Stress Intensity Factors and Order of Stress Singularities at the Tips of a Crack Normal to the Bimaterial Interface*

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The plane elastostatic problem for two bonded elastic half planes consisting of isotropic and anisotropic materials is considered. It is assumed that the anisotropic half plane contains a straight crack normal to the bimaterial interface, and that the bonded plane is subjected to a constant strain away from, and perpendicular to, the crack. Singular stress fields at the tips of a crack fully embedded in the anisotropic half plane and terminating at the interface are investigated. Mathematical formulations are made using the method of continuous distribution of dislocations and singular integral equation method. Numerical calculations are carried out for the stress intensity factors and the order of stress singularities at the tips of the crack.

Key Words: Composite Materials, Crack Normal to Interface, Plane Elastostatic Problem, Stress Intensity Factors, Order of Stress Singularity

1. Introduction

In our previous paper⁽¹⁾ a simple two dimensional elastostatic problem of a straight crack normal to a bonded interface between isotropic and orthotropic half planes, which models a matrix phase and a carbon fiber with elastic anisotropy, was considered, to investigate the fracture process of a unidirectional carbon fiber reinforced composite material from a micromechanical point of view, by regarding it as a crack propagation phenomenon and to show the influence of the interface between fiber and matrix on the singular stress field near the crack tips. We were concerned only with the case where the crack was embedded in the isotropic half plane, since crack propagation in the matrix phase was assumed. On the other hand, crack propagation in the carbon fiber is also important. In this case the problem can be treated

using the foregoing two dimensional elastostatic model and assuming the crack is in the orthotropic half plane. Now, in order to consider this problem using the same procedure as in the previous paper, first of all, it is necessary to analyse the fundamental stress field due to an edge dislocation in the orthotropic half plane of the bonded planes as mentioned above. Therefore, this paper considers a solution to the problem of an edge dislocation in an unbounded homogeneous orthotropic plane, from which the stress field as mentioned above can be obtained by the use of Fourier integral transforms. Then, application of the method of continuous distribution of dislocations to the present crack problem using the fundamental stress field due to this edge dislocation leads to a singular integral equation.

Numerical calculations are carried out for cracks embedded in the orthotropic half plane, including as their extreme a crack terminating at the interface.

The stress intensity factors at the crack tips, the order of stress singularity and the effect of elastic constants on them are presented.

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2. Method of solution

In Cartesian coordinates (x, y) a stress field in a two dimensional orthotropic elastic body is governed by a biharmonic type stress function satisfying the equation⁽²⁾

$$\left(\frac{\partial^2}{\partial x^2} + \mu_1 \frac{\partial^2}{\partial y^2}\right) \left(\frac{\partial^2}{\partial x^2} + \mu_2 \frac{\partial^2}{\partial y^2}\right) \chi(x, y) = 0 \quad (1)$$

where μ_1 and μ_2 are roots of

$$c_{11}c_{66}\mu^2 + \{c_{12}(2c_{66} + c_{12}) - c_{11}c_{22}\}\mu + c_{22}c_{66} = 0 \quad (2)$$

and c_{ij} 's are independent elastic constants of the orthotropic elastic body⁽³⁾.

The stress components corresponding to this stress function $\chi = \chi(x, y)$ are given by

$$\sigma_x = \frac{\partial^2 \chi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 \chi}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y} \quad (3)$$

On the other hand, the biharmonic type stress function $\chi(x, y)$, which satisfies Eq.(1), may be expressed with a linear combination of the sum and the difference of two harmonic stress functions $\chi_1(x, y)$ and $\chi_2(x, y)$ which satisfy Eq.(4).

$$\left(\frac{\partial^2}{\partial x^2} + \mu_i \frac{\partial^2}{\partial y^2}\right) \chi_i(x, y) = 0, \quad (i=1, 2) \quad (4)$$

That is,

$$\chi(x, y) = \chi_s(x, y) + \chi_d(x, y) \quad (5)$$

where

$$\begin{aligned} \chi_s(x, y) &= \chi_1(x, y) + \chi_2(x, y), \\ \chi_d(x, y) &= \frac{\chi_1(x, y) - \chi_2(x, y)}{\sqrt{\mu_1} - \sqrt{\mu_2}} \end{aligned} \quad (6)$$

Now consider a stress field due to an edge dislocation with its Burgers vector parallel to the y axis and located at the origin of the Cartesian coordinates (x, y) in an unbounded homogeneous orthotropic plane. By choosing a multi-valued function as one of the solutions satisfying Eq.(4), i.e.,

$$\chi_i(x, y) = r_i \log r_i \cos \theta_i - r_i \theta_i \sin \theta_i \quad (i=1, 2) \quad (7)$$

Fig. 1 Stress field due to an edge dislocation

we can express $\chi(x, y)$ with the unknown coefficients A_1 and A_2 as follows:

$$\chi(x, y) = A_1 \cdot \chi_s(x, y) + A_2 \cdot \chi_d(x, y) \quad (8)$$

where r_i and θ_i in Eq.(7) are given by

$$r_i^2 = \mu_i x^2 + y^2, \quad \theta_i = \tan^{-1}(y/\sqrt{\mu_i}x) \quad (9)$$

By substituting Eq.(8) into Eq.(3) and taking Eqs.(6) and (7) into account we have the stress components, and further, using the constitutive equations and the relationships of displacement and strain, we have the displacement components expressed with the unknown coefficients A_1 and A_2 . Then, A_1 and A_2 are determined as follows under the conditions that the displacement in the y direction has a dislocation of b_y after it rotates around an edge dislocation located at the origin and that the resultant force around the dislocation vanishes.

$$A_1 = 0, \quad A_2 = \frac{c_{66}b_y}{2\pi} \cdot \frac{(1+k_1)(1+k_2)(\sqrt{\mu_1} - \sqrt{\mu_2})}{k_1 - k_2} \quad (10)$$

in which $k_i = (c_{11}\mu_i - c_{66})/(c_{12} + c_{66})$, $(i=1, 2)$.

Using this result, we consider a fundamental stress field due to an edge dislocation with its Burgers vector parallel to the y axis and located at $(\xi, 0)$ on the x axis in the orthotropic half plane of the isotropic/orthotropic bonded half planes, as shown in Fig.1.

In this case Airy stress function $\chi^*(x, y)$ ⁽⁴⁾ may be written as follows for the isotropic half plane (I):

$$\chi^*(x, y) = \int_0^\infty \{x \cdot f_1(\lambda) + f_2(\lambda)\} e^{\lambda x} \cos \lambda y d\lambda \quad (11)$$

where $f_1(\lambda)$ and $f_2(\lambda)$ are unknown functions which are to be determined from the boundary conditions. For the orthotropic half plane (II), in addition to the above-mentioned results, i.e., the stress function

$$\chi(x, y) = A_2 \cdot \chi_d(x, y) \quad (12)$$

the following should be added:

$$\chi(x, y) = \int_0^\infty \{g_1(\lambda) \cdot \chi_s(x, y) + g_2(\lambda) \cdot \chi_d(x, y)\} d\lambda \quad (13)$$

where A_2 in Eq.(12) is given by Eq.(10) and $\chi_d(x, y)$ is given by the second expression of Eq.(6) together with Eq.(7). Here r_i and θ_i are given as follows instead of Eq.(9):

$$r_i^2 = \mu_i(x - \xi)^2 + y^2, \quad \theta_i = \tan^{-1}\{y/\sqrt{\mu_i}(x - \xi)\} \quad (14)$$

In Eq.(13), $g_1(\lambda)$ and $g_2(\lambda)$ are unknown functions which are to be determined from the boundary conditions, and $\chi_s(x, y)$ and $\chi_d(x, y)$ are given by Eq.(6) with

$$\chi_i(x, y) = e^{-\sqrt{\mu_i}\lambda x} \cos \lambda y, \quad (i=1, 2) \quad (15)$$

By assuming a perfect bond between the two half planes, the continuity conditions at the interface ($x=0$) can be stated

$$\sigma_x^I = \sigma_x^{II} \quad (16 \cdot a)$$

Fig. 2 Continuously distributed dislocation model and crack

$$\tau_{xy}^I = \tau_{xy}^{II} \quad (16 \cdot b)$$

$$\frac{\partial u_x^I}{\partial y} = \frac{\partial u_x^{II}}{\partial y} \quad (16 \cdot c)$$

$$\frac{\partial v_y^I}{\partial y} = \frac{\partial v_y^{II}}{\partial y} \quad (16 \cdot d)$$

with the superscripts "I" and "II" referring to the isotropic and the orthotropic half planes, respectively.

By substituting the stress and displacement components corresponding to Eqs.(11)~(13) into the continuity conditions of Eq.(16) at the interface and by applying the method of Fourier integral transforms to them, we have a system of four simultaneous linear equations for the four unknown functions $f_1(\lambda)$, $f_2(\lambda)$, $g_1(\lambda)$ and $g_2(\lambda)$, from which all the unknowns are determined and the stress field due to the edge dislocations as shown in Fig.1 can be obtained.

For later analysis, only the stress components in the y direction for the isotropic and orthotropic half planes will be written here, that is

$$(\sigma_y^I)_{y=0} = \frac{c_{66} b_y}{2\pi} \cdot \frac{(1+k_1)(1+k_2)(\sqrt{\mu_1} - \sqrt{\mu_2})}{(k_1 - k_2)} \times \left\{ \frac{b_1}{(x - \sqrt{\mu_1} \xi)} + \frac{d_1 x}{(x - \sqrt{\mu_1} \xi)^2} - \frac{b_2}{(x - \sqrt{\mu_2} \xi)} - \frac{d_2 x}{(x - \sqrt{\mu_2} \xi)^2} \right\}, (x \leq 0) \quad (17)$$

$$(\sigma_y^{II})_{y=0} = \frac{c_{66} b_y}{2\pi} \cdot \frac{(1+k_1)(1+k_2)(\sqrt{\mu_1} - \sqrt{\mu_2})}{(k_1 - k_2)} \times \left\{ \frac{1}{(x - \xi)} - \frac{a_0}{(x + \xi)} - \frac{a_1}{(\sqrt{\mu_1} x + \sqrt{\mu_2} \xi)} - \frac{a_2}{(\sqrt{\mu_2} x + \sqrt{\mu_1} \xi)} \right\}, (x > 0) \quad (18)$$

where the coefficients a_0 , a_1 , a_2 , b_1 , b_2 , d_1 and d_2 are constants depending on the elastic constants of the isotropic and the orthotropic half planes.

Now, with these results, we consider a straight crack of length $2c$ normal to the interface and lying along the x axis in an orthotropic half plane as shown

in Figure 2, where a constant strain parallel to the interface is applied at infinity ($p_1/E = p_2/E_y$).

By continuously distributing an infinitesimal dislocation with density $f(\xi)$ from point A to B along the x axis and by using the equilibrium of stresses together with a stress free condition on the crack surfaces, we have the following singular integral equation:

$$\frac{c_{66}(1+k_1)(1+k_2)(\sqrt{\mu_1} - \sqrt{\mu_2})}{2\pi(k_1 - k_2)} \cdot \int_{\epsilon}^{2c+\epsilon} \left\{ \frac{1}{(x - \xi)} - \frac{a_0}{(x + \xi)} - \frac{a_1}{(\sqrt{\mu_1} x + \sqrt{\mu_2} \xi)} - \frac{a_2}{(\sqrt{\mu_2} x + \sqrt{\mu_1} \xi)} \right\} \times f(\xi) d\xi + p_2 = 0 \quad (19)$$

In addition, the equation

$$\int_{\epsilon}^{2c+\epsilon} f(\xi) d\xi = 0 \quad (20)$$

is used to ensure the single-valuedness of the displacement.

By making a change of variables such as

$$s = \frac{\xi}{c} - \frac{c + \epsilon}{c}, \quad t = \frac{x}{c} - \frac{c + \epsilon}{c}$$

$$\frac{c_{66}(1+k_1)(1+k_2)(\sqrt{\mu_1} - \sqrt{\mu_2})}{2(k_1 - k_2) \cdot p_2} f(cs + c + \epsilon) \equiv \phi(s) \quad (21)$$

Eqs.(19) and (20) reduce to

$$\frac{1}{\pi} \int_{-1}^1 \frac{\phi(s)}{s - t} ds + \frac{1}{\pi} \int_{-1}^1 k(s, t) \phi(s) ds = 1, |t| < 1 \quad (22)$$

$$\int_{-1}^1 \phi(s) ds = 0 \quad (23)$$

where, with $\delta = \epsilon/c$, the integral kernel $k(s, t)$ is given by

$$k(s, t) = \frac{a_0}{\{s + t + 2(1 + \delta)\}} + \frac{a_1}{\{\sqrt{\mu_2} s + \sqrt{\mu_1} t + (\sqrt{\mu_1} + \sqrt{\mu_2})(1 + \delta)\}} + \frac{a_2}{\{\sqrt{\mu_1} s + \sqrt{\mu_2} t + (\sqrt{\mu_1} + \sqrt{\mu_2})(1 + \delta)\}} \quad (24)$$

Consequently, by solving Eq.(22) under the condition of Eq.(23) we can determine the unknown function $\phi(s)$, from which all the stresses and displacements are determined.

Next, we consider the order of stress singularity and the stress intensity factors. First of all, we assume the unknown function $\phi(s)$ to be

$$\phi(s) = (1 + s)^\alpha \cdot (1 - s)^\beta \cdot F(s) \quad (25)$$

where α and β represent the order of stress singularity at crack tips A and B, respectively, and the unknown function $F(s)$ is continuous in $|s| \leq 1$. Then, when the crack is fully embedded in the orthotropic half plane, the integral kernel given by Eq.(24) turns out to be a regular one of Fredholm type due to $\delta \neq 0$ so the order of stress singularity is $\alpha = \beta = -1/2$. However, when crack tip A terminates at the interface the kernel $k(s, t)$ is of generalized Cauchy-type singularity due to $\delta = 0$ so the order of stress singularity is given as the

roots of the following characteristic equations which are derived using the function-theoretic method⁽⁶⁾.

$$\cos \pi \alpha + a_0 + \frac{a_1}{\sqrt{\mu_2}} \left(\frac{\mu_1}{\mu_2} \right)^{\alpha/2} + \frac{a_2}{\sqrt{\mu_1}} \left(\frac{\mu_2}{\mu_1} \right)^{\alpha/2} = 0 \quad (26)$$

$$\cos \pi \beta = 0 \quad (27)$$

From these equations we find that $\beta = -1/2$ and α depends only on the elastic constants.

On the other hand, the stress intensity factors are obtained as follows. When one of the crack tips does not terminate at the interface ($\delta \neq 0$), Eq.(22) with its right hand side replaced by $-(\sigma_y^{\text{II}})_{y=0}/p_2$ instead of unity holds for $|t| > 1$ as well as $|t| \leq 1$ and, since the integral part with the Cauchy kernel is dominant near the crack tips, it is approximated as follows:

$$\frac{(\sigma_y^{\text{II}})_{y=0}}{p_2} = -\frac{1}{\pi} \int_{-1}^1 \frac{\phi(s)}{s-t} ds \quad (t \leq 1, -1-\delta < t \leq -1) \quad (28)$$

Here, by substituting Eq.(25) with $\alpha = \beta = -1/2$ into Eq.(28) and using the mathematical formulae we have the following singular stress field near the crack tips:

$$\frac{(\sigma_y^{\text{II}})_{y=0}}{p_2} = \begin{cases} F(1)/\sqrt{t^2-1}, & (t \geq 1) \\ -F(-1)/\sqrt{t^2-1}, & (t \leq -1) \end{cases} \quad (29)$$

Then, by defining the stress intensity factors at points A and B by

$$\left. \begin{aligned} K_{I,A} &= \lim_{t \rightarrow -1} \sqrt{-2\pi c(t+1)} \cdot (\sigma_y^{\text{II}})_{y=0} \\ K_{I,B} &= \lim_{t \rightarrow 1} \sqrt{2\pi c(t-1)} \cdot (\sigma_y^{\text{II}})_{y=0} \end{aligned} \right\} \quad (30)$$

and substituting Eq.(29) into the above equation we get

$$\frac{K_{I,A}}{p_2 \sqrt{\pi c}} = -F(-1), \quad \frac{K_{I,B}}{p_2 \sqrt{\pi c}} = F(1) \quad (31)$$

On the other hand, when crack tip A terminates at the interface ($\delta = 0$), using the method of continuous distribution of dislocations together with Eq.(17), we have the following stress in the y direction on the x axis of the isotropic half plane ($x \leq 0$):

$$\begin{aligned} \frac{(\sigma_y^{\text{I}})_{y=0}}{p_2} &= -\frac{1}{\pi} \int_{-1}^1 \left\{ \frac{b_1}{\sqrt{\mu_1}(s-t_1)} - \frac{d_1(1+t_1)}{\sqrt{\mu_1}(s-t_1)^2} \right. \\ &\quad \left. - \frac{b_2}{\sqrt{\mu_2}(s-t_2)} + \frac{d_2(1+t_2)}{\sqrt{\mu_2}(s-t_2)^2} \right\} \phi(s) ds \end{aligned} \quad (32)$$

where Eq.(21) with $\varepsilon = 0$ is used in the derivation of the above equation except for x which is changed into

$$t_i = \frac{x}{c\sqrt{\mu_i}} - 1, \quad (i=1, 2) \quad (33)$$

By substituting Eq.(25) with $\beta = -1/2$ into Eq.(32) it reduces to⁽¹⁾

$$\begin{aligned} \frac{(\sigma_y^{\text{I}})_{y=0}}{p_2} &= -\frac{F(-1)}{\sqrt{2} \sin \pi(1+\alpha)} \left\{ \frac{b_1 - \alpha d_1}{\sqrt{\mu_1}(-1-t_1)^{-\alpha}} \right. \\ &\quad \left. - \frac{b_2 - \alpha d_2}{\sqrt{\mu_2}(-1-t_2)^{-\alpha}} \right\}, \quad (t_i \rightarrow -1, t_i < -1) \end{aligned} \quad (34)$$

near crack tip A, so by defining the stress intensity factor at point A by

$$K_{I,A}^* = \lim_{x \rightarrow 0} \sqrt{2\pi(-x)^{-\alpha}} \cdot (\sigma_y^{\text{I}})_{y=0} \quad (35)$$

and by substituting Eq.(34) into Eq.(35), and taking

into account Eq.(33), we have

$$\frac{K_{I,A}^*}{p_2 \sqrt{\pi} c^{-\alpha}} = -\frac{F(-1)}{\sin \pi(1+\alpha)} \left\{ \frac{b_1 - \alpha d_1}{(\sqrt{\mu_1})^{\alpha+1}} - \frac{b_2 - \alpha d_2}{(\sqrt{\mu_2})^{\alpha+1}} \right\} \quad (36)$$

Furthermore, in this case, for the stress intensity factor at point B we have the following stress by substituting Eq.(25) with $\beta = -1/2$ into Eq.(28):

$$\frac{(\sigma_y^{\text{II}})_{y=0}}{p_2} = 2^\alpha \cdot F(1)/\sqrt{t-1}, \quad (t \rightarrow 1, t > 1) \quad (37)$$

and by substituting this into the second expression of Eq.(30) we get

$$\frac{K_{I,B}}{p_2 \sqrt{\pi c}} = 2^{(1/2+\alpha)} \cdot F(1) \quad (38)$$

Hence, if $F(-1)$ and $F(1)$ are known, the stress intensity factors are obtained directly from Eqs.(31), (36) and (38).

3. Numerical calculations

Substitution of Eq.(25) into Eqs.(22) and (23) leads to the following singular integral equations for the unknown function $F(s)$:

$$\begin{aligned} \frac{1}{\pi} \int_{-1}^1 \frac{(1+s)^\alpha (1-s)^\beta F(s)}{(s-t)} ds + \frac{1}{\pi} \int_{-1}^1 (1+s)^\alpha \\ \times (1-s)^\beta F(s) k(s, t) ds = 1 \end{aligned} \quad (39)$$

$$\int_{-1}^1 (1+s)^\alpha (1-s)^\beta F(s) ds = 0 \quad (40)$$

It seems difficult to solve these equations analytically; so we use the Lobatto-Jacobi integration rule⁽⁶⁾ as in the previous paper⁽¹⁾, to obtain the numerical values of $F(s)$. The merit of this rule is that $F(-1)$ and $F(1)$ can be obtained directly. The order of stress singularity α at crack-tip point A terminating at the interface is numerically evaluated by applying the method of regula falsi to Eq.(26).

4. Numerical results

Numerical results are presented for carbon fiber reinforced epoxy resin or aluminum composites, as in the previous paper⁽¹⁾. In the numerical calculations, the elastic constants of epoxy resin ($E = 3.13$ GPa, $G = 1.16$ GPa, $\nu = 0.35$) or aluminum ($E = 69.92$ GPa, $G = 26.78$ GPa, $\nu = 0.3$) are used for the isotropic half plane and also those of two kinds of carbon fibers (CF1, CF2), as shown in Table 1, for the orthotropic one. In addition, the elastic constants of glass fiber

Table 1 Elastic constants of carbon and glass fibers

	E_x	E_y	G_{xy}	ν_{yx}
CF1	8.62	400	2.8	0.35
CF2	15	232	5.02	0.28
GF	68.96	68.96	28.73	0.2

(GPa)

Fig. 3 Stress intensity factor as a function of distance from the interface

(GF), an isotropic fiber, are also used for comparison.

Figure 3 shows the stress intensity factors $K_{I,A}$ and $K_{I,B}$ at crack tips A and B as a function of the distance $\delta (= \epsilon/c)$ from the interface when a crack is fully embedded in the orthotropic half plane which models a carbon fiber. The stress intensity factors at both points, when the matrix is an epoxy, are larger than $p_2\sqrt{\pi c}$ for a crack in an unbounded orthotropic plane, i.e., $K_{I,A}/p_2\sqrt{\pi c}$, $K_{I,B}/p_2\sqrt{\pi c} > 1$ and they increase as the crack approaches the interface, whereas, in aluminum, both $K_{I,A}/p_2\sqrt{\pi c}$ and $K_{I,B}/p_2\sqrt{\pi c}$ are smaller than unity and they decrease as the crack approaches the interface. One reason why the situation is reversed in the cases of epoxy and aluminum matrices seems to be due to the fact that the relative stress intensity factor depends on relative magnitudes of the shear modulus ratio between matrix and fiber, because we find $G/G_{xy} < 1$ for the combinations of CF 1, CF2 and GF/epoxy, $G/G_{xy} > 1$ for CF 1 and CF2/Al, and $G/G_{xy} \approx 1$ for GF/Al. It is also found that these tendencies are completely opposite to those obtained in the

Fig. 4 Stress intensity factor as a function of elastic constants

previous paper⁽¹⁾, where the crack is assumed to be in the matrix side.

Relating to the foregoing discussion, Figure 4 shows the influence of the relative magnitude of the shear modulus G and G_{xy} or Young's modulus E and E_y of the matrix and fiber on the stress intensity factors for three different combinations of $G_{xy}/E_y = 0.1, 0.05$ and 0.01 with $E_x/E_y = 0.05$ and $\nu_{yx} = \nu = 0.3$,

Table 2 Stress intensity factors and order of stress singularity when the crack is embedded in the matrix or fiber

Fiber/Matrix	$\frac{K_{I,A}^*}{p_1 \sqrt{\pi c}^{-\alpha}}$	$\frac{K_{I,B}^*}{p_2 \sqrt{\pi c}^{-\alpha}}$
CF1/Epoxy	1.5659 (-0.3450)	0.4415 (-0.6845)
CF2/Epoxy	2.1023 (-0.3217)	0.3191 (-0.7343)
GF/Epoxy	5.1328 (-0.2906)	0.1837 (-0.8236)
CF1/Al	0.2652 (-0.6920)	7.3773 (-0.3159)
CF2/Al	0.3610 (-0.6344)	4.1332 (-0.3614)
GF/Al	0.9453 (-0.5048)	1.0563 (-0.4948)

() ; Order of stress singularity

where the distance from the interface to one of the crack tips is fixed at $\delta=1$. It is seen from Figure 4(a) that the non-dimensional stress intensity factors $K_{I,A}/p_1\sqrt{\pi c}$ and $K_{I,B}/p_2\sqrt{\pi c}$ at points A and B are smaller than unity for a smaller G_{xy}/G , the former being smaller than the latter, and they increase gradually with an increase of G_{xy}/G , to the same value of unity, and that the stress intensity factor at point A becomes larger than that at point B for a larger G_{xy}/G .

It is also seen from Figure 4(b) that the stress intensity factor as a function of E_x/E_y is almost the same as that of G_{xy}/G , but $K_{I,A}/p_1\sqrt{\pi c}$ and $K_{I,B}/p_2\sqrt{\pi c}$, where the former is smaller than the latter, are generally smaller than unity except for a sufficiently large E_x/E_y , i.e., small E/E_y .

Table 2 shows the stress intensity factors $K_{I,A}^*$ and the order of stress singularity at crack tip point A terminating at the interface. The results are given for two cases when crack tip point B is embedded in the matrix (previous paper⁽¹⁾) or in the carbon (present paper) and they are normalized by $p_1\sqrt{\pi c}^{-\alpha}$ and $p_2\sqrt{\pi c}^{-\alpha}$, respectively. It is interesting to note that the sum of the stress singularities α is approximately "-1" in all combinations of materials, irrespective of whether a crack is embedded in the matrix or fiber.

Figure 5 shows the order of stress singularity as a function of the shear modulus ratio G_{xy}/G of the fiber and matrix for three different values of $E_x/E_y=1, 0.1$ and 0.05 with $G_{xy}/E_y=0.1$ and $\nu_{yx}=\nu=0.3$ fixed. The order $(-\alpha)$ increases gradually with an increase of

Fig. 5 Order of stress singularity as a function of shear modulus ratio

Fig. 6 Stress intensity factor as a function of shear modulus ratio

G_{xy}/G and it rapidly increases to 1 for a sufficiently large G_{xy}/G , i.e., small G/G_{xy} .

Figure 6 shows the stress intensity factors at point A terminating at the interface and at point B as a function of the shear modulus ratio G_{xy}/G of the fiber and matrix for three different values of $E_x/E_y = 1, 0.1$ and 0.05 with $G_{xy}/E_y = 0.1$ and $\nu_{yx} = \nu = 0.3$. The non-dimensional stress intensity factors $K_{I,A}/p_2\sqrt{\pi C}^{-\alpha}$ at point A are very large for a sufficiently small G_{xy}/G and they rapidly decrease to vanish with an increase of G_{xy}/G , whereas the corresponding ones $K_{I,B}/p_2\sqrt{\pi C}$ at point B tend to increase gradually with an increase of G_{xy}/G . Although the present results are obtained by assuming a plane stress state, there is no difference between the present and the plane strain state-assumed results.

5. Concluding remarks

A simple two dimensional elastostatic problem of a straight crack normal to a bonded interface between isotropic and orthotropic half planes, which model, respectively, a matrix and a carbon fiber with elastic anisotropy in a composite material, is theoretically analyzed, and is related to the propagation of a cracklike defect existing in, and normal to, a fiber in a unidirectional carbon fiber reinforced composite material. In the analysis, by using a fundamental stress field due to an edge dislocation in an orthotropic half plane, such as in the bonded half planes model, and the method of continuous distribution of disloca-

tions, the problem is formulated as a singular integral equation. The results obtained by the numerical calculations are summarized as follows:

- (1) As one of the crack tips in a fiber approaches the interface, the non-dimensional stress intensity factors at both ends increase when the matrix is an epoxy resin and decrease when it is aluminum.
- (2) They also tend to increase with an increase of the shear modulus ratio G_{xy}/G or Young's modulus ratio E_y/E of the fiber and matrix.
- (3) The order of stress singularity α at the crack tip point terminating at the interface is generally not $-1/2$ and depends only on the elastic constants. The value $(-\alpha)$ increases with an increase of G_{xy}/G .
- (4) The non-dimensional stress intensity factor defined at that point decreases rapidly with an increase of G_{xy}/G .

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