624.042.7:534.015.001.36

On the Response Spectrum of the Building-Machine Structure System to the Strong Motion Earthquake*

By Hisayoshi SATO**

The building-machine structure system is simulated by an idealized two-degrees-of-freedom system with small mass ratio, the response spectrum of the system is obtained by the analog computation. It can be seen that the coincidence of the periods of both systems has to be evaded from the view point of the design of the machine structure, and it is proposed that the two-degrees-of-freedom system can be examined easily through T_b - T_m response spectrum. The width of the two-degrees-of-freedom system around $T_b = T_m$ is discussed to know the shape approximately of the peak of the spectrum. The response spectrum of a single-degree-of-freedom system to the acceleration recorded upon a building is computed. A statistical computation in case of the two-degrees-of-freedom system is performed and the result is compared with the response spectrum by the analog computation.

1. Introduction

The author has studied about the aseismic design of machine structures in relation to the construction of the atomic power plant⁽¹⁾. In the reference the response computations of a single- and a two-degrees-of-freedom system with small mass ratio to earthquake records were performed. They assume the building and the building-machine structure system respectively.

Some results that would be useful for the dynamic aseismic design were obtained. And it was also shown that the maximum of the response of the system could be presumed by a statistical computation which simulated the earthquake by a single-degree-of-freedom system of ground model and random vibration, so it was approximately equal to the results by analog computation.

In this report more precise discussions will be made on the response of the building-machine structure system to strong motion earthquake records.

2. The two-degrees-of-freedom system acceleration response spectrum

When machine structures, piping systems and appendages are attached to the building structure

which has the characteristic of a single-degree-of-freedom system, the whole structure system can be simulated by a two-degrees-of-freedom system with small mass ratio.

When the machine structure has many natural frequencies as piping systems, it is supposed that each frequency included in the range of earthquake bandwidth corresponds to that of the machine structure respectively, and then the response can be computed for each two-degrees-of-freedom system and the result is summed up for instance by the method of square root of sum of squares. Fig. 1 shows a model of the system.

The equation of the system can be given as

Fig. 1 The model of the building-machine structure system

* Received 9th June, 1965.

** Associate Professor, Institute of Industrial Science, University of Tokyo, Minato-ku, Tokyo.

$$\left. \begin{aligned} \ddot{x}_m + 2\omega_m h_m \dot{x}_m + \omega_m^2 x_m - (2\omega_m h_m \dot{x}_b + \omega_m^2 x_b) &= -\alpha(t) \\ -\gamma(2\omega_m h_m \dot{x}_m + \omega_m^2 x_m) + \ddot{x}_b + (2\omega_b h_b + \gamma \cdot 2\omega_m h_m) \dot{x}_b + (\omega_b^2 + \gamma \omega_m^2) x_b &= -\alpha(t) \end{aligned} \right\} \dots\dots\dots (1)$$

where x : the relative displacement between mass and ground, h : the damping coefficient, ratio of the system damping constant to the critical damping constant, $\gamma (=m_m/m_b)$: the mass ratio, $\alpha(t)$:

the earthquake acceleration, suffix m : the machine structure, the upper system, suffix b : the building structure, the lower system.

Fig. 2 shows an example of the response spectra of this system for El Centro earthquake, which is obtained by using analog computer. Such results for some records are also given, but here the discussion will be made principally on the example of El Centro.

In Fig. 2 as the abscissa the period of the machine structure T_m and as the ordinate the maximum value of each response are taken and the parameter of several lines is the period of the building structure T_b . About the damping coefficients, $h_b=0.07$ and $h_m=0.02$ are selected as a standard value. They are typical values for the concrete structure and the piping system respectively.

As for the mass ratio $\gamma=0$, 0.01 and 0.1 are adopted, $\gamma=0$ means that the upper mass is small enough comparing with the lower mass. In case of $\gamma=0.1$ the first and the second natural frequencies are divided, so the abscissa of T_m does not represent the characteristic of the system well, but it is used to compare with those of $\gamma=0$, 0.01. Making the problem simpler h_m is fixed constant when the period of upper system is varied.

When $T_b=T_m$, the acceleration of the upper

(a) The acceleration response spectrum of the two-degrees-of-freedom system

(b) The velocity response spectrum of the two-degrees-of-freedom system

(c) The displacement response spectrum of the two-degrees-of-freedom system

El Centro, $\gamma=0$, $h_b=0.07$, $h_m=0.02$

Fig. 2

system generally shows the maximum in the form of sharp peak for almost all cases of T_b .

On the other hand the response spectrum to the single-degree-of-freedom system has obvious peaks at 0.2, 0.5 and 0.9 sec and the acceleration magnification factor in case of 0.2 and 0.5 sec shows the same value. Then it can be thought that the factor for two-degrees-of-freedom system might show same value also at $T_b=T_m=0.2, 0.5$ sec, but really the latter is smaller than the former in Fig. 2.

It can be given as the reason for these results that the structure system has a low pass filter characteristic. The result is analysed as this. When $T_b=T_m=0.2$ sec, the machine structure system receives the effect of the component of the building structure period and that of the dominant period in earthquake record, 0.5 sec which is longer than that of the building structure. But in case of $T_b=T_m=0.5$ sec the dominant period of 0.2 sec is blocked by the building structure, then the machine system is only affected by the period of the building system. Consequently the acceleration magnification factor is larger for the former.

This effect can be seen also in the form of response spectra for $T_b=0.2, 0.5$ sec. The spectrum for $T_b=0.2$ sec shows a flat tendency around $T_m=0.5$ sec though the level is rather low. It means that the influence of the dominant period in the earthquake record appears in the machine structure through the building structure. On the other hand when $T_b=0.5$ sec, $T_m=0.2$ sec the increase of the spectrum is hardly seen and it decreases continuously.

The magnification factor at 0.3 sec of the single-degree-of-freedom system for El Centro earthquake makes a valley, and it is considered that 0.3 sec is the period where the period component in the earthquake is little. Then in Fig. 2 the value at $T_b=T_m=0.3$ sec is also less than those of $T_b=T_m=0.2, 0.4$ sec. This result tells us that as for the period of which component is little in the earthquake, the magnification factor can be small. But it will be difficult to conjecture such period for a site as a practical problem.

When γ becomes pretty small, it is presumed that the magnification factor grows large as the reacting force between two systems decreases. Generally for the change of mass ratio from 0.01 to 0 the factor can be as large as 1.3~1.5. In case of $\gamma=0.1$ the sharp peak for $T_b=T_m$ changes to wide and dull one. And the magnification factor decreases, too.

3. The velocity and displacement response spectrum of the two-degrees-of-freedom system

The velocity response spectra for various T_b 's also generally show the maximum when $T_b=T_m$, but there is such exception as the case of $T_b=0.3$ sec which has the maximum at $T_m=0.4$ sec. Perhaps this is because of less period component at $T=0.3$ sec in the earthquake.

The spectrum of $T_b=0.2$ sec has two peaks at $T_m=0.6, 1.0$ sec which are considered to correspond to the dominant period of the earthquake. This means that the influence of the dominant period in the earthquake on the machine structure system appears more sensitive than in the case of the acceleration response spectrum. On the contrary when an earthquake has a dominant period which is shorter than T_b , there cannot be seen the effect on the machine structure system.

Now let us consider the displacement response spectrum. Where $T_b=T_m$ at rather short period the spectrum does not show the maximum but the extreme, and in range of long period it becomes the maximum. For example the spectrum for $T_b=0.2$ sec shows an extreme at $T_m=0.2$ sec, but it is smaller than the value at $T_m=1.0$ sec, the line for $T_b=0.3$ sec increases monotonously as T_m becomes larger.

Moreover let us look at the spectra for $T_b=0.4, 0.5$ and 0.6 sec, the values at $T_b=T_m$ and $T_m=0.9$ sec are almost same. This means that the influence of the dominant period in the earthquake on the machine structure, which is longer than that of the building structure, cannot be neglected in the displacement spectrum. This is slightly seen in the acceleration spectrum.

This makes us understand that it is also undesirable for both system periods to coincide as in the dynamic aseismic design from the view point of the displacement.

4. T_b - T_m response spectrum

As mentioned above, $T_b=T_m$ gives the worst condition as the principle of the aseismic design. Figs. 2 and 3 show a notable tendency of the spectrum when only the points $T_b=T_m$ are selected, namely it is the envelope for several spectra of the two-degrees-of-freedom system. Here let us call the curve " T_b - T_m response spectrum", which is made of the value of $T_b=T_m$. Fig. 3 shows an example by the parameter of h .

The spectra look like those of the single-degree-of-freedom system. The characteristics can

be explained like this. When $\gamma=0$, two natural periods of this system are equal, so the system falls nearly into resonant state and two systems oscillate as if they were one system. Then the nature of the response spectrum for the machine structure seems the same as that of the single-degree-of-freedom system. It can be said that when γ is small, this tendency will not change.

Here the author pays attention to the acceleration T_b - T_m response spectrum of Fig. 3 (a). The line for large h_m is rather smooth and its shape is very similar to that of the single-degree-of-freedom system. When h_m decreases, the spectrum is shifted in the direction of the larger value. The ratio, the value for large h_m to that for small h_m , is larger where the spectrum makes a peak than where it does a valley. And about the peak with same height for large h_m , the one in shorter range is conspicuous for the spectrum and it is considered that the magnification factor

can be suppressed by attaching a damper even if the periods of both systems coincide with each other.

According to Fig. 4 the maximum magnification factor is 32 when $h_m=0.007$. It is in the range

Fig. 3 The acceleration response spectrum of the two-degrees-of-freedom system

of 30~40 for almost all earthquake records when the parameter is same, and the worst value is 60 exceptionally. It is about 10 for $h_m=0.1$, nearly 7 for $h_m=0.2$. They are nearly constant for the response spectra of several earthquake records.

The T_b - T_m response spectra as for the velocity show the characteristic so calls the constant velocity, and the displacement spectra increases with period. The tendency of both spectra is also similar to the nature of the single-degree-of-freedom system. The displacement spectrum of small h_m is sensitive for the ground dominant period which is rather long. To attach the damper is effective in this case, too.

5. On the width of the response spectra for $T_b=T_m$

By computing the spectrum of the two-degrees-of-freedom system for the various combinations of γ , h and T , more accurate information can be obtained, but it would consume much time and expense for an enormous number of cases to be calculated. On the contrary the T_b - T_m response spectrum can be computed by less points for the same γ and h .

What difference between T_b and T_m should be taken to make the magnification factor one half of that for $T_b=T_m$ is examined by analysing the spectrum. Using the results, the general shape of the two-degrees-of-freedom system can be presumed from the T_b - T_m response spectrum.

When the magnification factor for $T_b=T_m$ is normalised as 1.0, the relation between T_b and the period where it becomes 0.5 is given as Table 1 by making use of Fig. 2. T_l and T_u are the periods which have such meaning as above mentioned for $T < T_b$ and $T > T_b$ respectively. Here the meaning of these values will be discussed

ed taking the ground dominant period T_g into consideration.

At first about T_u/T_b the discussion will be made. When $T_b < T_g$, it is predicted that the width of the response spectrum for $T_b=T_m$ becomes large by the effect of dominant period in earthquake, so the value T_u/T_b really becomes large. For example let us think of the case $T_b=0.5$ sec where the response spectrum of the single-degree-of-freedom system has a peak.

About the two-degrees-of-freedom system for $T_b=0.2, 0.3$ and 0.4 sec which correspond to the case $T_b < 0.5$ sec, $T_u/T_b=1.45, 1.6$ and 2.0 , otherwise for $T_b < 0.5$ sec $T_u/T_b=1.1\sim 1.3$ and the latter is comparatively smaller than the former. The characteristic of the spectrum for $T_b=0.8$ sec which is near one of the dominant period $T_g=0.9$ sec also shows the same tendency. Thus when it seems that there are dominant periods longer than that of the building structure, the width around $T_b=T_m$ extends to the longer period. From these considerations and Table 1 the next summary is derived as a standard,

$$\left. \begin{array}{l} T_b > T_g : T_u/T_b = 1.2 \\ T_b < T_g : T_u/T_b = 1.6 \end{array} \right\} \dots\dots\dots (2)$$

Secondly the observation is done for T_l/T_b . When $T_b=0.3$ and 0.6 sec which are 0.1 sec longer than $T_g=0.2$ and 0.5 sec, and for the cases $T_l/T_g=0.60$ and 0.63 , generally through out the examples for $T_g < T_b$ discrepancies are little.

As for El Centro earthquake it is supposed that the period component is least at 0.3 sec, then the value $T_l/T_b=0.8$ for $T_b=0.4$ sec is one of the closest to 1.0. And the value for $T_b=0.2$ sec is 0.8 in the case of any earthquake. These examples are the cases where there is no dominant period in $T < T_b$ or T_b is far from it even if it exists. Then the result is obtained as

Table 1 On the period where the magnification factor becomes one half of the maximum

	T_b s	0.2	0.3	0.4	0.5	0.6	0.8	1.0
El Centro $r=0$	T_l/T_b	0.80	0.70	0.85	0.74	0.77	0.76	0.70
	T_u/T_b	1.35	1.63	1.60	1.14	1.17	1.28	1.17
$r=0.01$	T_l/T_b	0.80	0.60	0.78	0.74	0.63	0.71	0.76
	T_u/T_b	1.35	2.00	1.45	1.12	1.25	1.31	1.18
Saitama $r=0$	T_l/T_b	0.85	0.67	0.60	0.62	0.70	0.73	0.66
	T_u/T_b	1.45	1.23	1.68	1.70	1.50	1.21	1.18
$r=0.01$	T_l/T_b	0.80	0.60	0.60	0.82	0.72	0.68	0.65
	T_u/T_b	1.65	1.30	—	1.64	1.40	—	1.17
Kushiro $r=0$	T_l/T_b	0.80	0.80	0.60	—	0.75	0.89	0.92
	T_u/T_b	1.70	1.20	1.18	—	1.28	1.08	1.10
$r=0.01$	T_l/T_b	0.80	0.77	0.60	—	0.77	0.63	0.65
	T_u/T_b	1.80	1.27	1.18	—	1.28	1.10	1.18

$$\left. \begin{array}{l} T_b > T_g : T_l/T_b = 0.6 \\ T_b < T_g : T_l/T_b = 0.7 \end{array} \right\} \dots\dots\dots (3)$$

By these discussions the influence of the ground dominant period on the machine structure through the building structure can be given, and from the response spectrum of the single-degree-of-freedom system and the T_b - T_m response spectrum the approximate shape of the two-degree-of-freedom system near $T_b = T_m$ can be estimated preliminarily.

6. On the response spectrum to acceleration records upon building structures

Now, to look into the result of the two-degrees-of-freedom system a computation of response by the single-degree-of-freedom system to the acceleration which is recorded upon the building structure is tried⁽²⁾. The magnification factor of this case ought to correspond to that of the machine structure of the two-degrees-of-freedom system with $\gamma=0$.

The absolute acceleration response is composed of the ground acceleration and the restoring force, but in the computation above mentioned the former is not summed up, because the really recorded ground acceleration amplitude is so small that it cannot be used as precise input for the computation and there is only one apparatus which reads the acceleration wave as the input of the analog computer. Really the difference is not so large as will be mentioned.

When $\alpha(s)$ is representative of earthquake acceleration, and X_b and X_m are the Laplace transform of the relative displacement of each structure to the ground motion, the magnification factor of the machine structure system by the two-degrees-of-freedom system can be written as

$$\frac{s^2 X_m + \alpha(s)}{\alpha(s)} = \frac{s^2 X_b + \alpha(s)}{\alpha(s)} \frac{s^2 X_m + \alpha(s)}{s^2 X_b + \alpha(s)} \dots\dots (4)$$

on the other hand the value which is computed in this section is

$$\frac{s^2 X_m}{s^2 X_b + \alpha(s)} \dots\dots\dots (5)$$

then the difference between the two factors is given as the ratio of the second term of Eqs. (4) to (5)

$$\frac{s^2 X_m + \alpha(s)}{s^2 X_m} = 1 + \frac{\alpha(s)}{s^2 X_m} \dots\dots\dots (6)$$

when $T_b = T_m$ and h_m is small, the magnification factor obtained from Eq. (6) is so large that the second term of right half of Eq. (6) may be small enough comparing with 1, namely Eq. (6) is nearly equal to 1.

Fig. 5 (a) is an example of the response spectrum for the single-degree-of-freedom system to the acceleration record upon a building during an earthquake. The magnification factor between the acceleration records on the building and that at the basement is about 3.5 and it seems that the natural period of the building is 0.5 sec from Fig. 5. In Fig. 5 the magnification factor reaches 10 in case of $h_m = 0.02$. Then that of the machine structure system amounts to 35 as the multiplication of the two factors. This may be compared with that of the two-degrees-of-freedom system though h and the ground dominant period are not known.

Table 2 gives the width for $T = T_b$ of such

Table 2 The width around the period where the magnification factor is maximum about the response spectrum upon a building

	A	B	C	D
T_l/T_b	0.92	0.68	0.84	0.75
T_u/T_b	1.25	1.36	1.34	1.68

(a) The acceleration response spectrum to the record upon a building

(b) The displacement response spectrum to the record upon a building

Fig. 5

spectra to four records. The magnification factor for the building structure is not included in the spectrum, but it is nearly constant for various periods when γ is approximately equal to zero, so the width can be compared with same analysis for the two-degrees-of-freedom system, actually with the results of Table 1.

The spectra about displacement, i.e., the relative displacement between the machine structure and the building, are also shown in Fig. 5. In Table 4 displacements for the period where the factor reaches the maximum are calculated by using the value of Fig. 5 and several assumptions that the maximum of the acceleration record is $0.33g$, the magnification factor of the building is about 3.

These results are not obtained by the records of the strong motion earthquake, so it is possible that the magnification factor of the machine structure may become larger and more severe from the view point of wave trains.

7. The presumption of the maximum by statistical computation

As for the single-degree-of-freedom system

Table 3 The displacement at the period where the acceleration is maximum in adjusting the scale of the earthquake

	A	B	C	D
The period where the acceleration response leads to the maximum	0.5	0.5	0.5	0.55
Displacement	54.9	37.8	43.1	36.8

Table 4 The time when the maximum occurs

$T_b \backslash T_m$	0.15	0.2 (0.007)	0.2	0.2 (0.1)	0.3	0.4	
0.2	3.3	3.0	3.0	2.6	2.4	2.6	
$T_b \backslash T_m$	0.2	0.3 (0.007)	0.3	0.3 (0.1)	0.4	0.5	
0.3	2.4	8.6	3.6	2.4	2.4	2.2	
$T_b \backslash T_m$	0.2	0.3 (0.007)	0.4	0.4 (0.1)	0.5	0.6	
0.4	2.5	2.6	6.8	3.8	2.7	3.2	2.4
$T_b \backslash T_m$	0.3	0.4 (0.007)	0.5	0.5 (0.1)	0.6	0.7	
0.5	2.4	2.6	3.5	3.5	2.9	2.5	2.6
$T_b \backslash T_m$	0.4	0.5 (0.007)	0.6	0.6 (0.1)	0.7	0.8	
0.6	2.5	2.5	14	4.4	2.5	6.5	—
$T_b \backslash T_m$	0.6	0.7 (0.007)	0.8	0.8 (0.1)	0.9	1.0	
0.8	2.7	7.1	11.1	7.6	6.4	6.5	4.9
$T_b \backslash T_m$	0.8	1.0 (0.007)	1.0	1.0 (0.1)	1.1	1.2	
1.0	4.3	6.3	5.0	5.1	5.6	5.3	4.8

which simulates the building structure the statistical computation was performed on several assumptions as bellows. The maximum value corresponds to the site the extreme probability density function is small enough, the ground characteristic is represented by a single-degree-of-freedom system, the acceleration at the base layer is random vibration with Gaussian distribution and band limited constant power spectrum. Then the acceleration magnification factor in this sense is computed.

Here this method will be extended to the two-degrees-of-freedom system which assumes the building-machine structure system. And the results will be compared with the response spectra by analog computation. It is also expected that the table which may be convenient for design will be composed of the results for various parameters. Eq. (1) can also be written by using an operator.

$$\left. \begin{aligned} (s^2 + 2\omega_m h_m s + \omega_m^2) X_m \\ - (2\omega_m h_m s + \omega_m^2) X_b = -\bar{\alpha}(s) \\ - \gamma(2\omega_m h_m s + \omega_m^2) X_m \\ + \{s^2 + (2\omega_b h_b + \gamma \cdot 2\omega_m h_m) s \\ + (\omega_b^2 + \gamma \omega_m^2)\} X_b = -\bar{\alpha}(s) \end{aligned} \right\} \dots\dots\dots (7)$$

it can be solved as for X_m and X_b

$$\left. \begin{aligned} X_m = -\bar{\alpha}(s) [s^2 + \{2\omega_b h_b + (1+\gamma)2\omega_m h_m\} s \\ + \{\omega_b^2 + (1+\gamma)\omega_m^2\}] / \Delta \\ X_b = -\bar{\alpha}(s) [s^2 + (1+\gamma)2\omega_m h_m \\ + (1+\gamma)\omega_m^2] / \Delta \\ \Delta = s^4 + \{2\omega_b h_b + (1+\gamma)2\omega_m h_m\} s^3 \\ + \{\omega_b^2 + (1+\gamma)\omega_m^2 + 2\omega_b h_b \cdot 2\omega_m h_m\} s^2 \\ + (\omega_b^2 \cdot 2\omega_m h_m + \omega_m^2 \cdot 2\omega_b h_b) s + \omega_b^2 \cdot \omega_m^2 \end{aligned} \right\} \dots\dots\dots (8)$$

The transfer function of the acceleration for the machine system is given from Eqs. (7) and (8) as

$$\frac{s^2 X_m + \bar{\alpha}(s)}{\bar{\alpha}(s)} = \frac{1}{\Delta} (2\omega_b h_b s + \omega_b^2) (2\omega_m h_m s + \omega_m^2) \dots\dots\dots (9)$$

as for the building system

$$\begin{aligned} \frac{s^2 X_b + \bar{\alpha}(s)}{\bar{\alpha}(s)} \\ = \frac{1}{\Delta} (s^2 + 2\omega_m h_m s + \omega_m^2) (2\omega_b h_b s + \omega_b^2) \end{aligned} \dots\dots\dots (10)$$

when $\gamma=0$, Eqs. (9) and (10) can be written as

$$\frac{s^2 X_m + \bar{\alpha}(s)}{\bar{\alpha}(s)} = \frac{2\omega_b h_b s + \omega_b^2}{s^2 + 2\omega_b h_b s + \omega_b^2} \frac{2\omega_m h_m s + \omega_m^2}{s^2 + 2\omega_m h_m s + \omega_m^2} \dots\dots\dots (11)$$

$$\frac{s^2 X_b + \bar{\alpha}(s)}{\bar{\alpha}(s)} = \frac{2\omega_b h_b s + \omega_b^2}{s^2 + 2\omega_b h_b s + \omega_b^2} \dots\dots\dots (12)$$

Eq. (11) shows that the whole transfer function can be given by the multiplication of that of each structure and the acceleration is transferred only

from the lower system to the upper, Eq. (12) means that the transfer function is represented by the building structure itself and has no reaction from the upper system.

The computation is performed by

$$H_g(s) = \frac{2\omega_g h_g s + \omega_g^2}{s^2 + 2\omega_g h_g s + \omega_g^2} \frac{s^2}{(\phi_1 s + 1)^2 (\phi_2 s + 1)^2} \quad \dots\dots\dots (13)$$

$$H_m(s) = \frac{(2\omega_m h_m s + \omega_m^2)(2\omega_b h_b s + \omega_b^2)}{A} H_g(s) \quad \dots\dots\dots (14)$$

$H_g(s)$ is the transfer function of the ground model. On the other hand a probability density function of the extreme can be written as

$$p(y) = \frac{1}{\sqrt{2\pi}} \varepsilon \exp\left(-\frac{y^2}{2\varepsilon^2}\right) + \frac{\sqrt{1-\varepsilon^2}}{2} y \exp\left(-\frac{y^2}{2}\right) \left\{1 + \operatorname{erf} \frac{\sqrt{1-\varepsilon^2}}{\sqrt{2}} y\right\} \quad \dots\dots\dots (15)^{(3)}$$

where

$$y = \frac{I(t)}{\sqrt{I_0}} \quad \dots\dots\dots (16)$$

$I(t)$ is the original time variable random function.

$$\left. \begin{aligned} I_0 &= \frac{1}{2\pi} \int_0^\infty |H(s)|^2 d\omega, \\ I_2 &= \frac{1}{2\pi} \frac{1}{4\pi^2} \int_0^\infty \omega^2 |H(s)|^2 d\omega, \\ I_4 &= \frac{1}{2\pi} \frac{1}{16\pi^4} \int_0^\infty \omega^4 |H(s)|^2 d\omega \end{aligned} \right\} \quad \dots\dots\dots (17)$$

$$\varepsilon^2 = 1 - \frac{I_2^2}{I_0 I_4} = 1 - \left(\frac{N_0}{N_m}\right)^2 \quad \dots\dots\dots (18)$$

N_m : the number of extremes per unit time

N_0 : the number of zeros with the positive slope per unit time

Substituting $H_g(s)$, $H_m(s)$ into Eq. (17), the integral⁽⁴⁾ is performed respectively. Making the maximum correspond to the point where $p(y)$ is small enough, let y_g and y_m be the value that is given by solving for each system.

$$p(y) = 0.01 \quad \dots\dots\dots (19)$$

then

$$(I_g)_{\max} = \sqrt{I_0} y_g, \quad (I_m)_{\max} = \sqrt{I_0} y_m \quad \dots\dots\dots (20)$$

so the magnification factor is given by

$$\alpha_m = \frac{\sqrt{I_0} y_m}{\sqrt{I_0} y_g} \quad \dots\dots\dots (21)$$

In Fig. 6 the number of extremes for every section which is divided appropriately along the amplitude axis is shown as for El Centro earthquake. The maximum occurs at the section where the number of extreme shows the minimum. ε which is defined by Eq. (18) is computed also for the earthquake record, then Eq. (15) is

computable for the ε . So the two curves as for the extreme probability density function can be compared with each other by making the maximum of two densities equal.

Fig. 7 shows the ratio of the maxima of earthquakes to the point where $p(y)=0.01$, and it has some width around 1.

In Fig. 8 the response spectra by the statistical computation and the analog computer are

Fig. 6 The extreme density of an earthquake

Fig. 7 The comparison of y where $p(y)=0.01$ with the maxima of earthquake records

(a) $h_m=0.2$

(b) $h_m=0.1$

Fig. 8 The comparison of the response spectrum

shown in the same figure. It can be said that the latter covers the former as the shape of an envelope when h_m is rather large. But when only h_m decreases with other parameters held constant, the latter becomes much larger than the former.

The time where the maximum occurs about the response of the upper system of the two-degrees-of-freedom system to El Centro earthquake is shown in Table 4. The value becomes considerably large when $T_b = T_m$ and h_m is small, which means that the system is in a state of pseudo-resonance and it needs considerable time for the amplitude to grow up. Taking the difference between two methods mentioned above into consideration, it seems that it is caused by the as-

Fig. 9 An example of the response spectrum by the statistical computation

sumption that the theoretical random vibration is stationary, although the real earthquake is nonstationary. So for making the discrepancy small it is necessary to analyse the system considering the nonstationarity of the earthquake.

Fig. 9, the spectra of the two-degrees-of-freedom system by the statistical computation, is an example showing how the magnification factor of the machine system changes by the combination of the ground dominant period and the natural period of building structure model. It is natural that the spectrum for $T_b/T_g = 1.0$ has the maximum factor at $T_m/T_g = 1.0$. The line for $T_b/T_g = 0.5$ gives two peaks at $T_m/T_g = 0.5$, that is, $T_b = T_m$ and $T_m = T_g$. The latter means that the effect of the ground dominant period appears in the magnification factor of the structure system through the building structure system, but its value is smaller than that of $T_b = T_m$. In case of $T_b/T_g = 2.0$ any effect like this cannot be seen. These characteristics agree with the results of analog computation.

Table 5 is an example of the magnification factors for the machine structure system of the two-degrees-of-freedom system by the statistical computation. Then the maximum acceleration during an earthquake could be presumed by knowing characteristic values of the vibration system. But in such a case as h_m is small, it has to be

Table 5 An example of the table of the magnification factor

$r=0, h_b=0.07, T_b/T_g=1.0, h_g=0.4$									
h_m	T_m/T_g	$p(y)$	0.01	0.001	$\rightarrow 0$	h_m	T_m/T_g	$p(y)$	
0.2	0.2		3.664	3.476	3.072	0.02	0.2		3.631
	0.4		4.149	3.936	3.479		0.4		4.154
	0.6		5.263	4.991	4.406		0.6		5.945
	0.8		7.403	7.015	6.186		0.8		13.84
	0.9		8.488	8.041	7.087		0.9		26.63
	1.0		8.721	8.262	7.281		1.0		44.61
	1.1		7.899	7.484	6.597		1.1		31.88
	1.2		6.675	6.326	5.579		1.2		20.64
	1.5		3.975	3.772	3.334		1.5		9.605
	2.0		2.221	2.113	1.879		2.0		5.200
0.1	2.5		1.548	1.477	1.321	0.007	2.5		3.731
	3.0		1.206	1.154	1.038		3.0		3.000
	0.2		3.661	3.474	3.073		0.2		3.578
	0.4		4.164	3.952	3.497		0.4		4.168
	0.6		5.491	5.210	4.604		0.6		6.771
	0.8		9.257	8.772	7.735		0.8		19.79
	0.9		12.68	12.01	10.58		0.9		42.98
	1.0		14.69	13.91	12.26		1.0		82.01
	1.1		12.57	11.90	10.49		1.1		54.30
	1.2		9.570	9.068	7.994		1.2		34.22
	1.5		4.902	4.652	4.114		1.5		15.85
	2.0		2.607	2.483	2.214		2.0		8.684
	2.5		1.811	1.733	1.558		2.5		6.305
	3.0		1.419	1.364	1.237		3.0		5.117

given by using a decreasing rate for the value of the the table.

8. Conclusions

Thus the principle of the aseismic design of a building-machine structure system is made clear by analog computer working and a method of the acceleration magnification factor by a statistical computation by making use of the knowledge of the system characteristics.

Some results are concluded as this.

(1) The response of the machine structure shows the worst condition as a rule, when the natural periods of both systems are equal. This is conspicuous in rather short period about the acceleration and in long period about the displacement.

(2) When we call $T_{b\max}$ the period where the acceleration magnification factor leads to the maximum about the single-degree-of-freedom system response spectrum, the factor at $T_m = T_{b\max}$ shows the maximum about the two-degrees-of-freedom system.

(3) Even if the response value becomes large by the reason of coincidence of both system periods, it can be diminished by increasing the damping coefficient of the machine structure system.

(4) The response spectrum which can be given by those values of the spectrum of the two-degrees-of-freedom system where the natural pe-

riods of both systems are equal shows the same nature as that of the single-degree-of-freedom system.

(5) The width for $T_b = T_m$ of the response spectrum of the two-degrees-of-freedom system can be given as the formula (2) and (3).

(6) Computing the response spectrum of the single-degree-of-freedom system which is obtained by using some records upon the building and presuming the acceleration and displacement of the machine system by using the results, they agree with those of the original two-degrees-of-freedom system.

(7) The response spectrum by a statistical computation coincides with that by analog computation in the sense of their envelope.

The author would like to thank Professors S. Fujii, A. Watari and H. Shibata, University of Tokyo for their comments. He is also grateful to emeritus Professor K. Muto and other members of SERAC for their kindness in permitting use of the analog computer.

References

- (1) H. Sato: *Jour. Japan Soc. Mech. Engrs.*, Vol. 65, No. 520 (1962), p. 686.
- (2) *Strong Motion Earthquake Records*, Vol. 1 (1960-3), Earthquake Research Institute, University of Tokyo.
- (3) S.O. Rice: *Bell Syst. Techn. Jour.*, Vol. 23 (1944), p. 282; Vol. 24 (1945), p. 46.
- (4) H.M. James et al.: *MIT R.L. Series*, Vol. 25 (1947), McGraw-Hill.
- (5) G.W. Housner et al.: *Bull. Seism. Soc. Amer.*, Vol. 43, No. 2 (1953), p. 97.