A Study on Improvement of the Accuracy of a Three-coordinate Measuring Machine*  
(A Method of Error Correction)

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A method for correcting measured coordinates with the error vector is proposed. The discrete distribution of the error vector, which is already known for the three-coordinate measuring machine used in the experiment, is utilized in this method. The correction method is as follows: (1) The measuring range is divided into the elements of a rectangular prism; (2) The error vector at any position in the element is calculated from interpolation; (3) and Measured coordinates are corrected with the error vector presumed from interpolation. Furthermore, measurements are made of both distance and flatness. The measured values obtained are corrected with the proposed method, and are found to be closer to the true values than those without correction. This correction makes the measuring error less than 3 \( \mu \)m, and hence verifies the usefulness of the proposed method.

Key Words: Measurement, Three-coordinate Measuring Machine, Error Correction, Three-dimensional Element, Interpolation Polynomial

1. Introduction

When a three-coordinate measuring machine (3CMM) is used in measurements, the measuring probe is moved independently in each of the coordinate axes to determine the coordinate positions. The accuracy of this measuring instrument depends on the following factors; the geometrical error of the structure, the deformation caused by the movement of the carriage, and the error in the detecting scale of the coordinates. These factors cause the systematic errors of the 3CMM.

The purpose of this study is to present a method for improving the accuracy of the 3CMM by correcting the systematic error in the measured coordinates. In a previous paper, the authors introduced the ideas of a standard rectangular coordinate system and an error vector, and proposed a basic method to determine the error vector*. On the basis of this method, a discrete distribution of the error vector has been experimentally determined. If the error vector at any position is already known, the true position vector can be obtained by the correction; however, it is impossible to determine the continuous distribution of the error vector over the measuring range. Therefore, in the present paper, the error vector at any position is calculated from interpolation, for which the discrete distribution of the error vector obtained is utilized. The measured coordinates are corrected with the error vector presumed from interpolation.

Measurement is made of both distance and flatness using a gauge block and a surface plate, respectively. The measured values are corrected by the proposed method. As a result, it is clarified that the proposed error correction method improves the accuracy of the 3CMM.

2. Method of error correction

2.1 Fundamental consideration of the error vector

To determine the probe position with theoretical
accuracy, the standard coordinate system ($O$-$XYZ$) is used instead of the measuring coordinate system ($O$-$xyz$). The directions of the $x$-, $y$-, and $z$-axes of the measuring coordinate system agree with the moving directions of the carriages. On the other hand, as shown in Fig. 1, the standard coordinate system is defined with three datum points; namely, the point $O$, or the origin of each system; the point $P$ on the $x$-axis; and the point $Q$ on the $y$-axis. The datum $XY$-plane coincides with the plane in which the three datum points lie and determines the direction of the $Z$-axis. The datum $X$-axis is the line connecting the two points $O$ and $P$. The $Y$-axis is consequently determined in the direction perpendicular to these axes.

The true position vector $X(t) = (X_x, Y_x, Z_x)$ of the measuring probe at point $R$ is unknown in the standard coordinate system. The measured position vector $x(t) = (x, y, z)$ is obtained by the measuring coordinate system. If the point $R'$ with measured coordinates $(x, y, z)$ is positioned in the standard coordinate system as shown in Fig. 1, its position is usually different from the probe position $R$. The deviation between these points $(R, R')$ is defined as the error vector, and is denoted by $\Delta x = (\Delta x_x, \Delta y, \Delta z)$. The relation of these three vectors is expressed as follows:

$$X(t) = x(t) - \Delta x$$

We have the following equation for the distance between two points in the standard coordinate system:

$$N(t)(X(t) - x(t)) = L$$

where, as shown in Fig. 2, vectors $X(t)$ and $x(t)$ are the true position vectors at points $R$ and $R'$, respectively; vector $N(t)$ is the unit vector in the direction of $R; R';$ and scalar $L$ is the distance between $R$ and $R'$. The vectors that we can obtain from the coordinate measurements are only the measured position vectors. The measuring error $M(t)$ arising in the distance measurement is expressed using $x(t)$ and $x(t)$ as follows:

$$M(t) = N(t)(x(t) - x(t)) - L$$

The measuring error in Eq. (3) has an immediate connection with systematic errors at the probe positions. It is determined when the distance is known.

The measuring error can be expressed in another form. Since the relation of the true position vector to the measured position vector and the error vector is as shown in Eq. (1), we obtain the following equation from Eq. (2):

$$M(t) = N(t) \cdot (\Delta x - \Delta x)$$

The left-hand side of Eq. (4) is calculated from Eq. (3). Therefore, from the view point of determining the error vector, Equation (4) is an observation equation containing the error vectors at $R_i$ and $R_j$ as the unknowns.

The observation equation such as Eq. (4) can be obtained from the measurement of a gauge block. In the measurement, the observation 'points' are approximately replaced by the the 'planes' for experimental convenience. In this case, the two measuring planes of the gauge block are positioned at $R_i$ and $R_j$, the length $L_{ij}$ of the gauge block is known, and the unit vector $N_{ij}$ is determined to be perpendicular to the measuring planes. In addition, the measuring error of the distance corresponds to the observed value $M_{ij}$ in Eq. (4).

Including the three standard points $(O, P, Q)$, several points are used as the observation points of the error vectors. The number of points is more than four and is denoted by the sign $n$. The distance measurement is carried out using a gauge block whose measuring planes are located at several positions corresponding to two out of $n$ points. Consequently, we obtain $n(n - 1)/2$ observation equations. The error vectors at $n$ points are determined by the least squares method.

That is, let the residual $r_{ij}$ be defined as follows:

$$r_{ij} = M_{ij} - N_{ij} \cdot (\Delta x - \Delta x)$$

the values of the error vectors are calculated to satisfy the following condition:

$$\sum_{i,j} \sum_{i'j'} (r_{ij})^2 \rightarrow \text{minimum}$$

In actual practice, first, vertices of a rectangular prism are taken as the observation points. The prism coincides with the measuring range of the CMM used in the experiment. Its size is 320 mm along the $x$-axis, 240 mm along the $y$-axis, and 160 mm along the $z$-axis.
axis. Distance measurements are made 28 times to obtain the observation equations. The error vectors at eight vertices are determined under the condition of Eq.(6). Next, to obtain the discrete distribution of the error vector in detail, 60 nodal points in the measuring range are taken as the observation points. They are regularly arranged at intervals of 80 mm as shown in Fig. 3. Each axis component of the error vector is observed independently in the axis direction; namely, the measurement of the positioning error along the each axis direction is made on all three sides of the measuring range, and then the flatness measurement is made at the cross sections indicated by the thick solid line in Fig. 3.

2.2 Method of error correction

The true position vector $X_p$ at any point $R$ in the standard coordinate system can be determined by correcting the measured coordinates $x_p$ with the error vector $\Delta x_p$; but it is almost impossible to determine the continuous distribution of the error vector. In this study, the measuring range is divided into the elements shown in Fig. 3. Calculation of the error vector at any position is attempted by interpolation in the element. The values of the error vectors obtained at 60 nodal points are utilized for the method. Error correction is made using the presumed error vector.

2.3 Interpolation method

The measuring range is divided into 24 cubic elements as shown in Fig. 3. The side length of the element is equal to 80 mm and each side is parallel to one of the three axes of the measuring coordinate system. Each vertex of the elements coincides with one of 60 nodal points.

An element together with its local coordinate system is indicated in Fig. 4. This system is denoted by $P_i=UVW$. The origin of the system is the point $P_i$. The measured position vectors $(x_{i1}, \ldots, x_{i6})$ and the error vectors $(\Delta x_{i1}, \ldots, \Delta x_{i6})$ at eight vertices $(P_i, \ldots, P_8)$ are already known. The axis length of the system is normalized with the side length of the element. The local position vectors at eight vertices are consequently expressed as $U_i(0,0,0), \ldots, U_i(1,1,1)$ in this system.

The local position vector at any point $P$ in the element is calculated from the measured coordinates. Each component of the error vector at point $P$ is determined from the interpolation polynomial as follows;

\[
\begin{align*}
\Delta x_i &= \{E\} \{a\}, \\
\Delta y_i &= \{E\} \{b\}, \\
\Delta z_i &= \{E\} \{c\},
\end{align*}
\]  

(7)

where

\[
\begin{align*}
\{E\} &= \begin{pmatrix} U_p & V_p & W_p & U_p & V_p & W_p & U_p & V_p & W_p \end{pmatrix}, \\
\{a\}^T &= \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \end{pmatrix}, \\
\{b\}^T &= \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & \beta_6 & \beta_7 & \beta_8 \end{pmatrix}, \\
\{c\}^T &= \begin{pmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 & \gamma_7 & \gamma_8 \end{pmatrix}.
\end{align*}
\]

In Eq.(7), the row vector $\{E\}$ depends only on the probe position. The column vectors $\{a\}, \{b\}$ and $\{c\}$ are the coefficient column vectors, which determine the relation between the error vector and the probe position. Each axis component of the error vector is calculated to be linear in the axis direction.

The coefficient column vectors are determined with the known error vectors at eight vertices. For example, elements $a_i (i=1, \ldots, 8)$ agree with the solutions of the following equation;

\[
\{C\} \{a\} = \{\Delta x\},
\]  

(8)

where

\[
\begin{align*}
\{\Delta x\}^T &= \begin{pmatrix} \Delta x_1 & \Delta x_2 & \Delta x_3 & \cdots & \Delta x_8 \end{pmatrix}, \\
\{C\} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}
\end{align*}
\]

In Eq.(8), the values of $\Delta x_1, \Delta x_2, \ldots$, and $\Delta x_8$ are the $x$
axis components of the error vectors at nodal points $P_1, P_2, \ldots, P_n$, respectively. The vectors $[\beta]$ and $[\gamma]$ are determined in the same way. Therefore, Eq. (7) can be rewritten as follows:
\[
\begin{align*}
\Delta x &= [E][C]^{-1}[dx], \\
\Delta y &= [E][C]^{-1}[dy], \\
\Delta z &= [E][C]^{-1}[dz].
\end{align*}
\]  

(9)

In Eq. (9), inverse matrix $[C]^{-1}$ is the same in every element if its shape is the same. The vectors $[C]^{-1}[dx], [C]^{-1}[dy],$ and $[C]^{-1}[dz]$ are previously calculated for each element.

The procedure for calculating the true position vector $\mathbf{x}_p$ is as follows:

1. The element in which the measuring probe is positioned is determined from the measured position vector $\mathbf{x}_p$.
2. The local position vector $\mathbf{u}_p$ and the vector $[E]$ at point $P$ in the element are calculated.
3. The error vector at point $P$ is presumed from the interpolation polynomial indicated in Eq. (7).
4. The true position vector is calculated from Eq. (1).

3. Results of Error Correction

To certify that the proposed error correction method is of use for improving the accuracy, measurements of distance and flatness are made. Gauge blocks and a surface plate are used for the standards of distance and flatness, respectively. The correction method is applied to the data obtained. The measured value and the corrected value correspond to the observed value without correction and the one with correction, respectively. All measurements are made at 20 ±1°C. The values of the error vectors at 60 nodal points employed for the method are those determined previously.

3.1 Distance measurement

Several gauge blocks are measured in various directions and positions in the measuring range. Their lengths range from 50 mm to 400 mm. The gauge block used in the measurement is set in close contact to an optical flat. The coordinate measurements at nearly 15 points are carried out on two planes; one is the plane of the optical flat with which the gauge block is in contact; the other is the measuring plane of the gauge block opposite the optical flat. The distance between the two planes is calculated. The measuring error is determined by subtracting the gauge length from the distance obtained. Then the measured value is corrected with the error vector presumed to be at the center position of each measuring plane.

The distance measurement is carried out about 700 times. Figure 5 shows the probability density variations of the corrected values and the measured values. The measured values are distributed widely in the range from $-8.6 \mu m$ to 16.1 km and their standard deviation is 4.1 km. The corrected values are distributed from $-3.9 \mu m$ to 4.8 km and the standard deviation is 1.2 km. Figure 6 shows the dispersion of the corrected value with the measured value. The effect of the error correction does not depend on the size of the measuring error. In the distance measurement, more than 98% of the corrected values are within the range of $\pm 3 \mu m$; however, they are slightly shifted in a positive direction.

3.2 Flatness measurement

A surface plate placed at various positions and directions in the measuring range is used for the standard of flatness. Three parameters of the standard plane on the surface plate are determined by coordinate measurements at three positions. The distances between the standard plane and the measured coordinates at other measuring positions are calculated, then the measured values are corrected with the presumed error vector. One of the experimental results without and with correction is shown in Fig. 7(a) and 7(b), respectively.

The flatness measurement is carried out about 120 times and the measuring positions in the experiment
amount to 1.200. The probability density variations of the observed values with and without correction are shown in Fig. 8. The measured values and the corrected values range from \(-1.16 \mu m\) to \(2.8 \mu m\) and from \(-2.6 \mu m\) to \(1.5 \mu m\), respectively. Their standard deviations are \(2.8 \mu m\) and \(0.5 \mu m\), respectively. Error correction is more effective in flatness measurement than in distance measurement.

3.3 Accuracy of the presumed error vector

The error vectors at 18 positions shown in Fig. 9 are examined and their values are determined\(^\text{2}\). The first of these points is positioned at \((60,20,60)\) mm in the measuring coordinate system. Each position is arranged with a 100 mm separation in each axis direction. The experiment is performed by placing the gauge block between each set of 2 of 18 points. Consequently, the number of gauge block measurements made to determine the error vectors is 153. It can be considered that these values are nearly true, because they are calculated to satisfy Eq. (6). Therefore, the presumed error vector is compared with the 'true' value.

The difference between the presumed value and the true value for each axis component is about \(3 \mu m\) at its maximum. The presumed value is generally smaller than the true value in the \(x\)-axis component. It seems that the positive shift of the distribution curve of the corrected value in Fig. 5 is due to this fact.

3.4 Subdivision of the element

Subdivision of the element makes the presumed error vector more accurate. For the 3CMM used in the experiment, only the \(x\)-axis component of the error vector varies in proportion to the \(z\)-axis coordinates; other components scarcely vary in the \(z\)-axis direction. Therefore, the side length of the element is divided in half along the \(x\)- and \(y\)-axes, leaving the \(z\)-axis untouched. Accordingly, the measuring range is divided into 8, 6, and 2 in the \(x\), \(y\), and \(z\) directions, respectively. The number of nodal points and ele-

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Table 1: Difference between the error vector component presumed with 96 elements and its true value at 18 positions.

<table>
<thead>
<tr>
<th>no.</th>
<th>(\Delta x)</th>
<th>(\Delta y)</th>
<th>(\Delta z)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>-0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>-0.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.8</td>
<td>-0.2</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>-0.2</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>7</td>
<td>-0.7</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>-0.7</td>
<td>-0.7</td>
<td>1.0</td>
</tr>
<tr>
<td>9</td>
<td>-0.9</td>
<td>-0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>10</td>
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<td>0.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>11</td>
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<td>-0.2</td>
<td>-0.5</td>
</tr>
<tr>
<td>12</td>
<td>-0.3</td>
<td>-0.4</td>
<td>-0.7</td>
</tr>
<tr>
<td>13</td>
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<td>1.1</td>
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</tr>
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<tr>
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<td>0.7</td>
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<td>17</td>
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<td>-0.3</td>
<td>-0.8</td>
</tr>
<tr>
<td>18</td>
<td>-1.1</td>
<td>0.3</td>
<td>-1.5</td>
</tr>
</tbody>
</table>

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Fig. 7: Experimental result of flatness measurement

Fig. 8: Probability density variation of the observed values with and without correction in flatness measurement

Fig. 9: Components of the error vector at 18 points determined by Eq. (6)

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ments amount to 189 and 96, respectively. Error correction is carried out on these 96 elements.

The unknown error vectors of 189 nodal points are determined by the following measurements; measurements of the positioning error at intervals of 40mm along the x- and y-axes, and flatness measurements in the cross sections indicated by the thin solid lines in Fig. 3. The comparison of the presumed and true values of the error vectors at 18 points is indicated in Table 1. The presumed value obtained from the correction with 96 elements is much closer to the true value than that with 24 elements.

In the measurements of distance and flatness, the correction method with 96 elements is applied to the data obtained. The mean value, the absolute mean value, and the standard deviation of the corrected values with 24 elements and with 96 elements as well as those of the measured values are shown in Table 2. The measuring accuracy is improved by the subdivision.

Corrected values with 24 elements exceed ±3 μm in 17 out of 700 distance measurements, but those with 96 elements exceed ±3 μm in only five cases.

4. Conclusion

A method for correcting measured coordinates with the error vector is proposed for the purpose of improving the accuracy of the 3CMM. The error vector at any position is presumed from the interpolation polynomial in the element of the rectangular prism. The method is applied to the data obtained from the measurements of distance and flatness. The proposed error correction method reduces the measuring error to less than 3 μm.

The main results obtained are as follows:

(1) By distance measurements using gauge blocks of several lengths, it is possible to determine the error vectors.

(2) Measured coordinates can be corrected with the discrete distribution of the error vector and the measuring accuracy is improved.

References

