concentration at the fillet, as will be shown in the answer (9).

(9) As shown in Append-Fig.3, when the measured results are temporarily extended in straight lines, the intersecting points of the lines with the calculated results are the positions of 12.7 mm and 14.0 mm, respectively from the tips of the teeth for both the pinion and the wheel. Accordingly, it is considered that the points which are not influenced by stress concentration at all are closer to the tips of the teeth than the positions above shown.

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Profile Modification of Helical Gear Teeth*

By Hajime KUGIMIYA**

Few investigations are available concerning the profile modification of helical gear teeth and there are many important problems which should be clarified.

The object of the present investigation is to find out the effect of the profile modification on the dynamic properties of helical gears, and furthermore to derive the method for determining the optimum amount of profile modification.

The profile modification of both non-crowned and crowned helical gear teeth was dealt with in this investigation.

As the result of the investigation, it was found that the vibration caused by a gear mesh was reduced by giving a suitable profile modification to the teeth, especially with regard to the crowned helical gears, and the optimum amount of profile modification to the teeth of both non-crowned and crowned helical gears could be determined from the tooth deflection under working load, similarly to the case of spur gears.

1. Introduction

The profile modification of spur gear teeth is necessary to prevent sudden fluctuation in angular velocity due to a sudden change in the length of line of contact, and the formulas for calculating the optimum amount of profile modification have been derived analytically(1) and experimentally(2) by many investigators.

However, few investigations(3)(4) are available.
concerning the profile modification of helical gear teeth and there are many important problems which should be clarified. For instance, the optimum amount of it used to be determined only by experiences.

The object of the present investigation is to find out the effect of the profile modification on the dynamic properties of helical gears, and furthermore to derive the method for determining the optimum amount of profile modification.

The profile modification of both the non-crowned and the crowned helical gears is dealt with in this investigation.

2. Method of experiment

The experiments were conducted by the power circulating type gear testing machine\(^\text{29}\), of which the maximum circulating power was 900 HP.

The dimensions of test gears are shown in Table 1.

The gears have normal module 8, normal pressure angle of 20 deg and helix angle of 20 deg. And numbers of teeth of the pinion and wheel are 29 and 63, respectively.

As regards the pinions, those without and with crowning were tested.

In the profile modification, four kinds of corrections were given successively to both the non-crowned and crowned pinion, and four series of tests were conducted respectively in the condition of meshing with the same one wheel.

The dynamic stresses in the teeth and the gear noise were measured in each test, and effects of profile modification on the dynamic properties of helical gear teeth were investigated by comparing the results of these measurements.

Four kinds of corrections given to the non-crowned pinion teeth were 0 micron, 15 microns, 30 microns, and 45 microns, respectively and all

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Fig. 1 Typical profile error diagrams of test gears

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Fig. 2 Typical lead error diagrams of test gears

Regarding b, crowning is given symmetrically to the center of face width.
Corrections were given to both the tip and the root of each tooth. For the crowned pinion teeth, 15 microns was given to only the tip, and 0 micron, 15 microns and 30 microns were given to both the tip and the root.

Corresponding amounts to these corrections were removed from the tooth profiles successively after each test by Maag gear grinding machine. The tip and root modification were extended respectively from the tips of each tooth down to points corresponding to a distance of a half module and from the meshing points with the tips of the mating teeth up to points corresponding to a distance of a half module.

Fig. 1 and Fig. 2 are the typical diagrams of tooth profile and lead errors of the test gears, which were measured by Maag gear testing machine, PH-60.

The tooth profile errors of the non-crowned and crowned pinion were 10 microns and 8 microns, respectively, covering those in all tests, and the error of the wheel was 12 microns. Four kinds of corrections which were given actually to the pinion teeth were nearly equal to the planned amounts respectively, though the tooth profiles with the planned 0 micron-correction both with and without crowning in test 1 resulted in the same profiles as modified by a little amount, owing to their tooth profile errors as shown in Fig. 1.

The lead errors of the non-crowned pinion and the wheel were 7 microns and 8 microns, respectively, and the given amounts of crowning were nearly equal in all tests, though the amount in test 2 was about 10 microns greater than those in other tests, as can be seen from Fig. 2.

The pinions with and without crowning had the maximum normal pitch errors of 15 microns and of 14 microns, respectively, including those in all tests, and normal pitch error of the wheel was 14 microns.

Dynamic stresses in the teeth were measured by the bakelite strain gauges of 3 mm gauge length. The strain gauges were cemented in recesses ground to about 1 mm depth at the tooth flanks on the compressive sides of both the pinion and wheel tooth. They were located at three points along the helix as shown in Fig. 3. A gear noise was measured by using a condenser microphone, audio frequency spectrometer, type 2111 and a high speed level recorder, type 2304 made in Bruel & Kjaer company.

Experiments were conducted under the peripheral speed, from 5.7 m/sec to 45.3 m/sec, and the peripheral force, from 400 kg to 4000 kg.

3. Experimental results

The variations of dynamic stresses in the test gear teeth were not so different among three points along the helix, in both the non-crowned and the crowned gears. Therefore, only the dynamic stresses in the teeth measured at the centers of each helix would be shown.

And as it was found that the noise of test gears could be represented by the noise levels of the tooth contact frequency, according to the investigations on the noise characteristic of the test gears, the comparison of the gear noise in each test was done by the noise levels of this frequency.

3-1 The dynamic properties of non-crowned helical gears

The variations of dynamic stresses in the test gear teeth measured in four series of tests were shown in Fig. 4. These variations are represented in the percentage of the maximum stresses in the teeth during a gear mesh, i.e., the maximum stresses in the teeth occurring at the minimum speed, 5.7 m/sec, are represented as 100 percent, and those at the other speeds and the same load are shown in percentage.

It will be seen from Fig. 4 that in the case of test 1, the variations of dynamic stresses in wheel teeth are different from those in pinion teeth. The reason for this difference is that the contact stresses in tooth surfaces of the wheel and pinion are remarkably changed by the initial engagement impulse and the meshing point corresponding to the maximum stress in the wheel tooth moves from the tip side down to the root side of the tooth. Owing to this phenomenon, it will be known that the vibration...
induced by a gear mesh is much greater.

It will be obvious from Fig. 4 that the vibration induced by a gear mesh is reduced by giving the profile modification to the teeth, and that the increase of dynamic stresses in the teeth is the least in the amount of profile modification of 15 microns.

However, in this case, the effect of profile modification on the dynamic stresses in the teeth will not be expected so much, because the variations of the dynamic stresses in the test gear teeth measured in four series of tests are obtained in the nearly equal condition.

The graphs in Fig. 5 show the noise levels of the tooth contact frequency of the test gears. These graphs show the relation between the noise levels and the gear speed under the applied peripheral force of 1200 kg, as a typical example. The gear noise induced by the peripheral force of 1200 kg is the least when the amount of profile modification is about 15 microns.

As wellknown, the optimum amount of profile modification should be given with consideration of the load condition applied to the gear tooth, and the range of the peripheral force, in which the noise level of the test gears with the amount of
In short, it will be clear from both the results measuring the dynamic stresses in the teeth and gear noise that the vibration caused by a gear mesh is reduced by giving profile modification to the teeth and that the optimum amount of profile modification of this helical gears without crowning is about 15 microns.

3-2 The dynamic properties of crowned helical gears

The graphs in Fig.6 show dynamic stresses in the test gear teeth with crowning measured in four series of tests in the same way as shown in Fig. 4.

It will be seen from Fig.6 that further improvement in the dynamic stresses in crowned helical gear teeth is obtained if a suitable profile modification is given, similarly to the non-crowned helical gears, and that the increase of dynamic stresses in the teeth is the least in the amount of profile modification of 15 microns to both the tips and roots of the pinion teeth.

In the same way as shown in Fig.5, the gear noise of the tooth contact frequency of the test gears measured under the applied peripheral force of 1200 kg is shown in Fig. 7. The gear noise induced under the peripheral force of 1200 kg was the least with the gears in test 3, in which modification to both the tips and roots of the pinion teeth amounted to about 15 microns. The range of the peripheral force giving the least noise of the test gears with the amount of correction of 15 microns was between 600 kg and 2000 kg.

In short, it is better to give a suitable profile modification with consideration of the load condition in crowned helical gears too, and the optimum amount of profile modification of the crowned test gears is about 15 microns, which is equal to the one of the non-crowned test gears.

Furthermore, it will be clear from comparison of Fig.4 with Fig.6, that the increase of the dynamic stresses in the helical gear teeth which is given both a crowning and a suitable profile modification is far less than that of gear teeth with a suitable profile modification only, and the vibration induced by a gear mesh of the former is less than that of the latter.

In the following chapter, the load distribution factor and the tooth deflection of both the test gears with and without crowning will be calculated, and the optimum amount of profile modification of helical gear teeth will be investigated theoretically.

4. The optimum amount of profile modification of non-crowned helical gear teeth

4-1 Load distribution factor and tooth deflection

The tooth stiffness per unit length of helical gear tooth will be calculated from the formula derived by C. Weber and K. Banaschek, and then the load distribution factor and tooth deflection will be calculated from the stiffness of a pair of helical gear teeth which can be obtained from the tooth stiffness per unit length.

On this occasion, the calculation was done based on the following assumptions.

(1) The stiffness at any point of contact on the helical gear tooth is equal to the stiffness per unit length of the tooth in the plane corresponding to the point of contact.

(2) The tooth deflection is equal at all points along the total length of lines of contact.

According to C. Weber and K. Banaschek, the sum of the deflections of a pair of teeth has been given by the following equation:

$$\delta_s = \frac{w}{E}q_s$$ \hspace{1cm} (1)

where

- $w$: normal tooth load per unit length,
- $E$: modulus of elasticity,
- $q_s$: deflection coefficient.

All the deflections, i.e., the deflection due to surface deformation at the point of contact, the deflections due to bending, compression and shear in the tooth and the deflection due to the inclination of the subsequent part of the tooth are considered in this equation.

From Eq. (1), the stiffness per unit length of a pair of teeth $k_s$ is
\[ \kappa' = \frac{\nu}{\partial_s} = \frac{E}{q_s} \]  \hspace{1cm} (2)

Now, in the helical gear teeth, if the center of the length of the line of contact, 2l, is taken as the origin, and taking z axis along the line of contact, the stiffness of a pair of helical gear teeth \( \kappa \) is calculated as follows from the assumption (1) and (2):

\[ \kappa = \int_{-l}^{l} \kappa' \, dx = \frac{1}{\partial_s} \int_{-l}^{l} \nu \, dx = \frac{P_i}{\partial_s} \]  \hspace{1cm} (3)

where \( P_i \) is the normal tooth load subjected to the line of contact at which the tooth stiffness is calculated.

Considering a simultaneous meshing of three pairs of teeth, and taking the loads split between each pair of teeth as \( P_1, P_2, \) and \( P_3 \) and the deflections of each pair of teeth as \( \delta_{1s}, \delta_{2s}, \) and \( \delta_{3s}, \) respectively, the stiffnesses of each pair of teeth \( \kappa_1, \kappa_2, \) and \( \kappa_3 \) are

\[ \kappa_1 = \frac{P_1}{\delta_{1s}}, \quad \kappa_2 = \frac{P_2}{\delta_{2s}}, \quad \kappa_3 = \frac{P_3}{\delta_{3s}} \]

from Eq. (3).

Here, since \( \delta_{1s} = \delta_{2s} = \delta_{3s} \), according to the assumption, the load distribution factors of each pair of teeth, \( f_1, f_2, \) and \( f_3 \) are determined as follows:

![Graphs showing variations of dynamic stresses in test gear teeth with crowning against peripheral speed of the gear.](image-url)
\[ f_1 = \frac{P_1}{P_1 + P_2 + P_3} = \frac{\kappa_1}{\kappa_1 + \kappa_2 + \kappa_3} \]
\[ f_2 = \frac{P_2}{P_1 + P_2 + P_3} = \frac{\kappa_2}{\kappa_1 + \kappa_2 + \kappa_3} \]
\[ f_3 = \frac{P_3}{P_1 + P_2 + P_3} = \frac{\kappa_3}{\kappa_1 + \kappa_2 + \kappa_3} \]

Consequently, the deflection of a pair of teeth \( \delta \) on this occasion is calculated from
\[ \delta = \delta_{sl} = \delta_{rl} = \delta_{sl} = \frac{P}{\kappa_1 + \kappa_2 + \kappa_3} \]  

The load distribution factor and the deflection of a pair of the test gear teeth calculated from Eqs. (4) and (5) were shown in Fig. 8(a) and (b), respectively.

In Fig. 8(a), the load distribution factors obtained by experiment\(^{(5)}\) were shown also. It will be obvious that the factor in calculation is comparatively in good agreement with the factors in experiment, and consequently that the assumptions applied to this calculation are nearly correct.

4-2 The optimum amount of profile modification

It will be obtained from the tooth deflection shown in Fig. 8(b) how the profile modification should be given to the helical gear teeth. There are almost no variations in the deflection of a pair of the test gear teeth during a gear mesh. However, the deflection is suddenly changed at the initial and the final engagement and it may be supposed that the vibration is induced by these engagement impulses at that time.

This suggests that the vibration induced by a gear mesh should be reduced by removing the materials by the amounts equal to the deflection at the initial and final engagement from the root and tip of the pinion tooth, respectively (otherwise, from the tips of both wheel and pinion teeth). These amounts of the test gears are about \( 6U/1000 \) microns at both the initial and final engagement, if the peripheral force is \( U \) in kg. When the peripheral force is between 600 kg and 2,800 kg, these are between 4 microns and 17 microns.

According to the experimental result, the optimum amount of profile modification of the test gears was 15 microns in the range of the same peripheral force as shown above. The amount of tooth deflection in calculation is in good agreement with the amount of correction in experiment.

That is, the optimum amount of profile modification of non-crowned helical gear teeth can be determined from the tooth deflection under working load, similarly to spur gear teeth\(^{(1)}\).

In spur gears, there are two meshing points at which the load subjected to the teeth and the tooth deflection are suddenly changed (the points at which one tooth begins and ends to carry the full load), and the corrections are generally started at...
these points (1). However, in helical gears, there is none of these meshing points generally, as will be seen also from Fig. 8, so that the starting points for corrections cannot be clearly determined. It may be enough in practice to give profile modification in the range of a half module from the tips and the meshing points with the tips of mating teeth as given in this experiments.

5. The optimum amount of profile modification of crowned helical gear teeth

5.1 The amount of relief due to profile modification and crowning

In order to apply for the calculation of load distribution factor and tooth deflection, the distance between tooth surfaces of wheel and pinion in contact, which is caused by the profile modification and crowning, and is at right angles to their line of contact (to be followed as the amount of relief) will be calculated, and then the relation between the tooth load and the amount of relief reduced under the load will be obtained.

Fig. 9 shows the result of calculation for the amounts of relief along the lines of contact which lay at the interval of a quarter of the length of action at the center of the face width of the test gear. The amounts of relief for the gear teeth with both the amount of profile modification of 15 microns and crowning are shown as a typical example.

If the amount of profile modification and that of crowning measured by Maag gear testing machine, PH-60 are put as $\varepsilon_m$ and $\varepsilon_c$ respectively, the amount of relief $\varepsilon$ is calculated (9) from

$$
\varepsilon = \varepsilon_1 + \varepsilon_2 = (\varepsilon_m + \varepsilon_c) \cos \beta_p
$$

where

$\varepsilon_1$: amount of relief at right angles to line of contact due to profile modification

$\varepsilon_2$: amount of relief at right angles to line of contact due to crowning

$\beta_p$: base helix angle

The equivalent lines for the amounts of relief of 10 microns and 20 microns are shown together in Fig. 9.

For the test gear teeth with both the crowning and the amount of profile modification of 30 microns as a typical example, the figure of the tooth bearing under the peripheral force of 3200 kg was shown in comparison with the figure of the equivalent line for the amount of relief of 20 microns in Fig. 10. These two figures are in good agreement with one another.

Therefore the relation between the tooth load...
and the amount of relief reduced under the load may be obtained by considering the maximum length measured to the direction of the face width of the tooth bearing under working load as equal to the maximum length of the equivalent line for the suitable amount of relief corresponding to the load.

In Fig. 11, the relation between the peripheral force and the maximum length of the tooth bearing obtained by experiment was shown together with the relation between the amount of relief and the maximum length of the equivalent line for the amount of relief obtained from Fig. 9, based on the consideration mentioned above.

According to Fig. 11, for instance, the peripheral force which should be subjected to the teeth for the approach by an amount of 10 microns of the distance between the tooth surfaces of wheel and pinion is 1,000 kg and the peripheral force for the approach by an amount of 20 microns is 3,200 kg.

The relation between the peripheral force, U and the amount of relief reduced under the load, y in this case, can be calculated approximately from

\[ U = 6.74y^2 + 27.48y \]  \hspace{1cm} (7)

5.2 Load distribution factor and tooth deflection

With regard to the helical gear teeth with both profile modification and crowning, it will be considered that under the load subjected to the teeth, the tooth surfaces of wheel and pinion contact to the extent of the equivalent line for the amount of relief corresponding to the applied load, together with the deformatins of the teeth and the subsequent parts of the teeth.

The load distribution factor and the tooth deflection of crowned helical gear teeth were calculated with the following assumptions in addition to the assumptions (1) and (2) shown previously.

(3) The load P acting on the helical gear teeth with profile modification and crowning can be divided into the two loads \( P_A \) and \( P_B = P - P_A \), where \( P_A \) is the load necessary only to let the tooth surfaces contact to the extent of the equivalent line for the amount of relief corresponding to the load \( P \), and \( P_B \) is the load necessary for the other deformations. Consequently, the tooth deflections can be calculated as the sum of the deflections caused by \( P_A \) and \( P_B \).

(4) The distributed load \( w_A \) at any point of contact regarding the load \( P_A \) is calculated from \( w_A = \kappa'(y_0 - y) \), where \( \kappa' \) is the stiffness per unit length of a pair of teeth at that point, \( y_0 \) is the amount of relief reduced by the load \( P \) and \( y \) is the initial amount of relief at that point.

(5) The load \( P_B \) acts alone to the extent of the equivalent line for the amount of relief corresponding to the load \( P \), similarly to the load \( P_A \) and the magnitude is determined from \( P - P_A \) when \( P_A \) is obtained from the assumption (4). Concerning the load \( P_B \) and the deflection \( \delta_B \) due to \( P_B \), these can be considered the same as the load and the deflection of the non-crowned helical gears, and the distributed load and the deflection can be calculated from the assumptions (1) and (2).

An example of this calculation is shown in Fig. 12. In this example, three pairs of teeth are in meshing simultaneously and the tooth surfaces of the gears contact to the extent of the equivalent line for the amount of relief of 20 microns. The load \( P \) acting on the tooth surfaces in this condition can be obtained from Fig. 11.

Fig. 12(a) is the result of calculating the loads \( P_A \), \( P_{AB} \) and \( P_{AB} \) taken with each line of contact for the load \( P_A \), and the deflection of a pair of teeth \( \delta_A \) in this case is determined from \( \delta_A = (20 - y_{\text{min}}) \) microns, where \( y_{\text{min}} \) microns are the mini-
maximum amounts of relief under no load along each line of contact.

The loads $P_{BI}$, $P_B$, and $P_{BS}$ calculated for the load $P_B$ are shown in Fig. 12(b). These shared loads and the tooth deflections are calculated by substituting $P$ with $P_B$ in Eqs. (4) and (5) respectively, as mentioned in the assumption (5).

The loads $P_A$, $P_B$, and $P_C$ taken with each line of contact in the crowned helical gear teeth are obtained by adding the loads on the corresponding lines of contact shown in the figure (a) and (b), as shown in Fig. 12(c).

When the load distribution factors in each line of contact are taken as $f_{SL}$, $f_{SB}$, and $f_{SA}$, and the tooth deflection as $\delta_{mi}$, these are calculated from

$$
\begin{align*}
 f_{SL} & = \frac{P_{AI} + P_{BI}}{P} = \frac{P_A}{P} \\
 f_{SB} & = \frac{P_{AS} + P_{BS}}{P} = \frac{P_B}{P} \\
 f_{SA} & = \frac{P_{AS} + P_{BS}}{P} = \frac{P_C}{P} \\
 P_A + P_B + P_C &= P
\end{align*}
$$

(8)

and

$$
\delta_{mi} = \delta_A + \delta_B = (y_0 - y_{min}) + \frac{P_B}{k_1 \cdot k_2 \cdot k_3} 
$$

(9)

It will be seen from Eq. (9) that the tooth deflection of the gears with profile modification and crowning becomes zero in the range of $y_{min} > y_0$, because the second term of the equation can be applied only to the extent of the equivalent line limited by $y_0$ from the assumption (5).

The load distribution factors and the deflections of a pair of teeth of the test gears with only crowning are shown in Fig. 13 (a) and (b) respectively. These figures are the typical results calculated under the peripheral forces of 1000 kg and 3200 kg. There is the range where the tooth

Fig. 14 Calculated result of deflections of a pair of test gear teeth with both crowning

Fig. 15 Calculated result of deflections of a pair of test gear teeth with both crowning and profile modification.

Position of initial engagement

Position of final engagement

Position of engagement

Position of initial and final engagement were shown regarding the case of tooth without both crowning and profile modification.

Fig. 13 Calculated result of load distribution factors and deflections of a pair of test gear teeth with only crowning.

Fig. 14 Calculated result of deflections of a pair of test gear teeth with both crowning and amount of profile modification of 15 microns.
deflections become zero at the initial and final engagements as already illustrated in Eq. (9).

In Fig. 13(a), the load distribution factors obtained by experiment are shown together. It will be clear that the assumptions applied to this calculation are nearly correct, since the factors in calculation are comparatively in good agreement with those in experiment.

Figs. 14 and 15 show the tooth deflections calculated by the same method for the gears with both the crowning and the profile modification of 15 microns and for the gears with both the crowning and the profile modification of 30 microns, respectively. These deflections are the typical results calculated under the peripheral forces of 1000 kg and 3200 kg, similarly to the case of Fig. 13.

5.3 The optimum amount of profile modification

The fluctuations of the tooth deflections in meshing which relate directly to the angular transmission error of the test gears were obtained from Figs. 13—15, and as the typical example, those under the peripheral force of 1000 kg are shown in Fig. 16.

As mentioned previously, comparison of the gear noise in each test was done in terms of the noise levels of the tooth contact frequency. Since the intensities of noise levels are proportional to the square of amplitudes when their frequencies are equal, as wellknown, the vibrations based on the fluctuations of the tooth deflections were developed to the Fourier series of which the basic vibrations had the same period as the tooth contact frequency and the amplitudes of the basic vibrations were compared with each other.

At first, the tooth deflections were approximated by the following polynomial expressions

\[ f(x)=c_0 x^7+c_0 x^5+c_0 x^3+c_0 x+c_0 \]

and then, after the expressions were developed to

\[ g_n(x)=\frac{1}{2} a_0+\sum_{k=1}^{n} \left( a_k \cos \frac{2\pi k x}{t_{eq}}+b_k \sin \frac{2\pi k x}{t_{eq}} \right), \]

Rang (0, t_{eq}) \hspace{1cm} (11)

in the range \((A, A+t_{eq})\) shown in Fig. 16, the amplitudes \(\sqrt{a_i^2+b_i^2}\) were compared with each other.

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<th>Table 2 Comparison of amplitudes</th>
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<td>Amount of profile modification (\mu)</td>
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<td>Peripheral force kg</td>
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<td>1000</td>
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These amplitudes calculated for the cases of each amount of profile modification were shown in Table 2. When the peripheral force is 1000 kg, the amplitude for the case of the amount of profile modification of 15 microns is the least and when the peripheral force is 3200 kg, the amplitudes for both the case of the amount of profile modification of 15 microns and of 30 microns are nearly equal. For both the peripheral force, the greatest amplitudes are obtained in the case of the amount of profile modification of 0 micron.

According to the experimental results, the noise levels of the test gears were the least with the amount of profile modification of 15 microns in the range of the peripheral force between 600 kg and 2000 kg. The results investigated above are in good agreement with those obtained by the experiments.

T. Nakada and others\(^{10,11}\) have made clear that the noise and dynamic loads of gears were in close connection with the angular transmission errors of gears.

As will be obvious from above investigation, the optimum amount of profile modification of crowned helical gear teeth can be determined from the tooth deflection under working load. That is, the deflections of a few kinds of teeth which are given different amounts of profile modification are calculated respectively, and the amount at which...
transmission error due to the tooth deflection is minimum, can be considered as the optimum one.

Though the deflection of a pair of teeth can be calculated from Eq. (9), this calculation is complicated a little, so that now the method for calculating approximately the condition of the tooth deflection will be derived.

It has been clear in another report\(^{123}\) that the distributed loads along the line of contact of helical gear teeth could be considered approximately as uniformly distributed loads. Therefore, when the stiffness per unit length of a pair of teeth and the total length of lines of contact are taken as \(k''\) and \(L\) respectively, the sum of the stiffness of teeth in meshing simultaneously is \(k''L(k''=\text{constant})\), and consequently Eq. (9) is given approximately by the following equation.

\[
\delta_{sc} = (y_0 - y_{\text{min}}) + \frac{P_B}{k''L}
\]  

(12)

It will be seen from Eq. (12) that the deflection \(\delta_{sc}\) is inversely proportional to the total length of lines of contact for a constant tooth load.

Fig. 17 shows the diagrams of \(y_0 - y_{\text{min}}\) and \(1/L\) calculated for the test gear teeth subjected to the peripheral force of 1 000 kg, as an example. The deflections calculated from Eq. (9) are shown also in Fig. 17. Though the diagram of \(1/L\) is different a little from the one of tooth deflection, the conditions of the tooth deflections can be obtained approximately by the diagrams of \(1/L\).

Next, as will be clear from Fig. 16, the angular transmission error is influenced by the decrease of the length of action due to the profile modification, even if the deflection of a pair of teeth in meshing does not fluctuate so much. Accordingly, the range of profile modification will be determined from the condition in which the transmission error is not influenced by the decrease of the length of action due to the profile modification. Referring to Fig. 17, if the imaginary lengths of action for tip and root modification are given by \(e_1\) and \(e_2\) respectively, the condition may be represented as follows:

\[
e_1 + e_2 = (e_{1s} - 1)t_{x0} + n \tan \beta_s
\]  

(13)

6. Conclusion

It was found experimentally that further improvement on the dynamic properties of crowned helical gears could be realized, if a suitable profile modification was given and that the vibrations caused by the meshing of the helical gear teeth with both a crowning and a suitable profile modification were less than those of the helical gear teeth with a suitable profile modification only.

Furthermore, it was found analytically that the optimum amounts of profile modification to the teeth of both non-crowned and crowned helical gears could be determined from the tooth deflections under working loads, similarly to the case of spur gears.

Acknowledgement

The author would like to express his gratitude to Dr. T. Nakada and Dr. J. Ishikawa for their valuable suggestions regarding this investigation.
References
(3) C. Weber und K. Banachek: Schriftenreihe Antriebstechnik, Bd. 11 (1955), VDMA.

Errata (Vol. 9, No. 35, August, 1966)

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