critical resolved shear stress is equal in all grains⁽⁶⁾. Since the deformation was almost uniform in all grains in plastic deformation (Figs. 13, 14), the ratio is given by

$$\frac{\tau}{\sigma} = \frac{\bar{\mu}_{\text{Ten}}}{\bar{\mu}_{\text{Tor}}} = 0.58 \quad \dots\dots\dots (8)$$

corresponding to the model of Fig. 15 (a) and taking the mean value $\bar{\mu}$ of orientation factor of all grains. This value is in agreement with that obtained from the von Mises' law [Eq. (7)].

On the other hand, in fatigue, the most favourably oriented grain has an important role (Figs. 10~12). Considering an ideal case of Fig. 15 (b), (namely, supposing that the fatigue limit is determined by the behaviour of the most favourably

oriented crystals) and taking the maximum values μ_{max} of orientation factor for tension and torsion (Fig. 4), we obtain

$$\frac{\tau}{\sigma} = \frac{(\mu_{\text{Ten}})_{\text{max}}}{(\mu_{\text{Tor}})_{\text{max}}} = 0.50 \quad \dots\dots\dots (9)$$

This value is in agreement with that obtained from the Tresca's law [Eq. (6)]. Thus, it is concluded that the fatigue limit and the yield strength under combined stresses are well explained in relation to the modes of slip in the grains in polycrystalline aggregates during fatigue and plastic deformation, as are shown schematically in Fig. 15.

Conversely speaking, it may be suggested that the assumption of the importance of microcrack is appropriate, and the series model of grains [Fig. 15 (b)] is valid in fatigue of polycrystalline aggregates. It may be remarked here incidentally that the Tresca's law is more representative in fatigue of fcc metals under combined bending and torsional stresses.

It was found by the authors that in the range of the applied stress higher than that of present study, the difference in work-hardening rate due to crystal orientation appeared in the case of fatigue⁽¹⁶⁾, similarly to the case of plastic deformation of relatively large strain range mentioned above. According to Wood et al.⁽¹⁸⁾, the S-N curve of fcc metal is divided into a few stages according to the stress amplitude. Therefore, it may be supposed that the data of the present study correspond to the F-region of S-N curve of fcc metals named by Wood et al., while the higher stress range in fatigue of our previous study⁽¹⁶⁾ is the case of the H-region.

6. Conclusions

The deformations of polycrystalline aggregates during fatigue and plastic deformation were studied, with emphasis on the relation between the half-value breadth of X-ray diffraction line and the orientation factor of diffracting crystals, and the following conclusions were obtained.

(1) In the fatigue of polycrystalline aggregates under a relatively low stress, slips seem to occur preferentially in the most favourably oriented grains.

(2) On the contrary, in plastic deformation ($\epsilon^* \approx 1\%$) deformation occurs nearly in the same amount in all grains irrespective of their orientations, which is possibly owing to the mutual restriction of grains. The difference in the mode of deformation between fatigue and plastic deformation is particularly noteworthy.

(3) It is suggested that the difference in

Fig. 16 Fatigue limit and plastic yield under combined stress (fcc metal)

the mode of deformation corresponds with the hypotheses that the Tresca's law is appropriate in the fatigue of fcc crystals, while in plastic deformation the von Mises' law is applicable.

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