621.822.033

Analysis of an Externally Pressurized Thrust Collar Gas-Bearing with Multiple Supply Holes*

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An externally pressurized thrust gas-bearing with multiple supply holes, which has no recess, is investigated by use of the complex potential theory. Using a series of complex potential functions, which satisfies the boundary conditions, the load capacity of this bearing can be calculated theoretically, and the volume rate of flow as well.

Experimental pressure distributions coincide very well with theoretical ones especially for the case with low supply pressure and small bearing clearance. Experiments are also made to investigate theoretically the load capacity, yielding good agreement between experiment and theory, with consideration of secondary effects due to the compressibility of the fluid and pressure loss for the acceleration of the fluid at supply holes and for the change of the flow velocity profiles, whose effects are estimated by assuming a flow pattern.

1. Introduction

In the practical designs of externally pressurized bearings, it is most desirable to calculate the exact pressure distribution in the bearing clearance in order to determine the load capacity of the bearing by integrating the pressure over the bearing area, and to make the quantity of flow clear from the pressure gradient which leads to the determination of the dimensions of an external restrictor such as a capillary or an orifice.

The complex potential theory is one of the most applicable methods for obtaining "point-source" solution of Reynolds equation, and is applied to the journal bearing⁽¹⁾ and the thrust bearings of some bearing configurations⁽²⁾⁽³⁾. The authors had applied the complex potential theory for the analysis of the externally pressurized thrust collar gas-bearing in the previous paper⁽⁴⁾. The bearing had multiple supply holes spaced circumferentially. The supply holes had no recess because it was apt to make the operation less stable due to the self-excited vibration for the gas-bearings. A suitable complex potential function had been found for this type of bearing, yielding theoretical pressure distribution, which was, however, hard to be integrated analytically with respect to the bearing surface in order to obtain

the load capacity. In order to overcome this difficulty, the complex potential function is expanded in the finite series in this paper, so that the pressure may be integrated term by term for the load capacity.

The theoretical results of pressure distribution and load capacity are investigated experimentally with consideration of the compressibility and inertia effect of the fluid.

2. Theoretical analysis

2.1 Fundamental equations

The pressure distribution in the clearance of externally pressurized bearing is governed by Reynolds equation, which reduces to Laplace equation (1) under the assumptions of parallel bearing surface, no relative surface motion and constant viscosity of the fluid⁽¹⁾,

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = 0 \quad \dots\dots\dots (1)$$

where

$$\left. \begin{aligned} P &= p && \text{(for an incompressible fluid)} \\ &= p^{(1+n)/n} && \text{(for a compressible fluid)} \end{aligned} \right\} \dots\dots (2)$$

in which

p : absolute pressure in the bearing clearance

n : polytropic index

x, y : coordinates

Therefore, the problem of obtaining the pressure distribution of the particular bearing configuration is reduced to the problem of finding an appropriate complex potential function of complex number Z , which satisfies the boundary conditions, where

* Received 2nd June, 1965.

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$$Z = x + iy = re^{i\theta}$$

If a complex potential function is obtained, the real part of it gives the form of the pressure distribution for an incompressible fluid case, because the incompressible solution p [which is P in Eq. (2)] satisfies Eq. (1).

For the first step, an externally pressurized thrust collar bearing with one supply hole as shown in Fig. 1 is analyzed; the bearing with multiple supply holes as shown later in Fig. 2 is investigated by an appropriate superposition of the results.

The suitable complex potential function was obtained in the previous paper⁽⁴⁾ as follows,

$$w(Z) = \frac{q}{2\pi} \left[\log \sin \frac{\pi \log Z/a}{2 \log r_0/r_1} - \log \sin \frac{\pi \log aZ/r_0^2}{2 \log r_0/r_1} \right] \quad (3)$$

where

q : strength of source (corresponding to the flow rate), a : radial position of supply hole
 r_1 : outer radius of bearing, r_0 : inner radius of bearing

The pressure distribution is obtained from the real part of Eq. (3) as

$$P = K_1 \{ \log (\sin^2 A' + \sinh^2 B) - \log (\sin^2 A + \sinh^2 B) \} + p_a \quad (4)$$

$$A' = A'(r) = \frac{\pi \log ar/r_0^2}{2 \log r_0/r_1}, \quad A = A(r) = \frac{\pi \log r/a}{2 \log r_0/r_1}, \quad B = \frac{\pi \theta}{2 \log r_0/r_1}$$

$$K_1 = (p_0 - p_a)/2 \log (\sin A_s'/\sin A_s), \quad A_s' = A'(r_s), \quad A_s = A(r_s) \quad (5)$$

in which

r_s : radius of supply hole, p_0 : pressure just after supply hole
 p_a : ambient pressure

2.2 Potential function in series

At the first step, a single point source of strength q is put at $Z=a$, where Z represents a complex plane containing the bearing surface. The complex potential function is

$$w_1(Z) = \frac{q}{2\pi} \log \left(1 - \frac{Z}{a} \right) \quad (6)$$

This point source function cannot satisfy the boundary conditions, or the potential cannot be constant at bearing peripheries C_0 and C_1 . For the first correction of potential, two point sinks of strength $-q$ are put; one is at the inverse point of a with C_0 ($Z=r_0^2/a$) in order to cancel the potential $w_1(Z)$ and to make constant, the other at $Z=r_1^2/a$ in order to cancel $w_1(Z)$ on the periphery C_1 , resulting in

$$w_{20}(Z) = -\frac{q}{2\pi} \log \left(1 - \frac{aZ}{r_0^2} \right) \quad (7)$$

$$w_{21}(Z) = -\frac{q}{2\pi} \log \left(1 - \frac{aZ}{r_1^2} \right) = -\frac{q}{2\pi} \log \left[1 - \left(\frac{r_1}{r_0} \right)^{-2} \left(\frac{aZ}{r_0^2} \right) \right] \quad (8)$$

respectively.

The $w_{20}(Z)$ compensates the function $w_1(Z)$ on C_0 , but its effect on C_1 is not considered; in order to cancel $w_{20}(Z)$ on C_1 , the following potential must be added

$$w_{31}(Z) = \frac{q}{2\pi} \log \left(1 - \frac{r_0^2 Z}{ar_1^2} \right) = \frac{q}{2\pi} \log \left[1 - \left(\frac{r_1}{r_0} \right)^{-2} \left(\frac{Z}{a} \right) \right] \quad (9)$$

Similarly in order to cancel $w_{21}(Z)$ on C_0 , a source must be added

$$w_{30}(Z) = \frac{q}{2\pi} \log \left(1 - \frac{r_1^2 Z}{ar_0^2} \right) = \frac{q}{2\pi} \log \left[1 - \left(\frac{r_1}{r_0} \right)^2 \left(\frac{Z}{a} \right) \right] \quad (10)$$

An array of infinite sources and sinks must be put according to the similar reduction so that the potential may be canceled by each other to be constant on the bearing peripheries C_0 and C_1 , that is

$$w_{40}(Z) = -\frac{q}{2\pi} \log \left(1 - \frac{ar_1^2 Z}{r_0^4} \right) = -\frac{q}{2\pi} \log \left[1 - \left(\frac{r_1}{r_0} \right)^2 \left(\frac{aZ}{r_0^2} \right) \right] \quad (11)$$

$$w_{41}(Z) = -\frac{q}{2\pi} \log \left(1 - \frac{ar_0^2 Z}{r_1^4} \right) = -\frac{q}{2\pi} \log \left[1 - \left(\frac{r_1}{r_0} \right)^{-4} \left(\frac{aZ}{r_0^2} \right) \right] \quad (12)$$

and so forth; $w_m(Z)$ is given as summation of these sources and sinks, namely

Fig. 1 Configuration of thrust collar bearing with single supply hole

$$w_m(Z) = w_1(Z) + w_{20}(Z) + w_{21}(Z) + w_{30}(Z) + \dots + w_{2(m+1)0}(Z) \\ = \frac{q}{2\pi} \sum_{j=-m}^m \log \left[1 - \left(\frac{r_1}{r_0} \right)^{2j} \left(\frac{Z}{a} \right) \right] - \frac{q}{2\pi} \sum_{j=-m}^m \log \left[1 - \left(\frac{r_1}{r_0} \right)^{2j} \left(\frac{aZ}{r_0^2} \right) \right] \quad (13)$$

When m tends to infinity, one can obtain the potential function which may be constant on both C_0 and C_1 .

Complementary solution of Eq. (1) may be added by the boundary condition that the pressure at the bearing edges is p_a (ambient pressure):

$$w_0(Z) = A_1 \log Z + A_2 \quad (14)$$

Then, potential function $w(Z)$ is obtained as

$$w(Z) = \lim_{m \rightarrow \infty} [w_m(Z) + w_0(Z)] \quad (15)$$

Eq. (15) may be considered as an expansion of Eq. (3) in series, and is used for the calculation of load capacity instead of Eq. (3).

The real part of function $w_m(Z)$ is

$$\Re\{w_m\} = \frac{q}{4\pi} \sum_{j=-m}^m \log \frac{1 - 2 \left(\frac{r_1}{r_0} \right)^{2j} \left(\frac{r}{a} \right) \cos \theta + \left(\frac{r_1}{r_0} \right)^{4j} \left(\frac{r}{a} \right)^2}{1 - 2 \left(\frac{r_1}{r_0} \right)^{2j} \left(\frac{ar}{r_0^2} \right) \cos \theta + \left(\frac{r_1}{r_0} \right)^{4j} \left(\frac{ar}{r_0^2} \right)^2} \quad (16)$$

Substituting $r=r_0$, one can obtain the potential on bearing periphery C_0 as

$$\Re\{w_m\}_{r=r_0} = \frac{q}{4\pi} \sum_{j=-m}^m \log \left(\frac{r_1}{r_0} \right)^{4j} \left(\frac{r_0}{a} \right)^2 \quad (17)$$

The potential is constant everywhere on C_0 because this equation is independent on θ .

On the other hand, by substituting $r=r_1$ into Eq. (16) the potential on C_1 becomes

$$\Re\{w_m\}_{r=r_1} = \frac{q}{4\pi} \sum_{j=-m}^m \log \left(\frac{r_1}{r_0} \right)^{4j} \left(\frac{r_1}{a} \right)^2 + \frac{q}{4\pi} \log \frac{1 - 2 \left(\frac{r_1}{r_0} \right)^{2m} \left(\frac{r_1}{a} \right) \cos \theta + \left(\frac{r_1}{r_0} \right)^{4m} \left(\frac{r_1}{a} \right)^2}{1 - 2 \left(\frac{r_1}{r_0} \right)^{2m} \left(\frac{ar_1}{r_0^2} \right) \cos \theta + \left(\frac{r_1}{r_0} \right)^{4m} \left(\frac{ar_1}{r_0^2} \right)^2} \quad (18)$$

The first term is constant independently of θ , and the potential deviates from the real bearing boundary condition by the second term, which contains the variable θ . But the deviation becomes negligible for a sufficiently large number of m because $(r_1/r_0)^{2m}$ can be neglected compared with $(r_1/r_0)^{4m}$.

Now, the complementary solution $w_0(Z)$ is added to make the potential match the ambient condition both on inner and outer bearing peripheries. Then the real part of $w_0(Z)$, $A_1 \log r + A_2$, should satisfy the following conditions:

$$A_1 \log r_0 + A_2 + w_m^0 = 0, \quad A_1 \log r_1 + A_2 + w_m^1 = 0 \quad (19)$$

where

$$w_m^0 = \Re\{w_m\}_{r=r_0} = \frac{q}{2\pi} (2m+1) \log \left(\frac{r_0}{a} \right) \\ w_m^1 = \Re\{w_m\}_{r=r_1, \theta=0} = \frac{mq}{\pi} \log \left(\frac{r_0}{a} \right) + \frac{q}{2\pi} \log \frac{1 - \left(\frac{r_1}{r_0} \right)^{2m} \left(\frac{r_1}{a} \right)}{1 - \left(\frac{r_1}{r_0} \right)^{2m} \left(\frac{ar_1}{r_0^2} \right)} \quad (20)$$

For a sufficiently large number of m , w_m^1 becomes

$$w_m^1 \cong \frac{q}{2\pi} (m+1) \log \frac{r_0}{a} \quad (20)'$$

Obtaining A_1 and A_2 from these equations, the complementary solution becomes

$$\Re\{w_0\} = A_1 \log r + A_2 = \frac{q}{2\pi} \frac{\log a/r_0}{\log r_1/r_0} \log \frac{rr_1^{2m+1}}{r_0^{2m+2}} \quad (21)$$

By using above results, the pressure distribution is given by the following equation, when the ambient pressure is p_a :

$$p = K_1 \lim_{m \rightarrow \infty} \left\{ \sum_{j=-m}^m \log \frac{1 - 2R_0^{2j} (R/R_a) \cos \theta + R_0^{4j} (R/R_a)^2}{1 - 2R_0^{2j} (R_a R/R_0) \cos \theta + R_0^{4j} (R_a R/R_0)^2} - \frac{2 \log (R_a/R_0)}{\log R_0} \log (R/R_0^{2m+2}) \right\} + p_a \quad (22)$$

where

$$R = r/r_1, \quad R_0 = r_0/r_1, \quad R_a = a/r_1 \quad (23)$$

When the boundary condition of

$$p = p_0 \quad \text{at} \quad r = a - r_s, \quad \theta = 0 \quad (24)$$

is applied, K_1 has the same value as Eq. (5). The strength q has the following relation with K_1 ,

$$K_1 = -\frac{q}{4\pi} \quad \dots\dots\dots (25)$$

2.3 Load capacity (incompressible fluid case)

Now consider the following potential function which represents a single source at $Z=\bar{a}$,

$$\bar{w} = \frac{q}{2\pi} \log \left(1 - \frac{Z}{\bar{a}} \right) \quad \dots\dots\dots (26)$$

The potential of this source is denoted as $\bar{p}$,

$$\bar{p} = \Re\{\bar{w}\} = \frac{q}{4\pi} \log \left\{ \left(\frac{r}{\bar{a}} \right)^2 - 2 \left(\frac{r}{\bar{a}} \right) \cos \theta + 1 \right\} \quad \dots\dots\dots (27)$$

Then, $\bar{p}$ is integrated on the annular region of inner radius r_0 and outer radius r_1 , that is

$$\begin{aligned} \bar{W} &= \int_{r_0}^{r_1} \int_0^{2\pi} \bar{p} r d\theta dr \\ &= \frac{q}{4\pi} \int_{r_0}^{r_1} r dr \int_0^{2\pi} \log \left\{ \left(\frac{r}{\bar{a}} \right)^2 - 2 \left(\frac{r}{\bar{a}} \right) \cos \theta + 1 \right\} d\theta \quad \dots\dots\dots (28) \end{aligned}$$

The integral with respect to θ is given as⁽⁵⁾

$$\left. \begin{aligned} \int_0^{2\pi} \log \left\{ \left(\frac{r}{\bar{a}} \right)^2 - 2 \left(\frac{r}{\bar{a}} \right) \cos \theta + 1 \right\} d\theta &= 0 & (r \leq \bar{a}) \\ &= 4\pi \log r/\bar{a} & (r > \bar{a}) \end{aligned} \right\} \quad \dots\dots\dots (29)$$

Hence, for $\bar{a} < r_0$

$$\left. \begin{aligned} \bar{W} &= q \int_{r_0}^{r_1} r \log \frac{r}{\bar{a}} dr \\ &= q \left\{ \frac{r_1^2}{2} \log \frac{r_1}{\bar{a}} - \frac{r_0^2}{2} \log \frac{r_0}{\bar{a}} - \frac{1}{4} (r_1^2 - r_0^2) \right\} \end{aligned} \right\} \quad \dots\dots\dots (30)$$

For $r_0 < \bar{a} < r_1$

$$\left. \begin{aligned} \bar{W} &= q \int_{\bar{a}}^{r_1} r \log \frac{r}{\bar{a}} dr \\ &= q \left\{ \frac{r_1^2}{2} \log \frac{r_1}{\bar{a}} - \frac{1}{4} (r_1^2 - \bar{a}^2) \right\} \end{aligned} \right\}$$

For $r_1 < \bar{a}$

$$\bar{W} = 0$$

The integral of complementary part is

$$\begin{aligned} \bar{W}_c &= \int_{r_0}^{r_1} 2\pi r (A_1 \log r + A_2) dr \\ &= 2\pi \left[A_1 \left(\frac{r_1^2}{2} \log r_1 - \frac{r_0^2}{2} \log r_0 \right) - \left(\frac{A_1}{2} - \frac{A_2}{2} \right) (r_1^2 - r_0^2) \right] \quad \dots\dots\dots (31) \end{aligned}$$

According to the arrangement of sinks and sources of Eq. (15), the results of Eq. (30) are summed up to yield the load capacity as follows:

$$W' = -\frac{q}{4} \left\{ (r_1^2 - r_0^2) \frac{\log a/r_0}{\log r_1/r_0} - (a^2 - r_0^2) \right\} \quad \dots\dots\dots (32)$$

The pressure distribution is truncated at the supply hole so that its effect must be subtracted from the load capacity of Eq. (32). The pressure distribution of truncated part can be approximated by

$$p = \frac{q}{4\pi} \log \frac{r}{r_s} + p_0 \quad \dots\dots\dots (33)$$

where the boundary condition of $p=p_0$ at $r=r_s$ is applied, and it is assumed that the coordinates are taken for the supply hole locate at the origin. Then, the effect of truncated part is

$$W_{tr} = \int_0^{r_s} 2\pi r (p - p_0) dr = -\frac{qr_s^2}{8} \quad \dots\dots\dots (34)$$

The load capacity W is obtained as

$$\begin{aligned} W &= W' - W_{tr} \\ &= -\frac{q}{4} \left\{ (r_1^2 - r_0^2) \frac{\log a/r_0}{\log r_1/r_0} - \left(a^2 - r_0^2 + \frac{r_s^2}{2} \right) \right\} \quad \dots\dots\dots (35) \end{aligned}$$

For the case with multiple supply holes, the load capacity can be obtained by the appropriate superposition; for the bearing with k supply holes, the load capacity is

$$\begin{aligned} W_{(k)} &= \frac{kq_{(k)}}{4} \left\{ (r_1^2 - r_0^2) \frac{\log a/r_0}{\log r_1/r_0} - \left(a^2 - r_0^2 + \frac{r_s^2}{2} \right) \right\} \quad \dots\dots\dots (36) \end{aligned}$$

2.4 Volume rate of flow

The volume rate of flow can be calculated from the outflow the bearing edges.

For the incompressible fluid, it becomes

$$Q_{\text{incomp}} = \frac{\pi k h^3 q_{(k)}}{3\mu} \quad \dots\dots\dots (37)$$

where

μ : coefficient of viscosity

h : bearing clearance

k : number of supply holes

For a compressible fluid case, the volume rate of flow corresponding to the ambient pressure is given by the following equation assuming isothermal expansion:

$$Q_{\text{comp}} = \frac{p_0 + p_a}{2p_a} Q_{\text{incomp}} \quad \dots\dots\dots (38)$$

3. Examples of theoretical bearing characteristics

Fig. 3 shows some theoretical bearing chara-

Fig. 2 Configuration of thrust collar bearing with multiple supply holes

cteristics, load capacity and volume rate of flow, with some bearing parameters, such as number of supply holes, and radius of supply holes, respectively. In these figures, the bearing characteristics are shown in dimensionless forms of $W_{(k)}/\pi(r_1^2 - r_0^2)(p_0 - p_a)$ and $Q_{incomp}/\pi h^3(p_0 - p_a)(3\pi)^{-1}$, respectively.

For any bearing, the supply holes must locate farther inside than the middle of the thrust collar in order to maximize the load capacity. The

volume rate of flow may also vary with the value of R_a but this is not so conclusive at the bearing design, because the flow rate is almost constant as long as the supply holes are arranged near the middle of the thrust collar.

4. Experimental investigations

4.1 Pressure distribution

Some experiments were made to verify the theoretical results concerning the pressure distribution and load capacity.

Fig. 4 shows the apparatus of the experiments for pressure distribution. The upper plate (No. 2) and the bearing plate (No. 1) are fixed parallel by adjusting screws to provide a bearing clearance which is measured by three dial gages. The compressed air is fed to the annular groove behind the bearing plate, and then to the bearing clearance through the supply holes spaced circumferentially. The pressure in the bearing clearance is measured by a U-tube mercury manometer connected to the clearance through a small hole of 0.5 mm diameter drilled in the upper plate. The under bearing surface can be rotated and slid by fine screws to measure the pressure distribution on the entire bearing surface. The experiments are made on the static condition.

The bearing configuration is similar to one shown in Fig. 2; its typical dimensions are

- outer bearing diameter : $2r_1 = 120$ mm
- inner bearing diameter : $2r_0 = 40$ mm
- radial position of supply hole : $a = 40$ mm
- diameter of supply holes : $2r_s = 1.6$ mm

First, the experiments were made with considerably small bearing clearances and low supply pressures ($h = 15 \mu$, $p_0 - p_a = 0.5$ kg/cm²), because, in these cases, the flow in the bearing clearance was considered to be fully viscous so that the assumption in the foregoing theory was justified. Fig. 5 (a) and (b) show the typical results of measured pressure distributions in radial and

Fig. 3 Theoretical bearing characteristics

Fig. 4 Experimental apparatus for pressure distribution

circumferential direction, respectively, for a case of single supply hole, in which the thick curves represent theoretical values calculated under $n=1$ (that is, isothermal condition). They show a good conformity which verifies the theoretical results.

But, in the cases with larger bearing clearance or higher supply pressure, the experimental results become somewhat different from the theoretical ones. In Fig. 6, pressure distributions are shown for some variations of bearing clearances under $p_0 - p_a = 0.5 \text{ kg/cm}^2$ with single supply hole. The pressure drop at the edge of the supply hole increases with increase of h . When the clearance becomes 50 microns, a pressure depression develops about the hole. This will be discussed later concerning the load capacity.

For a case of multiple supply holes, the experimental results are similar to those for the case

of single supply hole.

4.2 Load capacity

The experimental apparatus for load capacity is shown in Fig. 7. The floator has a thrust bearing with multiple supply holes through which the compressed air is supplied into bearing clearance. The floator is guided in the liner by an externally pressurized journal air bearing in order to prevent their contact. The load is put on the top of the floator by a lever and dead weight. The bearing clearance is measured by a dial gage placed on the top of the apparatus. The experiments are made under static condition.

The bearing dimensions are typically as follows:

Single supply hole
 $p_0 - p_a = 0.5 \text{ kg/cm}^2$, $h = 15 \mu$
(a)

Fig. 5 Theoretical and experimental results of pressure distributions

Single supply hole
 $p_0 - p_a = 0.5 \text{ kg/cm}^2$
Fig. 6 Pressure distributions for various bearing clearances

Fig. 7 Experimental apparatus for load capacity

outer diameter	: $2r_1=90$ mm
inner diameter	: $2r_0=30$ mm
radial position of supply holes	: $a=30$ mm
diameter of supply holes	: $2r_s=1.5$ mm
number of supply holes	: $k=6$

Some variations of bearing dimensions are also made in experiments.

Fig. 8 is an example of experimental results on load capacity. Experimental load capacity is shown in dimensionless form of $W/\pi(r_1^2-r_0^2)(p_0-p_a)$ against the bearing clearance h . The thin broken line in the figure is the theoretical value for an incompressible fluid, and it is constant independently of the bearing clearance. Experimental load capacity is, however, larger than this for a small bearing clearance because of compressibility of the fluid. But, for larger clearance, the pressure drop after supply holes becomes considerably effective as also seen in pressure distributions so that load capacity decreases under the incompressible theoretical one. This tendency becomes remarkable as the load, accordingly the supply pressure, increases.

4.2.1 Compressibility of fluid Assuming the isothermal expansion of gas in the bearing clearance ($n=1$), the compressible pressure distribution can be obtained as follows the incompressible solution p by using the "root-transformation":

$$p = \sqrt{(p_0 + p_a)(P - p_a) + p_a^2} \quad (39)$$

When the pressure $(P - p_a)$ in the bearing clearance is small enough compared with p_a , the pressure p can be expanded in series as

$$\begin{aligned} p - p_a &= \sqrt{(p_0 + p_a)(P - p_a) + p_a^2} - p_a \\ &= \frac{p_0 + p_a}{2p_a}(P - p_a) - \frac{(p_0 + p_a)^2}{8p_a^3}(P - p_a)^2 + \dots \end{aligned}$$

Neglecting the terms with higher order than $(P - p_a)^2$, one can approximate the compressible pressure distribution as

$$p - p_a \cong \frac{p_0 + p_a}{2p_a}(P - p_a) \quad (40)$$

Then, the load capacity with a compressible fluid

Fig. 8 Theoretical and experimental values of load capacity

is

$$\begin{aligned} W_{\text{comp}} &= \int_{r_0}^{r_1} \int_0^{2\pi} (p - p_a) r d\theta dr \\ &\cong \frac{p_0 + p_a}{2p_a} \int_{r_0}^{r_1} \int_0^{2\pi} (P - p_a) r d\theta dr \\ &= \frac{p_0 + p_a}{2p_a} W_{\text{incomp}} \quad (41) \end{aligned}$$

The error in the approximation of Eq. (41) increases with increase of P . But, the externally pressurized thrust gas-bearing as analyzed here has point supply holes so that higher bearing pressure may be limited to a comparatively small region near the supply holes, which shows that the approximation of load capacity of Eq. (41) is rather reasonable as long as the supply pressure is not so high.

4.2.2 Pressure drop at supply hole The pressure drop at supply holes may be explained by considering the flow pattern in the bearing clearance.

To analyze the flow, the flow pattern as shown schematically in Fig. 9 is assumed approximately, in which the gas flows into bearing clearance from supply holes (section A), then it flows out toward the bearing periphery (C-D). The B is the stagnation point without radial flow velocity. At section C, the velocity distribution may be uniform as shown in the figure. From section C, the boundary layers begin to develop to section D which refers to the fully developed profile of flow.

At supply hole, the pressure drop may occur through two causes; one is the accelerating energy of lubricant from the stagnation point (B) to the inlet flow (section C), and the other is the energy consumption for the development of the boundary layers and the change of velocity distribution profile.

For the former, i.e., the acceleration of the lubricant, Bernoulli's theorem can be applied between B and C in the figure neglecting the friction loss and compressibility effect, resulting in the following pressure drop;

$$p_0 - p_0' = \frac{1}{2} \rho_0 u_m^2 \quad (42)$$

where

Fig. 9 Schematic flow pattern in the bearing clearance

p_0 : pressure at the stagnation point B
 p_0' : pressure at section C
 ρ_0 : density of the fluid at section C
 u_m : average flow velocity at section C

For the latter, i.e., the change of velocity distribution, one can estimate approximately the pressure drop neglecting the entrance length for the development of the boundary layers, compressibility of the gas and divergence of flow area. At section C, the momentum of the fluid for a unit breadth, considering that the flow velocity profile is uniform, is as follows;

$$J_C = \int_0^h \rho_0 u^2 dz = \rho_0 u_m h \quad (43)$$

where

J_C : momentum of fluid
 h : bearing clearance
 ρ_0 : density of the fluid at section C
 u : flow velocity
 u_m : average flow velocity at section C
 z : ordinate normal to bearing surface

At section D where a fully developed flow profile may be expected, that is

$$u_D = 4u_0 \left(1 - \frac{z}{h}\right) \left(\frac{z}{h}\right) \quad (44)$$

where u_0 is flow velocity at the middle of bearing clearance. The momentum of the fluid is

$$J_D = \int_0^h \rho_0 u^2 dz = \frac{8}{15} \rho_0 u_0^2 h \quad (45)$$

From the continuity of the flow,

$$\int_0^h u_C dz = \int_0^h u_D dz \quad (46)$$

where the density change between section C and D is neglected. Then,

$$u_0 = \frac{3}{2} u_m \quad (47)$$

Using this equation and the momentum equation, the pressure loss for the change of velocity profile is

$$p_0' - p_0'' = \frac{1}{h} (J_D - J_C) = \frac{1}{5} \rho_0 u_m^2 \quad (48)$$

The pressure drop at supply holes can be estimated from the total effects of Eqs. (42) and (48), then

$$p_0 - p_0'' = \lambda \frac{1}{2} \rho_0 u_m^2 = \lambda \frac{1}{2} \rho_0 \left(\frac{Q_0}{2\pi k r_s h} \right)^2 \quad (49)$$

where λ is coefficient of pressure drop and is taken as $\lambda=1.4$ from the above investigations; Q_0 and ρ_0 are volume rate of flow and density of fluid, respectively, corresponding to pressure p_0'' approximately.

In Fig. 8, the thick curves are theoretical ones calculated with consideration of these effects. They coincide considerably well with experimental ones, even for comparatively large bearing clear-

Fig. 10 Load capacity for various numbers of supply holes

ance and high supply pressure though the above compensation terms do not concern the shock or pressure depression near the supply holes.

Experiments are also made with various bearing dimensions. In all cases, the results agree well with theoretical ones with consideration of secondary effects. For example, Fig. 10 shows the results for a varied number of supply holes.

5. Summary

In this study, the authors have investigated the externally pressurized thrust collar gas-bearing with multiple supply holes by use of complex potential theory. The appropriate complex potential function can be given by an infinite series, from which the load capacity is calculated by integration for an incompressible case.

Experiments are made to investigate the theoretical results, yielding good agreement of the results both qualitatively and quantitatively with consideration of secondary effects due to the compressibility of the fluid and pressure drop at supply holes. Pressure drop takes place at supply holes especially in the case with higher supply pressure and larger bearing clearance, because of the energy loss through acceleration of the fluid at the edge of the supply holes, and the change of the flow velocity profiles, whose effects are estimated by assuming a flow pattern.

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