The Constitutive Equations of Fiber Reinforced Metal Matrix Composites*

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The constitutive equations of unidirectionally fiber reinforced metals were derived. A modified fraction model concept was applied to the matrix and fiber, separately. Equilibrium and compatibility were considered between the matrix and the fiber for deriving the constitutive equations. The proposed theory is expected to describe the inelastic behavior of FRM and the strain rate sensitivity of stress-strain relations. Static and dynamic tensile tests were performed on unidirectional boron/aluminum composites and cross-ply laminated plates. Static and dynamic stress-strain curves were obtained and the effect of the strain rate on tensile strength was investigated. The tensile strength of the 0° and laminated specimens showed no strain rate sensitivity; but the tensile strength of the 45° and 90° specimens increased as the strain rate increased. The validity of the proposed constitutive equations was confirmed by comparing the calculated results with the experimental ones.

Key Words: Composite Materials, Metal Matrix Composites, Constitutive Equations, Fraction Model, Strain Rate, Tensile Strength

1. Introduction

Fiber reinforced metal matrix composites (FRM) are expected to be superior to fiber reinforced plastics (FRP) in impact strength due to the fact that the matrix is metal. In order to analyze the deformation and fracture behavior of FRM, anisotropic constitutive equations with strain rate dependency must be known. Dynamic tensile tests must be conducted to obtain the fracture behavior of FRM under dynamic loading conditions. Many dynamic tensile tests have been performed for FRP^{(1)~(3)}, but few for FRM.

In the present paper, the constitutive equations of unidirectional composites are proposed. They are based on the modified fraction model concept, and can describe the inelastic behavior of FRM and the strain

rate dependency of stress-strain relations. In order to experimentally investigate the validity of the proposed constitutive equations and the fracture behavior of FRM, static and dynamic tensile tests were performed on unidirectional boron/aluminum composites and laminated plates. Static and dynamic stress-strain curves were obtained and the effect of the strain rate on tensile strength was investigated. The results are shown in the following.

2. Strain rate dependent constitutive equations of unidirectional composites

2.1 Idealization of the representative volume element

A representative volume element of unidirectional composites is idealized as shown in Figure. 1. The matrix portion of the representative volume element is divided into three submatrices $m1$, $m2$, $m3$ with uniform stress fields (Fig. 2). The global stress and strain acting on the representative volume element are denoted by σ_{ij} and ϵ_{ij} , respectively. The stresses and strains of the submatrices are denoted by σ_{ij}^{m1} , σ_{ij}^{m2} , σ_{ij}^{m3} and ϵ_{ij}^{m1} , ϵ_{ij}^{m2} , ϵ_{ij}^{m3} , and those of the fiber by σ_{ij}^f and ϵ_{ij}^f , respectively.

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Equilibrium equations of the representative volume element are given by

$$\sigma_a = V_f \sigma_a^f + V_{m1} \sigma_a^{m1} + V_{m2} \sigma_a^{m2} + V_{m3} \sigma_a^{m3} \quad (1)$$

$$a = x, y, z$$

$$\sigma_x^f = \sigma_x^{m2}, \quad \sigma_x^{m1} = \sigma_x^{m3} \quad (2)$$

$$\sigma_y^f = \sigma_y^{m1}, \quad \sigma_y^{m2} = \sigma_y^{m3} \quad (3)$$

and conditions of compatibility by

$$a \cdot \varepsilon_x = c \cdot \varepsilon_x^{m1} + (a - c) \cdot \varepsilon_x^{m3} = c \cdot \varepsilon_x^f + (a - c) \cdot \varepsilon_x^{m2} \quad (4)$$

$$b \cdot \varepsilon_y = c \cdot \varepsilon_y^{m2} + (b - c) \cdot \varepsilon_y^{m3} = c \cdot \varepsilon_y^f + (b - c) \cdot \varepsilon_y^{m1} \quad (5)$$

$$\varepsilon_z = \varepsilon_z^f = \varepsilon_z^{m1} = \varepsilon_z^{m2} = \varepsilon_z^{m3} \quad (6)$$

where $V_f, V_{m1}, \dots, V_{m3}$ are volume fractions of the fiber and the submatrices.

2.2 Application of the fraction model

A modified fraction model⁽⁴⁾ is applied to the matrix and fiber, separately. The fraction model⁽⁵⁾, sometimes indicated as a sublayer model, describes the inelastic behavior of materials by a parallel arrangement of subelements with different properties.

The submatrices $m1, m2, m3$, and the fiber are idealized by the fraction models with N and M subelements, respectively. The stress and strain of the k -th subelement, constituting the submatrix ml ($l=1, 2, 3$), are denoted by $_{ml}\sigma_{ij}^k$ and $_{ml}\varepsilon_{ij}^k$, and those constituting the fiber by $_{f}\sigma_{ij}^k$ and $_{f}\varepsilon_{ij}^k$. Denoting the volume fraction of the subelement by $_{m}\psi^k$ (common to ml) and $_{f}\psi^k$, the following equations hold.

$$\dot{\varepsilon}_{ij}^{ml} = _{ml}\dot{\varepsilon}_{ij}^k, \quad k=1, 2, \dots, N \quad (7)$$

$$\dot{\sigma}_{ij}^{ml} = \sum_{k=1}^N _{m}\psi^k \cdot _{ml}\dot{\sigma}_{ij}^k \quad (8)$$

$$\dot{\varepsilon}_{ij}^f = _{f}\dot{\varepsilon}_{ij}^k, \quad k=1, 2, \dots, M \quad (9)$$

$$\dot{\sigma}_{ij}^f = \sum_{k=1}^M _{f}\psi^k \cdot _{f}\dot{\sigma}_{ij}^k \quad (10)$$

where $i, j=1, 2, 3, l=1, 2, 3, \sum_{k=1}^N _{m}\psi^k=1, \sum_{k=1}^M _{f}\psi^k=1$ and the "dot" denotes differentiation with respect to time. Total strain rate of the subelements is decomposed into elastic and time-dependent inelastic strains:

$$_{ml}\dot{\varepsilon}_{ij}^k = _{ml}\dot{\varepsilon}_{ij,e}^k + _{ml}\dot{\varepsilon}_{ij,i}^k \quad (11)$$

$$_{f}\dot{\varepsilon}_{ij}^k = _{f}\dot{\varepsilon}_{ij,e}^k + _{f}\dot{\varepsilon}_{ij,i}^k \quad (12)$$

where $(\cdot)_e$ and $(\cdot)_i$ denote elastic and inelastic

components, respectively. The elastic strains are given by

$$_{ml}\dot{\varepsilon}_{ij}^k = _{m}E_{ijkl} \cdot _{ml}\dot{\varepsilon}_{kl,e}^k \quad (13)$$

$$_{f}\dot{\varepsilon}_{ij}^k = _{f}E_{ijkl} \cdot _{f}\dot{\varepsilon}_{kl,e}^k \quad (14)$$

where $_{m}E_{ijkl}$ and $_{f}E_{ijkl}$ are the tensors of elastic constants of the matrix and fiber, respectively. The inelastic strains can be expressed by

$$\dot{\varepsilon}_{ij,i}^{ml} = \sum_{k=1}^N _{m}\psi^k \cdot _{ml}\dot{\varepsilon}_{ij,i}^k \quad (15)$$

$$\dot{\varepsilon}_{ij,i}^f = \sum_{k=1}^M _{f}\psi^k \cdot _{f}\dot{\varepsilon}_{ij,i}^k \quad (16)$$

The inelastic strains of the k -th subelement can be obtained by

$$_{ml}\dot{\varepsilon}_{ij,i}^k = \frac{_{ml}\dot{\varepsilon}^k}{2 \cdot _{ml}\bar{\sigma}^k} \cdot \frac{\partial (_{ml}\bar{\sigma}^k)^2}{\partial _{ml}\dot{\sigma}_{ij}^k} \quad (17)$$

$$_{f}\dot{\varepsilon}_{ij,i}^k = \frac{_{f}\dot{\varepsilon}^k}{2 \cdot _{f}\bar{\sigma}^k} \cdot \frac{\partial (_{f}\bar{\sigma}^k)^2}{\partial _{f}\dot{\sigma}_{ij}^k} \quad (18)$$

The matrix and the fiber being individually isotropic, the Mises type equivalent stress is used as $_{ml}\bar{\sigma}^k$ and $_{f}\bar{\sigma}^k$. As for the equivalent strain rates $_{ml}\dot{\varepsilon}^k$ and $_{f}\dot{\varepsilon}^k$, it is necessary to select functions which express the strain rate dependency of FRM.

2.3 Relation between global stress and global strain

There are two cases for obtaining the relation between global stress and global strain. Namely, the relation can be obtained by providing the strain rate $\dot{\varepsilon}_{ij}$ or the stress rate $\dot{\sigma}_{ij}$. The former is shown in the following.

In each step of the time increment, the initial values of the stresses $_{ml}\sigma_{ij}^k, _{f}\sigma_{ij}^k$, and the strains $_{ml}\varepsilon_{ij}^k, _{f}\varepsilon_{ij}^k$ are known. The inelastic strain rates of the k -th subelement $_{ml}\dot{\varepsilon}_{ij,i}^k$ and $_{f}\dot{\varepsilon}_{ij,i}^k$ are obtained from Eqs. (17) and (18), and from Eqs. (13) and (14) the stress rates $_{ml}\dot{\sigma}_{ij}^k$ and $_{f}\dot{\sigma}_{ij}^k$ are given by

$$_{ml}\dot{\sigma}_{ij}^k = _{m}E_{ijpq}(\dot{\varepsilon}_{pq}^{ml} - _{ml}\dot{\varepsilon}_{pq,i}^k) \quad (19)$$

$$_{f}\dot{\sigma}_{ij}^k = _{f}E_{ijpq}(\dot{\varepsilon}_{pq}^f - _{f}\dot{\varepsilon}_{pq,i}^k) \quad (20)$$

From Eqs. (8) and (10), the stress rates of the matrix and the fiber are expressed by

$$\dot{\sigma}_{ij}^{ml} = _{m}E_{ijpq} \left(\dot{\varepsilon}_{pq}^{ml} - \sum_{k=1}^N _{m}\psi^k \cdot _{ml}\dot{\varepsilon}_{pq,i}^k \right) \quad (21)$$

Fig. 1 Idealization of a representative volume element

Fig. 2 Matrix and fiber composing a representative volume element

$$\dot{\epsilon}_{ij}^f = {}_f E_{ijpq} \left(\dot{\epsilon}_{pq}^f - \sum_{k=1}^M {}_f \psi^k \cdot {}_f \dot{\epsilon}_{pq,i}^k \right) \quad (22)$$

In Eqs. (21) and (22), the inelastic strain rates of the k -th subelement are known, while the strain rates of the submatrices $m1$, $m2$, $m3$ and the fiber, i.e., $\dot{\epsilon}_{pq}^{m_l}$ ($l=1, 2, 3$) and $\dot{\epsilon}_{pq}^f$, are unknown. These unknown values can be obtained from the global strain rate $\dot{\epsilon}_{ij}$, the equilibrium equations and the conditions of compatibility shown in Section 2.1. A method for obtaining $\dot{\epsilon}_{pq}^{m_l}$ and $\dot{\epsilon}_{pq}^f$ is shown in the following.

In the first step, vectors $\tilde{\epsilon}_x, \tilde{\epsilon}_y, \dots, \tilde{\sigma}_x, \tilde{\sigma}_y, \dots$, $\tilde{\epsilon}_{x,i}, \tilde{\epsilon}_{y,i}, \dots$ are defined as follows:

$$\tilde{\epsilon}_x^T = (\dot{\epsilon}_x^{m1} \quad \dot{\epsilon}_x^{m2} \quad \dot{\epsilon}_x^{m3} \quad \dot{\epsilon}_x^f) \quad (23)$$

$$\tilde{\sigma}_x^T = (\dot{\sigma}_x^{m1} \quad \dot{\sigma}_x^{m2} \quad \dot{\sigma}_x^{m3} \quad \dot{\sigma}_x^f) \quad (24)$$

$$\tilde{\epsilon}_{x,i}^T = \left(\sum_k^N m \psi^k \cdot m1 \dot{\epsilon}_{x,i}^k \quad \sum_k^N m \psi^k \cdot m2 \dot{\epsilon}_{x,i}^k \quad \sum_k^N m \psi^k \cdot m3 \dot{\epsilon}_{x,i}^k \quad \sum_k^M {}_f \psi^k \cdot {}_f \dot{\epsilon}_{x,i}^k \right) \quad (25)$$

where “ T ” denotes a transposition of the matrix. The vectors $\tilde{\epsilon}_y, \tilde{\sigma}_y, \dots$ are defined similarly. Eqs. (21) and (22) are rewritten in the matrix form

$$\tilde{\sigma}_x = [A] \cdot \tilde{\epsilon}_x + [B] \cdot \tilde{\epsilon}_y + [B] \cdot \tilde{\epsilon}_z - [A] \cdot \tilde{\epsilon}_{x,i} - [B] \cdot \tilde{\epsilon}_{y,i} - [B] \cdot \tilde{\epsilon}_{z,i}, \dots \quad (26)$$

$$\tilde{\sigma}_{xy} = [C] \cdot \tilde{\epsilon}_{xy} - [C] \cdot \tilde{\epsilon}_{xy,i}, \dots \quad (27)$$

where $[A]$, $[B]$ and $[C]$ are 4×4 diagonal matrices with diagonal components $(2\mu + \lambda)_m, (2\mu + \lambda)_m, (2\mu + \lambda)_m, (2\mu + \lambda)_f$, $\lambda_m, \lambda_m, \lambda_m, \lambda_f$ and $\mu_m, \mu_m, \mu_m, \mu_f$, respectively, and μ and λ are given by

$$\mu = G, \quad \lambda = 2\nu G / (1 - 2\nu) \quad (28)$$

Eqs. (2) and (4) are written in the matrix form and substitution of Eq. (26) into these equations yields

$$[s_x] \cdot [A] \cdot \tilde{\epsilon}_x + [s_x] \cdot [B] \cdot \tilde{\epsilon}_y = \tilde{q}_x \quad (29)$$

$$[e_x] \cdot \dot{\epsilon}_x = \dot{\epsilon}_x \cdot \tilde{l}_x \quad (30)$$

where $[s_x], [e_x], \tilde{l}_x$ and $\tilde{q}_x$ are given by

$$[s_x] = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \quad (31)$$

$$\tilde{l}_x^T = (a \quad a) \quad (32)$$

$$[e_x] = \begin{bmatrix} c & 0 & a-c & 0 \\ 0 & a-c & 0 & c \end{bmatrix} \quad (33)$$

$$\tilde{q}_x = -[s_x] \cdot [B] \cdot \tilde{\epsilon}_z + [s_x] \times \{ [A] \cdot \tilde{\epsilon}_{x,i} + [B] \cdot \tilde{\epsilon}_{y,i} + [B] \cdot \tilde{\epsilon}_{z,i} \} \quad (34)$$

Similar equations are obtained for the y direction. The strain rate $\dot{\epsilon}_x$ in Eq. (30) and $\dot{\epsilon}_y$ in the corresponding equation for the y direction are known. From Eq. (6) the components of $\tilde{\epsilon}_z$, i.e., $\dot{\epsilon}_z^{m1}, \dot{\epsilon}_z^{m2}, \dot{\epsilon}_z^{m3}$ and $\dot{\epsilon}_z^f$, are equal to $\dot{\epsilon}_z$ and this is also a known value. When Eqs. (29) and (30) for the x direction, and the corresponding equations for the y direction, are put in order, this yields simultaneous equations with unknowns $\tilde{\epsilon}_x$ and $\tilde{\epsilon}_y$.

$$[M^*] \cdot \tilde{\epsilon}^* = \tilde{q}^* \quad (35)$$

where

$$[M^*] = \begin{bmatrix} [s_x] \cdot [A] & [s_x] \cdot [B] \\ [e_x] & \text{zero} \\ [s_y] \cdot [B] & [s_y] \cdot [A] \\ \text{zero} & [e_y] \end{bmatrix} \quad (36)$$

$$\tilde{\epsilon}^{*T} = (\tilde{\epsilon}_x^T \quad \tilde{\epsilon}_y^T) \quad (37)$$

$$\tilde{q}^{*T} = (\tilde{q}_x^T \quad \dot{\epsilon}_x \tilde{l}_x^T \quad \tilde{q}_y^T \quad \dot{\epsilon}_y \tilde{l}_y^T) \quad (38)$$

Eq. (35) can be solved for the unknowns $\tilde{\epsilon}_x$ and $\tilde{\epsilon}_y$. Similar equations are obtained for shear components. By determining $\tilde{\epsilon}_x, \tilde{\epsilon}_y, \dots$, i.e. $\dot{\epsilon}_{ij}^{m_l}$ and $\dot{\epsilon}_{ij}^f$, the stress rates of the matrix and the fiber constituting the representative volume element are obtained from Eqs. (26) and (27).

The global stress rates $\dot{\sigma}_x, \dots$ are given by

$$\dot{\sigma}_x = \tilde{\beta}^T \cdot \tilde{\sigma}_x, \dots \quad (39)$$

$$\tilde{\beta}^T = (V_{m1} \quad V_{m2} \quad V_{m3} \quad V_f) \quad (40)$$

Finally, the global stress σ_{ij} and the global strain ϵ_{ij} are given by

$$\sigma_{ij} = \sigma_{ij0} + \dot{\sigma}_{ij} \cdot \Delta t, \quad \epsilon_{ij} = \epsilon_{ij0} + \dot{\epsilon}_{ij} \cdot \Delta t \quad (41)$$

where subscript “0” represents initial values of the step and Δt denotes a time increment of the step.

3. Static and dynamic tensile tests of FRM

3.1 Experimental methods

A conventional testing machine was used for the static tensile tests, while a rotating disc type machine was used for the dynamic tensile tests. The latter machine employs a block-to-bar method⁽⁶⁾ (Fig. 3 and 6) and is installed at the Institute of Interdisciplinary Research, the University of Tokyo. Dynamic tensile stress $\sigma(t)$ and strain $\epsilon(t)$ in the specimen are calculated based on the strain $\epsilon_g(t)$ measured by semiconductor gauges located at a distance “ a ” from the end of the output bar as shown in Fig. 3. The formulae for $\sigma(t)$ and $\epsilon(t)$ are obtained using the one-dimensional stress wave propagation theory as follows:

$$\sigma(t) = \frac{S_0}{S} E_0 \epsilon_g \left(t + \frac{a}{c} \right) \quad (42)$$

$$\epsilon(t) = \frac{1}{l} \int_0^t \left\{ V(t) - c \epsilon_g \left(t + \frac{a}{c} \right) \right\} dt \quad (43)$$

where l and S are the length and the cross sectional

Fig. 3 Fundamentals of the dynamic tensile testing machine

area of the specimen, S_0 , E_0 and c are the cross sectional area, Young's modulus and the longitudinal elastic wave velocity of the output bar, and $V(t)$ is the velocity of the impact block.

Test specimens were cut from boron/aluminum-6061 composite plates (unidirectional composites and $[0^\circ/90^\circ]_s$ laminated plates) fabricated by the hot press method. The volume fraction of the fibers was 0.38. In the case of unidirectional composites, test specimens were cut with the fibers oriented at 0° , 45° and 90° to the long dimension of the specimens. Figures 4 and 5 show the dimensions of the test specimens for the static and dynamic tensile tests. The test specimens for the dynamic tensile tests were made by bonding FRM plates to screw parts as shown in Fig. 5.

In the case of the static tensile tests, strains were measured by three-element strain gauges mounted on both sides of the specimens. Figure 6 shows a schematic diagram of the measuring system for the dynamic tensile tests. The transient strain $\epsilon_g(t)$ was recorded with a transient recorder. Then, by combining it with the velocity of the impact block, $V(t)$, $\sigma(t)$ and $\epsilon(t)$ were calculated from Eqs. (42) and (43)

Fig. 4 Specimen for static tensile test

using a mini-computer.

3.2 Experimental results

Stress-strain curves for the unidirectional 0° , 90° and $[0^\circ/90^\circ]_s$ laminated specimens obtained from the static tensile tests are shown in Fig. 7 by the dotted lines. From the experimental results, the fracture strains of the 90° specimens were estimated to be about 2% and the 45° specimens over 20%.

Using the rule of mixtures and the lamination theory⁽⁷⁾, the stiffness and the strength of unidirectional composites and laminated plates were calculated from the properties of the composing

Specimen	L	B	t
0°	138	3.5	0.8
45°	108	3.2	3.2
90°	58	3.2	3.2
$[0^\circ/90^\circ]_s$	88	4.5	0.8

Fig. 5 Specimen for dynamic tensile test

Fig. 6 Schematic diagram of the measuring system for dynamic tensile tests

Fig. 7 Stress-strain curves for static and dynamic tensile tests

materials. The following values were employed in the calculation.

Boron fiber

Young's modulus : 392 GPa{40 000 kgf/mm²}

Poisson's ratio : 0.2

tensile strength : 3 580 MPa{365 kgf/mm²}

Aluminum alloy 6061-O

Young's modulus : 68.2 GPa{6 950 kgf/mm²}

Poisson's ratio : 0.3

tensile strength : 136 MPa{13.9 kgf/mm²}

The Tsai-Hill theory⁽⁷⁾ was employed as a failure criterion, and the following value for aluminum alloy 6061-O was used as the shear strength of the unidirectional composites.

shear strength : 125 MPa{12.7 kgf/mm²}

Figure 8 presents a comparison of the calculated results with the results of the static tensile tests for Young's modulus and tensile strength. The experimental values other than the tensile strength of the 0° specimens are the average of three specimens and are in good agreement with the predicted values. On the other hand, the experimental values of the tensile strength of the 0° specimens show a large scatter and are in poor agreement with the predicted values. The discrepancy may be attributed to the fabrication process of FRM.

Examples of the stress-strain curve obtained from the dynamic tensile tests are shown in Fig. 9. The stress-strain curves for the unidirectional 0°, 90° and [0°/90°]_s laminated specimens obtained from the dynamic tensile tests are shown in Fig. 7. They are shown by the solid lines. The results of the 0° specimens show a large scatter and a correspondence of the static tensile test results with those of the dynamic ones is not evident. On the other hand, the dynamic tensile test results correspond well with those of the static ones for the [0°/90°]_s specimens. A comparison

Fig. 8 Young's modulus and tensile strength

is made in Table 1 of the tensile strengths of the composites obtained by static and dynamic tensile tests. The values shown in the table are the average of several specimens. The results for aluminum alloy 6061-O are also shown in Table 1. The 45° and 90° specimens, where the tensile strength depends on the matrix, indicate the strain rate sensitivity, since the aluminum alloy shows an increase in tensile strength with increasing strain rate. On the other hand, the tensile strength of the 0° and laminated specimens shows no strain rate sensitivity. Consequently, the tensile strength of boron fibers is estimated to be independent of the strain rate.

For the strain rate sensitivity of the fracture

Table 1 Effect of strain rate on tensile strength

Specimen	Static		Dynamic		$\frac{\sigma_d}{\sigma_s}$
	Strain rate $\dot{\epsilon}$ (1/s)	Tensile strength σ_s (MPa)	Strain rate $\dot{\epsilon}$ (1/s)	Tensile strength σ_d (MPa)	
0°	8.33×10^{-5}	1170	1.67×10^3	1200	1.02
45°	8.33×10^{-4}	162	1.55×10^3	192	1.18
90°	8.33×10^{-5}	148	1.48×10^3	202	1.36
[0°/90°] _s	8.33×10^{-5}	554	1.74×10^3	554	1.00
Aluminum alloy 6061-O	1.04×10^{-3}	136	1.82×10^3	168	1.24

1 MPa = 0.102 kgf/mm²

Fig. 9 Examples of stress-strain curves for dynamic tensile tests

strain, no apparent conclusion is obtained due to the effects of adhesive bonding in the specimens.

4. Comparison of experimental and calculated results

A comparison was made between the results of static and dynamic tensile tests for the unidirectional composites and the predictions using the constitutive equations proposed in Section 2.

Material constants used in the computation were determined as follows. Boron fibers were assumed to deform elastically up to fracture and to show no strain rate sensitivity. The values shown in Section 3 were used for Young's modulus and Poisson's ratio of boron fibers and aluminum alloy 6061-O. The stress-strain curve for the aluminum alloy was represented by a piecewise linear curve composed of five segments in an inelastic region. From the static and dynamic tensile test results, the following values were employed as the yield stress of the material:

static tensile test

$$\sigma_y = 73.5 \text{ MPa} \{7.5 \text{ kgf/mm}^2\}, \quad \dot{\epsilon} = 1.04 \times 10^{-3} \text{ s}^{-1}$$

dynamic tensile test

$$\sigma_y = 132 \text{ MPa} \{13.5 \text{ kgf/mm}^2\}, \quad \dot{\epsilon} = 1.82 \times 10^3 \text{ s}^{-1}$$

The rate sensitivity of the yield stress was approximated by the power law as follows:

$$\dot{\epsilon} = 1.576 \times 10^{-49} \cdot \sigma_y^{24.55} \quad (\text{stress: MPa, strain rate: s}^{-1}) \quad (44)$$

Figure 10 presents a comparison of the experimental results with the calculated ones for the 0° and 90° specimens. While the dynamic tensile test results of the 0° specimens show a large scatter, a reasonably good agreement is obtained between theory and

experiment under both the static and dynamic loading conditions.

The stress-strain curves for the 90° specimens show a strain rate sensitivity, but no strain rate sensitivity is observed in the case of the 0° specimens where the stiffness is dominated by the fibers.

5. Conclusions

The constitutive equations of unidirectionally fiber reinforced metals were derived. A modified fraction model concept was separately applied to the matrix and fiber. Equilibrium and compatibility were considered between the matrix and fiber for deriving the constitutive equations. The proposed theory is expected to describe the inelastic behavior of FRM and the strain rate sensitivity of stress-strain relations.

Static and dynamic tensile tests were performed on unidirectional boron/aluminum composites and cross-ply laminated plates. Static and dynamic stress-strain curves were obtained and the effect of strain rate on tensile strength was investigated. The tensile strength of the 0° and laminated specimens showed no strain rate sensitivity; but in the case of the 45° and 90° specimens, the tensile strength increased as the strain rate was increased.

The validity of the proposed constitutive equations was confirmed by comparing the calculated results with experimental ones.

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Fig. 10 Stress-strain curves for static and dynamic tensile tests

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