Dimensional Synthesis of Dwell Function Generators*

(On 6-Link Stephenson Mechanisms)

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Dwell function generators are broadly applicable to several kinds of automatic machines because they do not need any special control equipment for positioning, and therefore have wider usage. In the present paper, Stephenson type 6-link crank-and-rocker mechanisms are studied. The conditions under which the output arms dwell at the edge points of rocking are analytically developed, and relationships between the dwell periods and mechanism parameters are clarified numerically by simulations. Such relationships are approximated by polynomials in the order of less than five, and using the dwell periods as design data, an automatic optimum synthesis of 6-link dwell mechanisms is carried out.

Key Words: Link Mechanism, Dimensional Synthesis, Automatic Conveying Machine

1. Introduction

Most dwell mechanisms generally used, such as geneva stops, cam mechanisms, irregularly formed gear trains, and so on, have higher pairs, which are easily designed and synthesized but are known to show some inferior dynamic characteristics. On the other hand, dwell mechanisms with lower pairs only have been studied, and some methods of graphical constructions^{(1),(2)} and syntheses⁽³⁾ have been suggested for the simplest planar 6-link dwell mechanism, the output arms of which dwell approximately at the edge points of rocking. However, these methods are rather qualitative and require much trial-and-error and computing time for obtaining optimum solutions.

One of the authors has already tried a precision-point-synthesis of a 6-link dwell mechanism in a series of studies on adjustable mechanisms⁽⁴⁾. A mechanism

similar to the above is studied in this paper, especial with regard to quantitatively dwell characteristics, and a precise synthesis with less trial-and-error is carried out.

2. Conditions for Dwelling

2.1 Conditions for dwelling at the edge points of rocking

We should define an approximate dwell quantitatively, because an output link of a mechanism with revolute pairs only cannot dwell precisely in a finite domain.

Figure 1 shows a typical and well known method for constructing a 6-link dwell mechanism, using the shape of an approximate arc on a 4-bar coupler curve $s^{(1)-(3)}$. The output link f may dwell approximately at the edge points of rocking during a uniform motion of the input link b , if the coupler point E describes arcs approximately in the neighbourhoods of the points E_1 and E_2 , whose radii ρ coincide with the link length e . Now, we consider the additional condition that the times necessary for the clockwise and anticlockwise motions of the output rocking link should coincide with each other. One of the methods satisfying this is

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use of a symmetric coupler curve s , which is formulated as follows.

$$c = c_2 = d \quad (1)$$

The typical shape of an input-output displacement curve $\Phi = F(\theta)$ of such a mechanism is shown in Fig. 2. Now we define an approximate dwell period as a domain of an input angle $\Delta\theta_k (k=1, 2)$, the corresponding output angle Φ of which satisfies⁽⁴⁾

$$\begin{aligned} \Delta\theta_k &= \{\theta | \Phi_{\min} \leq \Phi = F(\theta) \leq \Phi_{\max} + r\Delta\Phi \\ \text{or} \quad &\Phi_{\max} - r\Delta\Phi \leq \Phi \leq \Phi_{\max}\}, k=1, 2 \end{aligned} \quad (2)$$

Where $\Delta\Phi$ is the output rocking range. Let E_1 and E_2 , in Fig. 1, be the positions of the coupler joint E and further, F_1 and F_2 be the positions of the output moving joint F at $\theta=0^\circ$ and 180° , respectively, then the line E_1E_2 is the axis of symmetry and it passes through the fixed joint D. Although the curvature of a general 4-bar coupler curve is very complicated, that of a symmetrical one leads to such a simple form as follows. Let ρ_0 and $\rho_k = E_kO_k$ (positive means that the center of curvature exists on the same side as D) be the radii of curvature of the loci of joints B and D, respectively, and κ_0 and κ_k be those non-dimensional curvatures when joint E exists on $E_k (k=1, 2)$. Then, we have the following equation from Euler-Savary's equation⁽⁵⁾ and Eq. (1). ($d=1$: unit length)

$$\left. \begin{aligned} \frac{1-\kappa_k}{1-\kappa_0} &= \frac{\tan \psi_k}{\tan \psi_0} \\ \kappa_k &= \frac{E_kD}{\rho_k} = \frac{2 \cos \psi_k}{\rho_k}, \psi_0 - \psi_k = \frac{\lambda}{2} \\ \kappa_0 &= \frac{B_kD}{\rho_0} = \frac{a + \rho_0}{\rho_0} = \frac{2 \cos \psi_0}{\rho_0} \end{aligned} \right\} \quad (3)$$

Therefore

$$\left. \begin{aligned} \rho_k &= \frac{2\rho_0 \tan \psi_0 \cos(\psi_0 - \frac{\lambda}{2})}{\rho_0 \tan \psi_0 + a \tan(\psi_0 - \frac{\lambda}{2})} \\ \psi_0 &= \cos^{-1} \frac{a + \rho_0}{2} \end{aligned} \right\} \quad (4)$$

where $\rho_0 = -b$ or $+b$ when the joint E is at E_1 or E_2 . If the shape of the coupler curve satisfies $\rho_1 = \rho_2$, and the link length e is

$$e = \rho_m = (\rho_1 + \rho_2)/2 \quad (5)$$

and further more, if joints F_1 and F_2 exist on the axis of symmetry of the coupler curve, we obtain a mechanism which dwells at both edge points of rocking. Since the thickness of the coupler curve is

$$t = 2 \left\{ \cos \left(\psi_0 - \frac{\lambda}{2} \right) - \cos \left(\psi_0 - \frac{\lambda}{2} \right) \right\} \quad (6)$$

the output rocking range is given by

$$\Delta\Phi = 2 \sin^{-1}(t/2f) \quad (7)$$

The width w of the curve is expressed approximately as a double distance from the middle point E_0 of the segment E_1E_2 to the furthest point E_w on the curve.

Although this width has no direct relation with the displacement of the mechanism, its extremely large value, compared to the thickness t or link length e , causes an enlargement of the relative rocking range between links e and f and therefore a reduction of the transmissibility of the mechanism⁽⁶⁾.

Putting

$$BD^2 = l^2 = a^2 + b^2 - 2ab \cos \theta$$

$$\angle ABC = \theta = \sin^{-1} \left(\frac{a}{l} \sin \theta \right) + \cos^{-1} \left(\frac{l}{2} \right)$$

at the position of input angle θ , shown in Fig. 3 by solid lines, the distance ρ between joint E and the center of curvature O_k is obtained by

$$\begin{aligned} \rho &= -a + be^{i\theta} + 2 \sin \frac{\lambda}{2} e^{i(\theta - \theta)} \\ &\quad - \left\{ 2 \cos \left(\psi_0 - \frac{\lambda}{2} \right) - \rho_m \right\} e^{i[\pi - (\lambda/2)]} \end{aligned} \quad (8)$$

and the dwell period defined in Eq. (2) is approximately given by the range of the input angle θ which satisfies

$$|\rho - \rho_m| \leq rt \quad (9)$$

Fig. 1 6-1 link dwell mechanism

Fig. 2 Displacement curve of dwell mechanism

2.2 Ranges of the parameters

In the case of obtaining dwell mechanisms using the method suggested in Section 2.1, we impose the following constraints considering the shape and the transmissibility of the mechanism. Considering the conditions for

- (1) crank-and-rocker
- (2) maximum link lengths (not larger than 5)
- (3) maximum pressure angle
(not larger than 65°)

about a symmetric 4-bar linkage, the ranges of the link lengths a and b are shown as the inside of the quadrilateral enclosed by four lines in Fig. 4 ($d=1$)⁽⁴⁾.

Further more, considering the condition for

- (4) shape of the symmetric 4-bar curve
($t/w \geq 0.1$)

we have

$$a \leq 1.19 - (b - 0.76)^2 \quad (10)$$

$$\lambda_{\max} = 173.3(b - 0.76)^2 + 24.0 + 176.0(a - b - 0.43)$$

$$\lambda_{\min} = 109.4 - 253.3(b - 0.24)^2 - (a - b - 0.43)\{116.0(b - 0.27)^2 + 13.6\} \quad (11)$$

Fig. 3 Curvature of coupler curve

Fig. 4 Domain of parameters

and, finally, the domain of (a, b) is shown in the figure by oblique lines.

3. Approximation of Dwell Periods by Functions

Synthesizing the mechanisms satisfying the given dwell periods, as a general case, dwell periods are generally calculated using proper values for the parameters and compared to the given values. However, such a trial-and-error method requires a lot of computing time, especially for multiple-link mechanisms. In this paper, we suggest a method for reducing the computing time by representing graphically the relationships between the parameters and dwell periods obtained from Eq. (9) and approximating them with the use of some simple functions.

(a) $b=0.2$

(b) $b=0.4$

(c) $b=0.6$

Fig. 5 Relationships between parameters and dwell periods (No.1)

Given the values of (a, b) by splitting the domain shown in Fig. 4 at intervals of 0.05 for a and 0.1 for b , determine the range of the coupler angle λ from Eq. (11). Figures 5 (a)~(c) are some of the results of dwell periods $\Delta\theta_1$ and $\Delta\theta_2$ at the neighbourhoods of $\theta = 0^\circ$ and 180° , respectively, calculated from Eq. (9) with the use of the above values. The value of the constant r is, as an example, 0.02. Figure 6 shows the same data as Fig. 5 except for a difference in the method of representation. We have the following characteristics from these graphs.

(i) Both dwell periods $\Delta\theta_1$ and $\Delta\theta_2$ are not influenced by a and λ , and they are almost constant (40°) when the input link length b is short.

(ii) When b is long, $\Delta\theta_1$ and $\Delta\theta_2$ both vary

Fig. 6 Relationships between parameters and dwell periods (No.2)

parabolically with respect to λ , and they have a large difference in the domain where λ is large.

(iii) $\Delta\theta_1$ increases monotonously with respect to λ and has a maximum value of about 100° , but $\Delta\theta_2$ has the extreme value of about 65° .

Considering the above characteristics, we might approximate the dwell periods by the following quadratic functions.

$$\Delta\theta_k = \alpha_k(\lambda - \beta_k)^2 + \gamma_k, \quad k=1, 2 \quad (12)$$

The coefficients α_k , β_k , and γ_k are shown approximately as follows.

$$\alpha_k = \alpha_{k1}a + \alpha_{k2} \quad (13)$$

$$\beta_k = \beta_{k1}(b + \beta_{k2})^2 + \beta_{k3} \quad (14)$$

(Refer to Fig. 7)

$$\gamma_k = \gamma_{k1}a + \gamma_{k2}(b + \gamma_{k3})^2 + \gamma_{k4} \quad (15)$$

The optimum values of these coefficients obtained from Figs. 5, 6 and 7 are shown in Table 1. Substituting Eqs. (13)~(15) into Eq. (12), we have

Table 1 Coefficients of approximate functions

	$k=1$	$k=2$
α_{k1}	0.100 ($\lambda \geq \beta_1$)	0.02
	0.004 ($\lambda < \beta_1$)	
α_{k2}	-0.065 ($\lambda \geq \beta_1$)	-0.011
	0.0044 ($\lambda < \beta_1$)	
β_{k1}	-100.0	-150.0
β_{k2}	0.25	-0.05
β_{k3}	128.3	47.4
γ_{k1}	30.0	-40.0
γ_{k2}	90.0	-106.7
γ_{k3}	-0.57	-0.63
γ_{k4}	-1.50	72.9

Fig. 7 Coefficients of approximate functions

$$\Delta\theta_k = (\alpha_{k1}a + \alpha_{k2})\{\lambda - \beta_{k1}(b + \beta_{k2})^2 - \beta_{k3}\}^2 + \gamma_{k1}a + \gamma_{k2}(b + \gamma_{k3})^2 + \gamma_{k4} \quad (k=1, 2) \quad (16)$$

As a result, the dwell periods are representable quantitatively by an algebraic function with degree one of a , degree four of b and degree two of λ .

4. Dimensional Synthesis

4.1 Procedure of synthesis

The synthesis is carried out in accordance with the flow chart shown in Fig. 8. Given dwell periods

Fig. 8 Flow chart

$\Delta\theta_k (k=1, 2)$ and rocking angle $\Delta\Phi$, link lengths a and b are obtained from Eq. (12) by assuming λ , and they should be checked whether they satisfy Eq. (11) and Fig. 4. Then, the link lengths e, f , and position of the output fixed joint G are determined considering the conditions for link lengths and pressure angles about links e and f , as in Section 2.2. The precise values of the dwell periods of this mechanism should be obtained from Eq. (2), and they should be compared to the given values.

Table 2 Parameters of synthesized mechanism

Parameter	Value	Parameter	Value
a	1,05	f	0,68
b	0,60	λ	56,0°
c, c_2, d	1,00	x_G	0,0346
e	2,00	y_G	-0,6808

Fig. 9 Synthesized mechanism

Fig. 10 Real displacement curve

4.2 Examples

The dwell periods and rocking angle are given, as an example, as follows.

$$\Delta\theta_1=30^\circ, \Delta\theta_2=60^\circ, \Delta\phi=60^\circ$$

Carrying out the synthesis according to the above procedure, we have the mechanism shown in Table 2 and Fig. 9. The input-output displacement curve of this mechanism is shown in Fig. 10, the actual dwell periods, obtained from Eq. (2) when $r=0.02$, become as follows.

$$\Delta\theta_1=29^\circ(346\leq\theta\leq 360^\circ, 0\leq\theta\leq 15^\circ)$$

$$\Delta\theta_2=59^\circ(152\leq\theta\leq 211^\circ)$$

These results mean that the former methods⁽⁴⁾, where dwell periods could be given only as lower limits, have been improved and that a synthesis with high precision is now possible.

5. Conclusions

The results obtained in this paper are summarized as follows:

The characteristics and limits of dwell periods of six-link mechanisms, whose output link dwells at both edge points of rocking, have been clarified by restricting the existing domains of parameters, considering

the conditions for link lengths and pressure angles, and simulating the relationships among each parameter and dwell period.

Approximating the relationships simulated above by algebraic equations with degree 4 or less of each parameter, a dimensional synthesis of six-link dwell mechanisms has become possible whereby given dwell periods are satisfied not only at lower limits, but also at upper ones, with less trial-and-error.

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