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Study on Application of Dropwise Condensation to Steam Condensers*

(Experiment and Theory of Behavior of Single
Water Drop on Inclined Surface)

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This study has been undertaken to make clear the motion of condensate-drops falling down along a cooling surface in dropwise condensation. Since the condensate-drops in steam condensers have various weights, shapes and velocities and make the experimental results much complicated, an experiment has been carried out in the air by using a single water-drop of known weight on an inclined surface. Observation has been made with motion picture.

From this experiment and theoretical analysis, the behavior of a water-drop with constant weight moving on the inclined surface has been made clear and formulas predicting the velocity, moving distance and length of the water-drop have been derived.

1. Introduction

In general, it is usual for the surface condensers of steam to be designed on the assumption that steam condenses as a continuous film. If the steam condenses as discrete drops, it leaves no doubt from the numerous literatures dealing with dropwise condensation that these condensers come to operate with the high condensing rates and to improve their performance.

As recently long life good promoters to induce the dropwise condensation have been discovered and tested⁽¹⁾, practical condensers operating with dropwise condensation have come to be available.

However, the flowing behavior of condensate from the dropwise condensation, compared with that from the filmwise must be considerably different, and this behavior must affect the heat transfer of condensation. Accordingly, for designing the best construction of the surface condenser applying the dropwise condensation only, the behavior of the condensate-drops must be considered.

This paper, first, describes the behavior of a condensate-drop moving on a vertical flat condensing surface. It was difficult to make clear the

behavior of the condensate-drop on the condensing surface because of its weight and shape changing all the time.

Therefore, next, in the air the behavior of a single water drop with constant weight on an inclined surface was investigated by using motion picture and then analysed theoretically.

2. Nomenclatures

The following nomenclatures are used in this paper.

- A : area of longitudinal and vertical section of drop
- $a_R = \gamma \cdot b_R^2 / f(\theta_m)_s$: weight of unit length of tail part of tailed drop
- B : contact area of drop
- b : contact diameter or breadth of drop
- G, G_0, G_C : weight of drop, critical weight of resting drop and critical weight of tailless drop at angle α of inclination of surface
- g : acceleration of gravity
- H, J : quantity defined by Eq.(36)
- k_1, k_2, k_3 : coefficient defined by Eqs.(3), (6) and (26) respectively
- l : length of drop
- n, L : quantity defined by Eq.(36)
- R : force of frictional resistance against motion
- t : time
- $\Delta t_{1P} = t_1 - t_P$

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v, v_{RC}, v_{∞} : velocity, critical velocity, velocity defined by Eq. (10)

x : distance

Y : quantity defined by Eq. (11)

y : height of drop

α : angle of inclination of surface

γ : specific weight

$$\theta = \cos \theta_R - \cos \theta_A$$

$$\Delta\theta_{C0} = \theta_C - \theta_0$$

θ : contact angle

$$f(\theta_m)_1 = 2 \sin \theta_m /$$

$$\left\{ \frac{\pi}{3} (1 - \cos \theta_m)^2 (2 + \cos \theta_m) \right\}^{1/3}$$

$$f(\theta_m)_2 = \frac{1}{2} \left(\frac{\pi \theta_m}{90} - \sin 2\theta_m \right) /$$

$$\left\{ \frac{\pi}{3} (1 - \cos \theta_m)^2 (2 + \cos \theta_m) \right\}^{2/3}$$

$$f(\theta_m)_3 = f(\theta_m)_1^2 / f(\theta_m)_2$$

ζ : constant

μ : viscosity

σ : surface tension

τ : shearing stress

subscript

A : foremost edge of drop or dropwise head of tailed drop

C : critical velocity

i : segment of sphere

dl : elongating velocity

m : mean

P : point indicated in Fig. 26

R : hindmost edge of drop or tail of tailed drop

v : velocity

0 : just before motion

$1, 2$: points indicated in Fig. 26

3. Behavior of a condensate drop

In order to observe the behavior of condensate-drops in dropwise condensation, an oblong vertical cooling surface (a brass plate of 120 cm high and

5 cm wide) coated with oleic acid as a promoter was exposed to steam and the behaviors of the condensate-drops moving on it were taken in pictures by a 16 mm cine camera.

Fig. 1 gives successive motion pictures (the time intervals between two successive frames 0.055 sec) showing how a naturally falling drop begins to fall off near the top of the vertical surface. Picture (1) shows the state of distribution of the adherent growing drops before a sudden fall of a drop marked with $\times$ by coalescing with the adjacent adhering drops. Pictures (2) and (3) show the falling drop growing in size very quickly by coalescing with other adhering drops on its path. Such a falling drop, starting near the top of the surface, reaches the lower part of the surface with increasing weight.

Fig. 2 gives successive motion pictures showing the behavior of the falling drop at the lower part of this surface. Picture (1) shows the head of the falling drop, picture (2) the central part of it and the lower part of picture (3) the tail of it. The falling drop, thus, varying the shape on its downward journey along the vertical cooling

Fig. 1 Fall of condensate-drop at the top of vertical cooling surface

Fig. 2 Falling condensate-drop at the lower part of vertical cooling surface

surface comes to have an elongated tail. The extremely long tail, however, was almost torn off and made into some of new falling drops. Such a fact also is shown in Fig. 2, where the drop appearing at the upper part of picture (3) is disrupted from its tail through processes as shown by pictures (4) and (5).

Fig. 3 shows the relation between the moving time t and distance x of a typical falling condensate-drop measured at the foremost and hindmost edges of it. The vertical distances between two curves represent the lengths of the drop. Therefore, the shapes of the drop at some instants are shown in Fig. 3. Fig. 4 shows the relation between the moving time t and the fore-

Fig. 3 Relation between moving time t and distance x of a falling condensate-drop

Fig. 4 Typical falling velocity of a condensate-drop

most or hindmost edge velocity, v_A or v_R , for the same drop as shown by Fig. 3. This result suggests the order of magnitude of the velocity of condensate drops in dropwise condensation.

4. Behavior of a water drop on an inclined surface

4.1 Experimental apparatus and method

Since, in practice, each of condensate-drops falls with increasing weight and changing shape by coalescing with other droplets on its path, it is quite difficult to analyze this behavior. For this reason, in the air, a single water drop of known weight was placed on an inclined surface and pictures of its moving state were taken by a 16 mm cine camera and a 35 mm camera. The results, thus, obtained were analyzed.

The surfaces used were a Ni-Cr plated brass surface and a sheet glass surface coated with paraffin-wax, both being 60 cm long and 6 cm wide.

For drop-materials, soft water obtained through an ion exchanger and several kinds of surface tension-water added with a surface active agent, Noigen ET 190 (manufactured by Daiichi Kogyo Seiyaku, Japan) to alter contact angle, were used.

Fig. 5 shows the result of measurements representing the relation between the concentration of Noigen and surface tension σ of the water added with Noigen. From this graph, the concentration of the agent for drop-water used was determined.

Fig. 6 shows the result of measurement representing the relation between the average angle θ_m and the surface tension σ of the Noigen soluble water drop placed on both horizontal surfaces, paraffin coated and Cr-plated.

To obtain the experimental water drops having desired weight, an injection needle was used and the weights of the drops were determined from the number of the dripping drops from it. When placing a water drop on an experimental surface, the drops dripping from the needle were first received on a special spoon coated with paraffin wax, and, holding the spoon tip as close as possible to the surface, the spoon was tilted and a water

Fig. 5 Surface tension σ of water added Noigen

drop was moved slowly on to the surface from the tip of spoon.

In investigating the behavior of water drop on a surface the contact angle is an important factor. For this measurement, an inclined surface was placed in parallel rays made by a combination of a zircon lamp for a point light source and a convex lens (14 cm in diameter and 90 cm in focal distance), and the silhouette of a resting drop or a moving drop on this surface was directly photographed. And the film negatives were enlarged onto a screen for measurement or printing. The components of the silhouette photography equip-

ment, the light source, lens, test surface and camera, all were fixed on a same axis and the inclinations of the surface were altered by rotation of this axis, so that the surface inclination was not recorded in films.

4.2 Experimental results

When placing water drops with various weights G on the surface with a constant angle α of inclination, the water drops with $G \leq G_0$ attached and rested on the surface, while those with $G > G_0$ rolled down on it. Therefore, drops on an inclined surface may be separated into either resting drops or rolling drops by weight. And the weight G_0 of a drop just before rolling down on the surface with the angle α of inclination is called a critical weight of resting drop at the surface-angle α .

4.2.1 Resting drop To find the critical weight G_0 of a resting drop at the angle α of inclination of a surface, a drop of known weight was placed on a horizontal surface and by increasing this angle gradually, the minimum angle, where the drop begins first to move, was obtained.

Fig. 7 represents the experimental results for two kinds of surface tension water on the Cr-plated surface, where plots are seen to be scattered widely, especially for smaller drops. This is considered to be due to lack of uniformity of the surface roughness, that is, the plated-surface, looking mirror-like smooth macroscopically, has microscopical roughness, which is apt to exert a great influence on smaller drops. Fig. 8 also shows the results for the Noigen-soluble-water of several concentrations on a paraffin-coated surface. In both figures, showing the curves drawn on $-3/2$ slope, the experimental points are expressed approximately by

$$G_0 \sin^{3/2} \alpha = \text{const.} \quad (1)$$

Fig. 6 Surface tension σ and average contact angle θ_m for water drop added with Noigen

Fig. 7 Measurements of critical weight of resting drop

Fig. 8 Effect of G_0 on concentration of Noigen

and it will be seen that, at the same α , G_0 for low surface tension is smaller than that for high one.

4.2.2 Rolling drop When placing a water drop with weight $G > G_0$ on an inclined surface, the drop with $G < G_c$ rolled down in a lump with a little deformation, while the drop with $G > G_c$ rolled down trailing a long tail elongated gradually, though having the same shape as with $G < G_c$ in the early stage of falling. Therefore, the rolling drop can be classified into two kinds of shapes called a tailless and a tailed drop respectively, and G_c may be referred to as the critical weight of a tailless drop for α .

Both shapes of a drop are shown in Fig. 9 as parts of successive motion pictures. The left edge of a vertical wide black line on a scale in each frame is a base line. A drop was placed on the surface at the point of an x cm distance away from this line to the upper side of the inclined

(a) $x=0$ (b) $x=5$ (c) $x=10$ (d) $x=20$
Fig. 9 Successive motion pictures illustrating the change of shape of a rolling drop

Fig. 10 Moving distance x for pure water drop

surface (left side in the picture). Picture (a) shows the appearances in the neighbourhood of $x=0$ where the water drop was moved from a spoon on to the surface and began to roll down, and (b), (c) and (d) show the appearances when the drop passing the neighbourhood of $x=5$, 10 and 20 cm respectively, indicating how the tail of

(b) Length
Fig. 11 Velocity and length of pure water drops for $\alpha=90^\circ$

drop was being elongated with time and distance. This example refers to the pure water drop having the highest surface tension in this experiment. Different behaviors were indicated by the drops with different surface tensions. Therefore the following description will be given by separating rolling drops into two kinds of surface tension.

(1) Water drop with high surface tension

The pure water drops with $G > G_0$ were placed on a Cr-plated surface at $\alpha = 30, 60$ and 90 deg. respectively and the behavior of these drops rolling down over a 50 cm distance was taken in motion pictures. Typical results obtained are shown in Figs. 10, 11 and 12. The tailed drops, remarkably elongated, were torn off at an instant next to the final point, suffering a remarkable change in weight and changing in velocity discontinuously. Therefore these changes are not shown in the graphs.

Fig. 12 Velocity of pure water drops for $\alpha = 30^\circ$

(a) Shapes

(b) Changes of θ_R and l

Fig. 13 Shapes and contact angles of a rolling drop at different x

From these graphs, it can be suggested that soon after beginning to move, the hindmost edge velocities v_R of the tailed drops reach a constant velocity v_{RC} regardless of the weight of drops, whereas the foremost edge velocities v_A continue to be accelerated even after reaching v_{RC} and then reduce after reaching the maximum velocity v_{Amax} .

In case of the tailed drops, therefore, it will be assumed that v_A is equal to the sum of v_R and dl/dt (called elongating velocity), and when reaching a terminal velocity without being torn off, the states of $dl/dt=0$ and $v_A=v_R$ may be attained. From a comparison of Fig. 11 and 12, it is clear that reducing α for the same weight G of drops, the values of v_{Amax} and v_R reduce, and the times, which v_A and v_R require to attain v_{Amax} and v_{RC} respectively, reduce.

In Fig. 13 (a) are sketches, taken from successive motion pictures, of a pure water drop with $G=0.1$ g rolling down at several positions on a paraffin-wax-coated surface at $\alpha=45^\circ$, in which the sketched drops are shown to coincide with each hindmost edge. (b) shows the relations of (a) by way of the curves. From these two Figures, it will be suggested that in the motion of a rolling drop on an inclined surface, the advancing contact angle θ_A remains constant independently of velocity throughout the experiment, while the receding contact angle θ_R , in the early stage of falling, decreases rapidly and, in the next stage, remains constant at a minimum value after reaching it, the tail of the drop elongating rapidly.

Fig. 14 Successive motion pictures illustrating a 0.1% Noigen-added-water drop beginning to move (Cr-plated surface, $\alpha=90^\circ$, $G=0.04$ g, time interval between frames 0.015 5 sec)

(2) Water drop with low surface tension

As an example of this case, experimental results using a 0.1 per cent Noigen-added-water as a drop-liquid on a Cr-plated surface are given. Fig. 14 shows how this drop placed on the surface will start to roll down.

In Fig. 15, (a) shows the relations of the moving times t and distances x_A of the foremost edge of drops for four different weights G and (b) shows the shapes of drop with $G=0.04$ g at four different moving times or distances, marked by A, B and C in (a), that is, in the early stage from starting point till B, which is shown with a solid line in (a), the drop is elongated with its hindmost edge fixed at starting point on the surface, namely $v_R=0$ and, in the next stage, it is torn off the elongated tail end, leaving small drops behind with a nearly constant pitch and finally it rolls down with the state shown by D. (c) shows the relations of v_A with time t for these drops.

From these data it will be seen that the reduction of surface tension of the rolling drops results in decrease of v_A and v_R , but inversely in increase of $v_A - v_R$.

4.2.3 Qualitative relations of rolling drops

The qualitative relations of rolling drops on inclined surfaces obtained from the above mentioned results are shown graphically in Fig. 16, where broken lines represent presumed state after the

Fig. 15 Experimental results for 0.1% Noigen-added-water drop

tail of the drop was torn off.

4.3 Analysis

A general equation of a water drop moving on an inclined flat surface is given as follows.

$$\frac{G}{g} \frac{dv_m}{dt} = G \sin \alpha - \sigma \int_0^b (\cos \theta_R - \cos \theta_A) db - R \quad (2)$$

Since in the above mentioned experiments it was shown that a tailless and a tailed drop acted in different manners from each other in motion, the following deals with the drop divided into these two kinds.

4.3.1 Tailless drop Since the tailless drop in motion changes slightly in shape from the segment of a sphere, $v_m = v_A = v_R$ and the contact diameter b and contact area B can be expressed by

$$b = k_1 b_i \quad (3)$$

$$B = \pi b^2 / 4 \quad (4)$$

where

$$b_i = f(\theta_m)_i (G/\gamma)^{1/3} \quad (5)$$

In the right side of Eq. (2), the 2nd term may be put as follows,

$$\int_0^b (\cos \theta_R - \cos \theta_A) db \equiv (\cos \theta_R - \cos \theta_A) k_2 b \quad (6)$$

Where θ_A and θ_R in the right side are values at the foremost and hindmost edge of the drop respectively. And referring to the experimental results that θ changed from θ_0 to θ_c while v_m varied from 0 to v_{RC} , the following may be assumed,

$$\cos \theta_R - \cos \theta_A = \theta_0 + \Delta \theta c_0 (v_m / v_{RC}) \quad (7)$$

Neglecting air resistance, R is only the force of frictional resistance caused at interface by

Fig. 16 Qualitative relations of rolling drops on an inclined surface

the viscosity of the drop-liquid and assuming the velocity distribution inside drop to be linear,

$$R = \tau B = \mu (dv/dt)_{y=0} B = 2\mu (v_m/y_m) B \\ = 2\gamma \mu v_m B^2 / G = \frac{\pi^2}{8} k_1^4 f(\theta_m) \left(\frac{G}{\gamma}\right)^{1/3} \mu v_m \quad \dots (8)$$

Substituting Eqs. (3)~(8) in Eq. (2) and integrating this with the initial condition as $v_m=0$ at $t=0$

$$v_m = v_\infty (1 - e^{-Yt}) \quad \dots (9)$$

where

$$v_\infty = \frac{G^{2/3} \gamma^{1/3} \sin \alpha - \sigma k_1 f(\theta_m)_1 k_2 \theta_0}{k_1 f(\theta_m)_1 \left\{ \sigma k_2 \frac{A \theta_{C0}}{v_{RC}} + \frac{\pi^2}{8} k_1^3 f(\theta_m)_1^3 \mu \right\}} \quad \dots (10)$$

$$Y = \frac{g k_1 f(\theta_m)_1}{G^{1/3} \gamma^{1/3}} \left\{ \sigma k_2 \frac{A \theta_{C0}}{v_{RC}} + \frac{\pi^2}{8} k_1^3 f(\theta_m)_1^3 \mu \right\} \quad \dots (11)$$

Futher, letting $v_m = dx/dt$

$$x = v_\infty \left\{ t - \frac{1}{Y} (1 - e^{-Yt}) \right\} \quad \dots (12)$$

The velocity and the moving distance of a tailless drop at any time can be calculated from Eqs. (9) and (12) respectively, where it is necessary to know the properties of material and the values of k_1 and k_2 in addition.

k_1 : When a drop began to move on an inclined surface, it was observed from the experiments to elongate in the direction of movement with the velocity. From this fact, k_1 will be considered as a function of the velocity. Here, k_1 under the two special conditions, just before beginning to move and just at the moment of attaining a critical velocity v_{RC} , i.e. $(k_1)_0$ and $(k_1)_C$ respectively, will be determined as follows.

The relation between the weight and shape of water drops placed on a horizontal surface can be calculated theoretically by using the same method as Takatama⁽²⁾. Fig. 17 shows the results obtained thus for $\sigma = 0.0739$ g/cm. Since the influence of gravity being changed with the shape of a drop on an inclined surface of the angle α is proportional to $\cos \alpha$, $(k_1)_0$ for any angle α is given as

$$(k_1)_0 = 1 + \cos \alpha \{ (k_1)_{0, \alpha=0} - 1 \} \quad \dots (13)$$

Fig. 17 Relation between G and k_1-1 on horizontal surface

Representing $(k_1)_C$ with area,

$$(k_1)_C = \frac{b_{C, \alpha}}{b_i} = \sqrt{\frac{B_{C, \alpha}}{B_i}} \\ = \sqrt{\frac{B_{0, \alpha}}{B_i}} \sqrt{\frac{B_{C, \alpha}}{B_{0, \alpha}}} = (k_1)_0 \sqrt{\frac{A_{C, \alpha}}{A_{0, \alpha}}} \quad \dots (14)$$

The experimental value of $\sqrt{A_{C, \alpha}/A_{0, \alpha}}$ was obtained as about 1.1 for water drops on a Cr-plated surface

k_2 : Putting $v_\infty = 0$ in Eq. (10)

$$(k_2)_0 = G_0^{2/3} \gamma^{1/3} \sin \alpha / \sigma \theta_0 (k_1)_0 f(\theta_m)_1 \quad \dots (15)$$

and putting $R=0$ and $G_0 = \gamma \int_0^b A db$ in Eq. (2)

$$\gamma A \sin \alpha = \sigma \theta_0$$

where A is nearly equal to the cross-sectional area of a segment of sphere with the contact angle θ_m .

Therefore

$$A = f(\theta_m)_2 (G_0 / \gamma)^{2/3}$$

Thus

$$G_0^{2/3} \gamma^{1/3} \sin \alpha = \sigma \theta_0 / f(\theta_m)_2 = \text{const.} \quad \dots (16)$$

This relation agrees with the experimental results as shown in Figs. 7 and 8.

Combining Eqs. (15) and (16)

$$(k_2)_0 = 1 / (k_1)_0 f(\theta_m)_1 f(\theta_m)_2 \quad \dots (17)$$

Putting $v_\infty = v_{RC}$ in Eq. (10)

$$(k_2)_C = \frac{G_0^{2/3} \gamma^{1/3} \sin \alpha - \frac{\pi^2}{8} (k_1)_C^4 f(\theta_m)_1^4 \mu v_{RC}}{\sigma \theta_0 (k_1)_C f(\theta_m)_1} \quad \dots (18)$$

Thus the values of k_1 and k_2 can be assumed from these relations, and consequently v_m and x of the tailless drops can be calculated from Eqs. (9) and (12). Table 1 shows an example of k_1 and k_2 calculated under the same condition that water drops roll down on a Cr-plated surface. Figs. 18 and 19 are examples of the calculated v and x of the tailless drops where the calculated curves are compared with experimental points. These curves have been obtained neglecting the experimental fact that a drop must change from a tailless drop to a tailed drop whenever its

Table 1 An example calculated k_1 and k_2

α	Observed		k_1 or k_2	Condition of drop			
	G_0	v_{RC}		Just before rolling down		Moment of critical velocity attained	
90°	0.032 g	58 cm/sec	k_1	$(k_1)_0$	1.0	$(k_1)_C$	1.5
			k_2	$(k_2)_0$	0.666	$(k_2)_C$	0.522
30°	0.091 g	45 cm/sec	k_1	$(k_1)_0$	1.055	$(k_1)_C$	1.16
			k_2	$(k_2)_0$	0.631	$(k_2)_C$	0.496

Conditions used for calculation: $\theta_m = 90^\circ$, $\theta_0 = 0.305$, $\theta_C = 1.4$, $\gamma = 1$ g/cm, $\sigma = 0.0739$ g/cm, $\mu = 1.3 \times 10^{-5}$ g sec/cm

velocity exceeds the critical velocity v_{RC} . It, nevertheless, seems from both figures that the calculated curves are generally in good agreement with the experimental points till about the maximum velocity of the tailed drops as well as in the range of the tailless drops.

4.3.2 Tailed drop A tailed drop consists of two parts called a dropwise head and an elongated tail. Now, consider only the tail part cut off from the tailed drop as shown in Fig. 20 and moving on an inclined surface under steady state. Since, then, $dv_m/dt=0$ and $\cos \theta_R - \cos \theta_A = 0$ in Eq. (2),

$$R = G_R \sin \alpha \quad (19)$$

On the other hand, in the same way that Eq. (8) was derived,

$$\begin{aligned} R &= \tau_R B_R = 2\gamma \mu v_m G_R (B_R/G_R)^2 \\ &= 2\gamma \mu v_m G_R \{f(\theta_m)_3 / \gamma b_R\}^2 \end{aligned}$$

Therefore,

$$v_R = v_m = \frac{\gamma \sin \alpha}{2\mu} \left\{ \frac{b_R}{f(\theta_m)_3} \right\}^2 \quad (20)$$

The calculated values v_R , obtained by substituting the experimental values into the right hand of Eq. (20), agreed almost with the observed tail end velocities.

From this fact and for the reason that the tail velocity v_R is independent of the tail length from Eq. (21), the average velocity v_m of a tailed drop may, in general, be put as follows

$$v_m = \left(1 - \frac{G_R}{G}\right) v_A + \frac{G_R}{G} v_R \quad (21)$$

And also

$$v_A = v_R + \frac{dl}{dt} \quad (22)$$

Fig. 18 Comparison of experimental values with calculated values from Eq.(12)

$$l = b_A + l_R \quad (23)$$

$$G_R = \gamma b_R^2 l_R / f(\theta_m)_3 = a_R l_R \quad (24)$$

$$b_A = (G - G_R)^{1/3} k_1 f(\theta_m)_1 / \gamma^{1/3} \quad (25)$$

Since $\theta_C = \text{const.}$ from the experiments

$$\int_0^t (\cos \theta_R - \cos \theta_A) db \equiv \theta_C b_A k_3 \quad (26)$$

Separating R into two parts of the head and

Fig. 19 Comparison of experimental values with calculated values from Eq.(9)

Fig. 20 Illustration of tail of a tailed drop

tail of the tailed drop, then

$$R = R_A + R_R \dots (27)$$

And in the same way as deriving Eq. (19)

$$R_R = G_R \sin \alpha = \gamma b_R^2 (1 - b_A) \sin \alpha / f(\theta_m)_3 \dots (28)$$

And

$$R_A = \tau_A \pi b_A^2 / 4 \dots (29)$$

In these relations, only τ_A can not be measured or assumed. Therefore, by substitution of Eqs. (20)~(29) into Eq. (2), the following is obtained,

$$\tau_A = \frac{4}{\pi b_A^2} \left\{ (G - G_R) \left(\sin \alpha - \frac{1}{g} \frac{d^2 l}{dt^2} \right) + \frac{\gamma b_R^2}{g f(\theta_m)_3} \left(\frac{dl}{dt} \right)^2 - \sigma \Theta c b_A k_3 \right\} \dots (30)$$

The typical examples of τ_A obtained by substituting observed and assumed values into the right hand of Eq. (30) are given in Fig. 21 which shows how τ_A relates with v_A by comparing the shapes of two curves. From this, it will be seen that τ_A increases sharply from about the time that the drop begins to elongate. It, therefore, may be considered that τ_A consists of $\tau_{A,vC}$ and $\tau_{A,dl}$ related to v_{RC} and dl/dt respectively, namely

$$\tau_A = \tau_{A,vC} + \tau_{A,dl} \dots (31)$$

where, in the same way as Eq. (7), $\tau_{A,vC}$ is derived as follows.

$$\tau_{A,vC} = \frac{\pi}{2} k_1^2 f(\theta_m)_1^2 \mu v_{RC} \gamma^{1/3} (G - G_R)^{-1/3} \dots (32)$$

Thus, from Eqs. (30)~(32), it is possible to determine $\tau_{A,dl}$.

Fig. 22 shows the relation of $\tau_{A,dl}$ with dl/dt for several weights G of water drops on a Cr-plated vertical surface. An arrow indicates the direction of lapse of the time and an open circle on each curve indicates a point where the drop attains the maximum velocity v_A or maximum elongating velocity dl/dt . Each of the curves, accordingly, is separated by the mark into the two

Fig. 21 An example of relation of τ_A and v_A with time

regions of acceleration and deceleration. In this figure, it will be seen that these two separated curves don't fall on a line. This fact is considered to be due to the inertia force of the head of drop.

Fig. 23 shows the relation of $\tau_{A,dl}$ with the maximum dl/dt for three kinds of surface inclinations obtained from the same experiment as shown in Fig. 22. This curve, representing almost the experimental points except some, corresponds

Fig. 22 Relation of dl/dt and $\tau_{A,dl}$

Fig. 23 Relation of $(dl/dt)_{\max}$ and $\tau_{A,dl}$

to $m=2$ in the following equation.

$$\tau_{A,dl} = \zeta (dl/dt)^m \quad \dots\dots\dots (33)$$

By substituting Eqs. (24) and (31)~(33) in Eq. (30) and letting $m=2$, the following is obtained

$$\left(\frac{G}{a_R} - l_R\right) \frac{d^2 l_R}{dt^2} + \left(\frac{g}{a_R} \zeta \frac{\pi}{4} b_A^2 - 1\right) \left(\frac{dl_R}{dt}\right)^2 - g \sin \alpha \left(\frac{G}{a_R} - l_R\right) + \frac{\pi}{2} k_1^2 f(\theta_m) i^2 \mu \nu_R \gamma^{1/3} a_R^{-4/3} \left(\frac{G}{a_R} - l_R\right)^{-1/3} + \frac{g}{a_R} \sigma \Theta c k_3 b_A = 0 \quad \dots\dots\dots (34)$$

Neglecting the fourth term which has smaller values than the other, and considering b_A to be a constant value, the solution of Eq. (34) with the boundary condition as $l_R=0$ and $dl_R/dt=0$ at $t=0$ is

$$\frac{dl_R}{dt} = H^{1/2} \left[J \left\{ 1 - \left(1 - \frac{l_R}{L} \right)^{2n} \right\} - \frac{l_R}{L} \right]^{1/2} \quad \dots\dots\dots (35)$$

where

$$n = \frac{g}{a_R} \zeta \frac{\pi}{4} b_A^2 - 1, \quad L = \frac{G}{a_R}, \quad H = \frac{2g \sin \alpha \frac{G}{a_R}}{2n-1}, \quad J = 1 - \left(1 - \frac{1}{2n} \right) \frac{\sigma \Theta c k_3 b_A}{G \sin \alpha} \quad \dots\dots\dots (36)$$

By differentiating Eq. (35) by l_R/L and equating this to zero, the relation that the value of dl_R/dt has the maximum $(dl_R/dt)_{\max}$ is obtained as follows.

$$l_R/L = 1 - (1/2nJ)^{1/(2n-1)} \quad \dots\dots\dots (37)$$

Therefore from Eq. (35)

$$\left(\frac{dl_R}{dt}\right)_{\max} = \left[\frac{2g \sin \alpha \frac{G}{a_R}}{2n-1} \left\{ 2n - \frac{4n^2-1}{2n} \left(\frac{\sigma \Theta c k_3 b_A}{G \sin \alpha} \right) \right\} \right]^{1/2} \quad \dots\dots\dots (38)$$

From this relation it seems that $(dl_R/dt)_{\max}$ is a function of n .

Then by searching for such value of n that the values of $(dl/dt)_{\max}$ in Eq. (38) may be nearly equal to experimental values, we obtained about 6 independently of the weights G of drops. This reason for $n \approx \text{constant}$ is probably the fact that n contains b_A/b_R as a factor and b_A/b_R tends to become a constant value, since each of b_A and b_R in practice is inversely proportional to the length of drop.

Fig. 24 shows the comparison between the calculated curves obtained from Eq. (35) given as $n=6$ and the experimental points. It is seen from this that Eq. (35) agrees well with experiments. In calculation, the term $\{1 - (l_R/L)\}^{2n}$ is nearly equal to zero when l_R increases in small degree, and therefore neglecting this term and integrating Eq. (35) with the initial condition as $l_R=0$ at $t=0$,

$$l_R = \sqrt{HJ} t - \frac{H}{4L} t^2 \quad \dots\dots\dots (39)$$

Substituting this in Eq. (35),

$$\frac{dl_R}{dt} = H^{1/2} \left[J \left\{ 1 - \left(1 - \frac{\sqrt{HJ}}{L} t + \frac{H}{4L^2} t^2 \right)^{2n} \right\} - \frac{\sqrt{HJ}}{L} t + \frac{H}{4L^2} t^2 \right]^{1/2} \quad \dots\dots\dots (40)$$

Fig. 25 shows the comparison between calculated curves predicted from Eq. (40) and experimental points obtained by the same experiment as represented in Fig. 24, where the calculated curves are placed so that their maximum points may agree with those of the experimental points. From this figure it seems that Eq. (40) represents the state of the drop after reaching the maximum velocity $(dl_R/dt)_{\max}$ and does not one before that. On the other hand, as shown in Figs. 18 and 19, v_A of the tailed drops agrees well with Eq. (9) till reaching about $v_{A\max}$. Therefore it may be possible to apply Eq. (9) for the process of v_A changing from v_{RC} till $v_{A\max}$. Thus, from the condition that two curves obtained from Eqs. (9)

Fig. 24 Comparison of experimental values with Eq. (35)

Fig. 25 Comparison of experimental values with Eq. (40)

and (40) should contact each other at 1 in Fig. 12, the following two equations are obtained.

$$e^{-Yt_1} = 1 - \frac{v_{RC}}{v_\infty} - \frac{H^{1/2}}{v_\infty} \left[J \left\{ 1 - \left(1 - \frac{\sqrt{HJ}}{L} \Delta t_{1P} + \frac{H}{4L^2} \Delta t_{1P}^2 \right)^{2n} \right\} - \frac{\sqrt{HJ}}{L} \Delta t_{1P} + \frac{H}{4L^2} \Delta t_{1P}^2 \right]^{1/2} \quad (41)$$

$$e^{-Yt_1} = \frac{H^{1/2}}{v_\infty Y} \frac{\left(\frac{\sqrt{HJ}}{L} - \frac{H}{2L^2} \Delta t_{1P} \right) \left\{ 2nB \left(1 - \frac{\sqrt{HJ}}{L} \Delta t_{1P} + \frac{H}{4L^2} \Delta t_{1P}^2 \right)^{2n-1} - 1 \right\}}{\left[J \left\{ 1 - \left(1 - \frac{\sqrt{HJ}}{L} \Delta t_{1P} + \frac{H}{4L^2} \Delta t_{1P}^2 \right)^{2n} \right\} - \frac{\sqrt{HJ}}{L} \Delta t_{1P} + \frac{H}{4L^2} \Delta t_{1P}^2 \right]^{1/2}} \quad (42)$$

where $\Delta t_{1P} = t_1 - t_P$. From the above two equations, the values of t_1 and t_P can be determined. And applying to Eq. (40) the conditions that dl_R/dt and the term of $2n$ -th power are equal to zero, the time t_2 indicated by a point 2 in Fig. 26 is given as follows,

$$t_2 = t_P + 2L \sqrt{\frac{J}{H}} \quad (43)$$

From the above mentioned, when $G > G_C$, v_A and x_A are given for the moving time t divided into three ranges as follows:

for $t = 0 \sim t_1$

v_A and x_A are obtained from Eqs. (9) and (12) respectively, for $t = t_1 \sim t_2$

$$v_A = \frac{dl_R}{dt} + v_{RC} = H^{1/2} \left[J \left\{ 1 - \left(1 - \frac{\sqrt{HJ}}{L} (t - t_1) + \frac{H}{4L^2} (t - t_1)^2 \right)^{2n} \right\} - \frac{\sqrt{HJ}}{L} (t - t_1) + \frac{H}{4L^2} (t - t_1)^2 \right]^{1/2} + v_{RC} \quad (44)$$

$$x_A = v_\infty \left\{ t_C - \frac{1}{Y} (1 - e^{-Yt_C}) \right\} + v_{RC} (t - t_C) + l_R \quad (45)$$

$$\left. \begin{aligned} \text{where } t_C &= \log \left(1 - \frac{v_{RC}}{v_\infty} \right)^{-1/Y} \\ l_R &= L \left[J - \left\{ J^{1/2} - \frac{\sqrt{H}}{2L} (t - t_P) \right\}^2 \right] \end{aligned} \right\} \quad (46)$$

and for $t = t_2 \sim \infty$

$$v_A = v_{RC} \quad (47)$$

$$x_A = v_\infty \left\{ t_C - \frac{1}{Y} (1 - e^{-Yt_C}) \right\} + v_{RC} (t - t_C) + LJ \quad (48)$$

Similarly v_R and x_R are given for t divided into two ranges as follows:

for $t = 0 \sim t_C$

$$v_R = v_A \quad (49)$$

$$x_R = x_A \quad (50)$$

and for $t = t_C \sim \infty$

$$v_R = v_{RC} \quad (51)$$

$$x_R = v_\infty \left\{ t_C - \frac{1}{Y} (1 - e^{-Yt_C}) \right\} + v_{RC} (t - t_C) \quad (52)$$

And, l_R , the tail length of the drop, is equal to $x_A - x_R$ at that time.

Fig. 26 shows an example that a solid curve obtained by calculation of the above equations is in good agreement with the experimental points for the pure water drop of $G = 0.06$ g on a Cr-plated vertical surface.

5. Conclusion

From both of the experiments and theoretical analysis mentioned above, the behavior of a water drop with constant weight moving on an inclined surface has been made clear, and formulas predicting the velocity, moving distance and length of the drop have been derived.

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Fig. 26 Comparison of experimental and calculated values