System Instability Caused by the Dynamic Behaviour of a Centrifugal Pump at Partial Operation*

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The system instability of a centrifugal pump and pipe network is encountered in the high pressure systems of a power plant. The frequency of the instability is limited to be very low (4Hz).

Judging from the observation that this instability occurs at the pump operating point with a negative gradient of head-flow characteristics (stable characteristics) and with sufficient NPSH (Net Positive Suction Head), it is concluded that this is a new type of system instability, different from classical surging or cavitation surging. A series of experiments to solve the mechanism of this instability was carried out. The research program consisted of the measurement of the dynamic behaviour of the centrifugal pump, a confirmation test using a pipe network similar to that of the plant and the stability analysis of the system. Our result shows that the matching of the frequency of unstable pump impedance and the pipeline's resonant frequency results in this new type of system instability.

Key Words: System Instability, Pump Dynamic Behaviour, Pump Impedance, Stability Analysis, Low Cycle Pulsation, Self-excited Oscillation

1. Introduction

System instability was encountered in the high pressure centrifugal pump and pipe network system of a power plant. This phenomenon had the following remarkable characteristics:
* The large system oscillation (pressure and flow pulsation) occurs only at very low flow rates of the pump, below 25% of its maximum efficiency point.
* Its frequency is very low (about 4 Hz).
* The pump has stable performance curve at this unstable operating region, and this phenomenon was not observed in the shop test.

By extensive investigation, it is concluded that this phenomenon is different from classical surging or cavitation surging[1].

Makay reported that pumps with droopy performance curves sometimes cause instabilities[5]. He recommended designing a minimum flow rate greater than 25% of the maximum efficiency, but he didn't refer to the mechanism of this instability.

This kind of instability is sometimes attributed to vortex phenomena, for example vortex formation at the branches[6] or the interaction of the check valve and the vortex[7]. However, these causes are denied by our field test because no instabilities were observed by stopping the pump and by pouring the same flow rate with the pressurized suction tank.

This paper describes the experimental and theoretical results of the research to clarify the mechanism of this system instability.

Nomenclature

\[ A_{eq} : \text{equivalent area in the pump} \]
\[ C_{p} : \text{pump compliance} \]
\[ f : \text{frequency} \]
\[ G_{0} : \text{transfer matrix} \]
\[ H : \text{head} \]
\[ H_{r} : \text{pressure ratio} \]
\[ \overline{i} : \text{\overline{I}} \]
\[ K_{e} : \text{bulk modulus of the water} \]
$L_P$: inertial effect of the fluid in the pump (inertance)
$L_{eq}$: pump equivalent length
$n$: rotational speed
$P$: static pressure
$p$: pressure pulsation
$Q$: pump flow rate
$q$: flow pulsation
$S$: Laplace transform
$S = s + i\omega$
$V$: steady velocity
$v$: velocity pulsation
$V_{eq}$: equivalent volume
$Z$: hydraulic impedance
$Z_p$: pump impedance
$Z_{pr}$: pump resistance
$\alpha$: stability index
$\beta$: angular frequency, $\beta = 2\pi \cdot f$
$\eta$: efficiency
$\theta$: angle
$\kappa$: ratio of specific heat
$\phi$: head coefficient
$\phi$: flow coefficient

suffix
$d$: discharge
$N$: rated value
$s$: suction

2. Dynamic Behaviour of a Centrifugal Pump

2.1 Pump Transfer Matrix

The pressure and flow pulsations at pump suction and discharge are connected by a $2 \times 2$ complex pump transfer matrix. The transfer matrix describes the dynamic behaviour of the pump and it is approximated by Eq. (1) at low frequencies.

\[
\begin{bmatrix}
    p_d \\
    q_d
\end{bmatrix} =
\begin{bmatrix}
    1 & Z_p \\
    -C_pS & 1
\end{bmatrix}
\begin{bmatrix}
    p_s \\
    q_s
\end{bmatrix}
\tag{1}
\]

$Z_p$ is called pump impedance and is a complex number. It is divided into pump resistance $Z_{pr}$ and pump inerterance $L_P$.

\[
Z_p = Z_{pr} - L_P \cdot S
\tag{2}
\]

The resistance at 0 Hz is equal to the slope of the pump performance curve at the operating point.

Pump inerterance $L_P$ means the inertial effect of the water column in the pump and is expressed as follows.

\[
L_P = L_{eq}/ (A_{eq} \cdot g)
\tag{3}
\]

where $L_{eq}$ and $A_{eq}$ are the equivalent length and area of the pump respectively.

The compliance term $-C_pS$ is the compressibility between two measuring sections and this means elasticity of the water or air volume in the pump.

The approximation shown in Eq. (1) is only applicable in the low frequency region. At a higher frequency range the duct effect in the pump becomes dominant.

2.2 Simplified Method

As $L_{eq}$ is negligible compared with the wavelength at low frequencies, the pump impedance $Z_p$ can be measured from equation (4).

\[
Z_p = \frac{p_d - p_s}{q_s}
\tag{4}
\]

Though this method is applicable only at low frequencies, it is an easier way to measure the pump impedance, so it is named the simplified method.

2.3 Strict Method

At higher frequencies, a more accurate method is necessary since the pump equivalent length cannot be neglected in these conditions. The pump transfer matrix $G_{12}$ is a $2 \times 2$ complex matrix. This can be measured using four signals: two pressure fluctuations and two velocity fluctuations at pump suction and discharge.

As the number of unknowns is four and only two equations are obtained at one measurement, tests with two different boundary conditions are necessary to get the pump transfer matrix. This measuring method, which we call the strict method, is shown in equations (5) and (6). In these measured values obtained from the different boundary conditions are indicated by putting dashes.

\[
\begin{bmatrix}
    p_d & p_d' \\
    q_d & q_d'
\end{bmatrix} =
\begin{bmatrix}
    G_{11} & G_{12} \\
    G_{21} & G_{22}
\end{bmatrix}
\begin{bmatrix}
    p_s & p_s' \\
    q_s & q_s'
\end{bmatrix}
\tag{5}
\]

where

\[
\begin{align*}
G_{11} &= \frac{1}{Z_p - Z_{pr} \left( \frac{Z_p - Z_{pr}}{Hr} \right)} \\
G_{12} &= \frac{Z_p}{Z_p - Z_{pr} \left( \frac{Z_p - Z_{pr}}{Hr} \right)} \\
G_{21} &= \frac{Z_p}{Z_p - Z_{pr} \left( \frac{Z_p - Z_{pr}}{Hr} \right)} \\
G_{22} &= \frac{Z_p}{Z_p - Z_{pr} \left( \frac{Z_p - Z_{pr}}{Hr} \right)}
\end{align*}
\tag{6}
\]

$Hr = p_s / p_d$ pressure ratio
$Z = p/q$ hydraulic impedance

$G_{12}$ is equal to the pump impedance of Eq. (4).

3. Experiment

3.1 Test Facility and Instrumentation

The experiment is carried out by utilizing the existing open loop test stand for the usual pump model test. Using this test stand we made a test circuit to measure the dynamic characteristics of the pump. The arrangement is shown in Fig.1. This facility is composed of the following components.

(1) Driving motor - a 375 kW variable speed motor is used for this test, whose output is far larger than the pump input power (18 kW at 2500 rpm), so the speed fluctuation due to pulsation is negligible.
(2) Test pump - The test pump is a two stage pump derived from a prototype ten stage pump. The impellers and casings are cast using the same wooden pattern that was used to make the prototype pump. Its sectional view is shown in Fig. 2 and its characteristics are shown in Fig. 3. The pump characteristics at low flow rates were carefully measured but no positive gradient was observed. The test pump has a balance sleeve to reduce axial thrust. At the latter stage of the test, the leak flow through this clearance proved to have a fairly large effect on the pump characteristics of this low specific pump, so the test with a closed balance line was also carried out to closely observe the characteristics at a low flow rate. These performance curves are compared in Fig. 3.

(3) Test sections - Two test sections are attached at the suction and discharge sides of the test pump. They are provided with transparent windows for LDV measurement. Strain gauge type pressure transducers are attached to the same sections where the LDV
measurements are made.

(4) LDV - Two He - Ne Laser Doppler Velocimeters are set at the suction and discharge measuring sections and the data is obtained by the forward scattering method.

(5) Fluid exciter - Two kinds of piston type fluid exciters are used for this test. One is driven by a variable speed motor and crank piston mechanism and the other utilizes a hydraulic exciter. The former is for low frequencies and the latter for higher frequencies.

3.2 The Distribution of Velocity and Velocity Pulsation Across the Pipe Section

The output of LDV is a velocity fluctuation of one point in the pipe section, so conversion of the velocity fluctuation to flow fluctuation is necessary to obtain the pump impedance. Figure 4 shows a few examples of the measured distribution of steady velocity and velocity pulsation. The ratio $K_v$, which is a ratio of the integrated value of velocity pulsation to the LDV measured output at the center of the pipe multiplied by pipe area, is experimentally measured.

$$K_v = \frac{\int_{0}^{L} 2\pi r vdV}{\pi r^2 v_L}$$

($r$: pipe inner radius, $v$: velocity pulsation, $v_L$: LDV output velocity pulsation) (7)

The ratio $K_v$ proved to be almost 1.0 regardless of the frequency.

4. Test Results

4.1 Simplified Method and Strict Method

At first, measurements are made both by the strict method and by the simplified method, and the pump impedance measured with the open balance line is compared.

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**Fig. 3** Pump Characteristics

**Fig. 4** Distribution of Velocity and Velocity Pulsation in Pipe Section

**Fig. 5** Pump Impedance $Z_p$ (Strict Method)
1. Strict method

Figure 5 shows the test results of pump impedance and compliance by the strict method. The following characteristics are observed.

(i) The measured pump resistance (Fig. 5(a)) shows a larger value, at a certain range of the frequency, than the value obtained from the slope of the performance curve which is equal to the pump resistance at 0 Hz. The maximum \( \text{Re}(Z_p) \) value which exceeds the gradient of this performance curve is almost the same at 10% and 20% flow rates.

(ii) The measured \( \text{Imag}(Z_p) \) (Fig. 5(b)) shows a linear increase up to 16 Hz which doesn't change as the flow rate changes. The equivalent pump duct effect is calculated from this as \( L_{ext}/A_{ext} = 501.7 \) (1/m). This is a fairly large value compared with the physical pump and pipe dimensions.

(iii) The measured compliance term (Fig. 5(c)) is a very small value but shows a linear relation with the frequency. The flow pulsations at the pump suction and discharge are connected in equation (8) by using the bulk modulus of the water \( K_w \) or the equivalent bulk modulus of the air \( \rho \cdot K_a \)

\[
q_a - q_e = -\frac{C_p}{C} \frac{dp}{dt} \tag{8}
\]

where \( C_p = V_{ext}/K_w \) (water) and \( C_p = V_{ext}/(\rho \cdot K_a) \) (air). From equations (8) and (1) the following relation is obtained.

\[
\text{Imag}(G_a) = \text{Imag}(-C_p \cdot S) = -2\pi C_p \cdot \omega \tag{9}
\]

The equivalent volume is obtained from the slope of Fig. 5(c), which results in \( 169 \ell \) for water and \( 2.2 \) cm\(^3\) for air. These values are reasonable considering pump construction and cavitation at the balance sleeve, which depressurizes the total pump head through a small clearance.

2. Simplified Method

The measured result of the simplified method is shown in Fig. 6. This figure shows the following characteristics. The resistance and inerterance are very different from those obtained by the strict method at frequencies greater than 7 Hz. Beyond this frequency the simplified method gives much smaller values than the strict method. This critical frequency depends on the pump size, pump type and number of stages.

4.2 Closer Experiment

The leak flow from the balance sleeve is found to have a fairly large effect on the pump characteristics of this low specific speed pump (Fig. 3). Therefore, in order to obtain the precise dynamic characteristics of the low flow region, a test with the balance pipe shut and with a larger pulsation is carried out by the simplified method below 7.0 Hz. The test result is shown in Figure 7 and 8.

Figure 7 shows the pump resistance as the flow rate is decreased. This pump resistance has the following unique tendencies.

(i) As the flow rate decreases, the pump resistance increases and it becomes positive at a certain frequency range, although \( \text{Re}(Z_p) \) at 0 Hz is negative which means a stable performance curve.

(ii) The maximum level of the pump resistance seems to be saturated at a 10% flow rate and its value is about 500 (s/m\(^3\)) at 2500 rpm.

Figure 8 shows the pump resistance when the pump rotational speed is changed. The pump resistance shows the following very interesting features.

(i) The peak frequency as well as peak level increase as the rotational speed is increased.

(ii) The region where the pump resistance is positive seems to expand and it shifts towards a

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Fig. 6 Pump Impedance \( Z_p \) (Simplified Method)

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higher frequency.

5. Stability Analysis and System Instability

5.1 Stability Analysis

In order to clarify the effect of the pump dynamic characteristics on system instability, stability analysis taking the pump impedance into consideration is carried out. The analysis is made of the simple line model shown in Fig. 10 and it is assumed that the pump impedance vector draws a circular locus around zero to examine the influence of pump resistance and inerterance on system instability.

In this analysis, the characteristic equation, which is a function of $S$, is constituted by the boundary conditions and the component transfer matrix; pump, pipe, branch and valves. The characteristic equation, which takes the form of a matrix, is solved by Muller's method and the eigenvalue vector $S(\alpha + i\beta)$ is obtained. The stability of the system is determined by the sign of $\alpha$; if $\alpha$ is positive the system is unstable. The resonant frequency is calculated from $\beta = 2\pi f$.

The calculated $\alpha$ for the first mode is shown in Fig. 9. This shows that $\alpha = -0.2$ and the system becomes stable when Re $(Z_p)$ is zero ($\theta = 90^\circ$ or $270^\circ$), which is equal to the condition without a pump. As Re $(Z_p)$ increases, $\alpha$ increases and it becomes positive when Re $(Z_p)$ exceeds a certain value. In this condition the system is unstable and the pulsation grows automatically. This also explains the surging due to an unstable performance curve. On the other hand Imag $(Z_p)$ only affects the resonant frequency as a duct effect and it doesn’t influence the stability.

In conclusion, this instability is caused by the unstable pump impedance (Re $(Z_p) > 0$), the relatively small damping of the pipe network and the relation between their frequencies. All of these conditions are needed for the occurrence of this phenomenon.

5.2 Model Pipe Network Test

In order to prove this mechanism of system instability, we constructed a pipe network whose first resonant frequency is within the unstable zone of the pump resistance and whose system damping is very low. The outline of the test pipe network is shown in Fig. 10. The test is carried out by pressurizing water around 2MPa with the test pump and depressurizing it with a valve to atmospheric pressure at the pipe end.

As shown in Fig. 11, the system instability appears at low flow rate and its frequency is 4.1Hz, which is very close to the instability experienced at the plant. In this way, the phenomenon at the plant is success-
fully reproduced at the test stand. The following typical features of self-excited oscillation are observed in particular.

- It takes a while for the oscillation to grow large enough to get into the limit cycle after the valve is closed suddenly.

- The valve resistance at the pump discharge damps the oscillation.

The FFT analysed results of this pressure pulsation are shown in Fig. 12. The pressure pulsation grows as large as 47 dB as the flow rate is reduced from 30% to 0%. This shows that the resonant frequency component is selectively amplified.

By these experiments, the mechanism of the instability explained herein is proved.

6. Conclusion

Through this research it is demonstrated theoretically and experimentally that this phenomenon is a new type of self-excited oscillation in which the pump dynamic behaviour and the pipe system characteristics play important roles.

The results are summarized as follows.

1. The pump we tested has dynamically unstable impedance at low flow rates and at a certain low frequency range (3 ~ 6 Hz) although it has a stable slope of the performance curve.

2. The system instability is successfully reproduced by a test pump and test pipe network.

3. From the experimental result, we conclude that this system instability occurs only when the following conditions are fulfilled.

(i) The pump is operated at a low flow rate where the pump resistance is positive (unstable) in a certain frequency range.

(ii) The resonant frequency of the pipe network coincides with this unstable frequency range.

(iii) The dynamic damping of the system is small.
References


