Pressure Losses and Flow Field Distortion Induced by Tip Clearance of Centrifugal and Axial Compressors *

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The flow field near the tip of compressor rotor blades is distorted by leakage through the tip clearance and the performance of the compressor is deteriorated. The literature regarding the tip clearance of compressor blades consists of computational fluid mechanics and experimental studies on the flow field and the pressure loss. Empirical equations expressing the pressure loss and the efficiency drop are varied. They relate to the lift coefficient in different ways, such as proportional to C_L , $C_L^{1.5}$, C_L^2 , or the sum of two terms, depending upon the ways of understanding the mechanics of pressure losses. These methods are examined and compared. Also, a brief discussion is made on the optimum value of the tip clearance.

Key Words : Fluid Machine, Axial Compressor, Centrifugal Compressor, Tip Clearance, Leakage, Impeller

1. Introduction

The performance of turbomachines is inherently deteriorated by leakage through the clearance, if such a clearance exists between the casing and the tip of rotor blades. In cases of high pressure ratio compressors, since the specific volume of gas is reduced considerably at the exit, the blades are short near the exit and the ratio of the tip clearance to the blade height is relatively large; that is, the tip clearance effect is significant. For design purposes, therefore, it is important to predict the influences of tip clearance on the performance of turbomachines. According to the literature, the degradation of compressor performance due to the tip clearance is expressed in different ways, and the empirical coefficients of these equations vary widely, depending upon the blade geometry and on the

operating conditions. This means that the physics of pressure loss due to tip clearance is not yet well understood.

In order to properly design the stator blade row behind an unshrouded rotor, it is necessary to predict the flow pattern behind the rotor. There has been much literature published regarding the effects of the tip clearance of blades on the flow pattern and the performance of turbomachines. The studies are classified as follows:

(1) Experimental study.

- (a) variation of input power, pressure loss and efficiency drop due to tip clearance,
- (b) study on the flow pattern, especially the spanwise distributions of circumferentially averaged flow-direction and total pressure.

(2) Macroscopic analysis on mechanics of flow and pressure loss.

- (a) pressure loss and efficiency drop due to tip clearance,

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- (b) assumption of a flow model which represents leakage, and prediction on spanwise distributions of velocity vector and total pressure.

(3) Flow analysis by means of computational fluid mechanics.

This paper reviews research activities in these fields.

2. Experimental studies on centrifugal compressors

2.1 Shrouded and unshrouded impellers

There are two types of centrifugal impellers. One is a shrouded impeller, where the shroud rotates with the impeller and there is no clearance between the shroud and the tip of blades, and the other is an open, or unshrouded impeller, where a stationary shroud is the casing, and there is a narrow clearance between the shroud and the tip of the blades. According to the literature^{(1)~(3)}, open impellers are superior to closed impellers of identical design, providing that the tip clearance is not large. These experimental data support the hypothesis that there is an optimum size of tip clearance for impeller blades.

In cases of shrouded impellers, there are secondary flows along the shroud from the pressure side of a blade to the suction side of the adjacent blade. In cases of open impellers, the secondary flows are intercepted by the leakage flow through the clearance and also by motion of the stationary shroud relative to the rotating blades, and it is presumed that the reduction of the secondary flow loss is larger than the pressure loss due to the tip clearance, providing that the tip clearance is fairly small. However, it is not possible to experimentally separate the amount of reduction in the secondary flow loss from the clearance loss.

2.2 Variation of impeller efficiency due to tip clearance

At the exit of the impeller, the velocity varies across the pitch as well as along the span⁽⁴⁾⁽⁵⁾, but diffuser blades at the exit of the impeller are either two-dimensional or pipe-type, because the blades are too short to twist. Therefore, correct predictions of the input head and the pressure drop due to tip clearance are more important than that of the spanwise distribution of flow angles at the diffuser inlet.

Variation of the compressor characteristics due to the tip clearance at the best efficiency condition is usually expressed as follows, where $\dot{G}$ is the flow rate, h is the head rise and η is the efficiency while c is the clearance, and $(b_1 + b_2)/2$ is the mean blade height at the inlet and the exit of the blades.

$$-\Delta \dot{G}/\dot{G} = \alpha 2c/(b_1 + b_2) \quad (1)$$

$$-\Delta h/h = \beta 2c/(b_1 + b_2) \quad (2)$$

$$-\Delta \eta/\eta = \gamma 2c/(b_1 + b_2) \quad (3)$$

In Ref. (6), $\alpha=0.5$ and $\beta=\gamma=0.9$, while in Ref. (7), $2\alpha=\beta=\gamma=1.5\sim 3.0$. If $\beta=\gamma$, the input head is hardly changed by the tip clearance. The wide variation of the coefficients in the literature means that these equations are not sufficient to express the tip clearance effects on the performances of the compressors, and a few other parameters must be involved.

The References (6) and (7) are dated. At the time of their publication, impellers had either short inducers or none at all. Fig. 1 shows the data of six centrifugal compressors for small gas turbines. The three full lines indicate the range of data and the mean line⁽⁸⁾, where the abscissa is the tip clearance/blade width ratio at the impeller exit and the ordinate is the difference of efficiency from the same impeller with zero clearance. The mean line agrees with Ref. (6) if $b_1=4b_2$ and $\eta=0.8$. The four dotted lines were drawn by the present author, who connected data belonging to the respective compressors. Gradients of these dotted lines are steeper than the gradient of the full lines, and none of the dotted lines pass through the origin. Since it is not possible to test a compressor with zero clearance, the origin of the ordinate is dubious. The gradients of these lines are 1.3 to 3 times the gradient of the full line, and they are within the range shown in Ref. (7).

Regarding the experiment on tip clearance effect, a very high accuracy is required because the variation in performance is not large. Since the velocity distribution is considerably distorted by tip clearance, the mass-averaged total pressure measured at the impeller exit is not accurate. At the diffuser exit, the total pressure is accurately measured because the velocity is low; however, the variation of impeller performance due to tip clearance is not correctly represented by the exit pressure unless the pressure recovery in the diffuser is not influenced by the velocity distribution at the diffuser inlet. Such difficulty in accurate measurement may be one reason why the empirical

Fig. 1 Efficiency drop due to axial tip clearance of centrifugal compressors⁽⁸⁾

equations in the literature have such wide variation.

In almost all experiments on centrifugal compressors, the tip clearance was changed by the axial movement of the shroud casing relative to the impeller. Therefore, the tip clearance of the inducer was not so much changed as was the clearance at the impeller exit. If the tip clearance is uniformly distributed from the inlet to the exit of the impeller, it is likely⁽⁹⁾ that the influence of tip clearance is considerably larger than the experimental results that the literature suggests. Such realistic experiments are welcomed by compressor designers.

In general, the input head becomes slightly smaller as the tip clearance becomes larger, and the trend is conspicuous in impellers with a large, backward-leaning blade angle, but the relation is not consistent⁽¹⁰⁾. Therefore, it is understandable that the input head was assumed to be independent of the tip clearance in Refs. (6) and (7).

2.3 Tip clearance effects at different operating conditions

In cases of compressible fluid, the density ratio of gas at the inlet and the exit of an impeller depends upon the shaft speed; therefore, the flow pattern in the impeller and the leakage through the tip clearance will not be similar if the shaft speed varies. In the case of a centrifugal compressor of 6:1 pressure ratio⁽¹¹⁾, the efficiency drop due to tip clearance became only one-half of that at the design speed when it was operated at 50% speed⁽¹²⁾. Furthermore, in general, as the shaft speed is increased, impellers are deformed by the centrifugal force in such a way that the tip clearance at the exit is reduced.

If a vaned diffuser is installed behind an impeller, the available flow range is narrowed at a given shaft speed. Therefore, it is not possible to examine the tip clearance effects of an impeller at various flow rates, unless a vaneless diffuser is used instead of a vaned diffuser. According to an experiment and the prediction⁽¹²⁾, the drop of impeller efficiency due to tip clearance became somewhat less as the flow rate was reduced.

2.4 Surge limit

In many cases, the minimum flow rate of a compressor is limited by the stall of either the impeller or the diffuser. If the inducer is the cause of impeller stall, the tip clearance of the inducer may have a direct influence on the surge limit. However, in many experiments in the literature, although the tip clearance of the impeller was changed by axial movement of the shroud casing, the tip clearance of the inducer was hardly changed. Therefore, experimental data in the literature should be examined carefully in this respect.

If surge is induced by the stall of the diffuser, the surge limit is influenced by the diffuser-inlet velocity distribution, which is modified by the tip clearance of impeller blades. In the cases of high pressure ratio compressors, the lower efficiency of impellers with a larger tip clearance results in a lower pressure ratio or a larger volume flow rate at the impeller exit; as a result, the incidence angle to the diffuser varies. According to experimental data of a 6:1 pressure ratio compressor⁽¹¹⁾, the flow rate at the surge limit was less as the tip clearance of the impeller blade was larger. At the same time, the pressure ratio was reduced, and the surge point moved approximately on the surge line of the original compressor of narrow tip clearance operating at different shaft speeds.

3. Experimental studies on axial compressors

3.1 Variation of impeller efficiency due to tip clearance

Similar to cases of centrifugal compressors, the velocity distribution at the exit of axial flow impellers is distorted by the tip clearance of impeller blades; as a result, the pressure recovery in the downstream stator blades is reduced. Therefore, it is difficult to examine correctly the influence of tip clearance on the performance of the impeller itself.

Pfleiderer⁽⁷⁾ assumed the ratio of the annular area made by tip clearance c to the annular area of the impeller blades as the representative value equivalent to $2c/(b_1 + b_2)$ in Eqs. (1), (2) and (3) for centrifugal impellers. This value is $2/(1 + \nu)$ times the ratio c/h , where ν is the hub/tip ratio and h is the blade height. He suggested that $\alpha=1.25$, $\beta=2.5$ and $\gamma=2.15$, which happened to be the mean values of the ranges suggested by him in the case of centrifugal impellers. Howell suggested that the stage efficiency drops by 3% for an increase of 1% in c/h , or $\gamma\eta=3$ in Eq. (3). According to the experimental data⁽¹³⁾⁽¹⁴⁾ of seven kinds of axial fans and pumps, the ratio $\gamma\eta$ varied between 1.4 and 2.8. The wide variation of the coefficients means that the tip clearance effect varies, depending upon the geometry of blades and the blade loading.

The blade geometry of an impeller including the tip clearance was modeled as a linear cascade with a gap at the middle of the span, and it was tested in a wind tunnel⁽¹³⁾. In such cases, the pressure drop due to the tip clearance is sometimes expressed as an increment of drag coefficient C_D . The efficiency drop of the impeller is related to the increment of C_D as follows:

$$-\Delta\eta = \frac{\Delta C_D}{C_L} \frac{\phi}{\cos^2 \beta} \frac{1}{1 + \varepsilon \tan \beta} \quad (4)$$

where $\varepsilon = (C_D + \Delta C_D)/C_L$, C_L is the lift coefficient of the blades, β is the direction of vector mean velocity

and ϕ is the ratio of the axial velocity to the tip velocity.

The relative flow in an impeller is different from the flow through a linear cascade in many respects, such as curvature of casing, secondary flow in the boundary layer along the impeller blades induced by rotation and the tangential motion of the casing relative to the blades. In a wind tunnel test of a linear cascade, a running belt was used to represent the motion of the casing relative to the cascade⁽¹⁵⁾.

The pressure losses measured in those experiments were expressed as increments of the pressure loss coefficient or of the drag coefficient of the blades, as well as reduction of the stage efficiency, and they were used for deriving empirical equations based on different hypotheses on loss mechanism, which will be reviewed in Chapter 5.

3.2 Secondary flow due to tip clearance

In cases of axial compressors blades are long and twisted. If the flow pattern at the exit of an impeller is distorted by the tip clearance, it is possible to design the downstream stator so that the blades match the local flow direction. Behlke⁽¹⁶⁾ was successful in improving the peak efficiency and widening the flow range considerably. It may also be possible to design the rotor blades so that they match the distorted spanwise distribution of the axial velocity, or so that

the blade loading near the tip is reduced and leakage is less. For these purposes it is essential to predict the velocity distribution at the impeller exit by including the tip clearance effects. There has been a considerable number of experimental studies performed on the flow in the rotor and immediately downstream.

For designing a blade row, the flow is handled as axisymmetric and circumferential variation is disregarded. A probe behind a rotor indicates the time mean value, which usually differs considerably from the circumferentially mass-averaged value if both the direction and the magnitude of velocity vary significantly. For measurement of the circumferentially mass-averaged velocity distribution and for understanding the behavior of blades in the distorted casing boundary layer, a considerable effort has been made for measuring the details of flow between blades which are influenced by the tip clearance.

By rotating various types of probes together with the impeller⁽¹⁷⁾ or by applying a phase-lock technique to the periodic output signal of a fixed probe behind an impeller^{(18)~(22)}, the relative velocity distributions between and behind the blades were measured, and it was revealed that leakage flow rolled up, forming a vortex between blades, and the velocity distribution was distorted within a layer of thickness about ten times the tip clearance. The flow patterns at off-

Fig. 2 Distributions of relative total pressure and of velocity-vector of secondary flow measured behind an axial impeller⁽²⁴⁾. τ is clearance, chord and pitch are 117.5 mm, blade length is 85.5 mm, impeller diameter is 449 mm.

design conditions were compared with those at the design conditions. The flow pattern behind the rotor was also measured using a laser velocimeter⁽²³⁾ and a periodic multisampling technique with a slanted hot-wire⁽²⁴⁾. The flow patterns for five values of tip clearance were measured by Inoue et al.⁽²⁴⁾ and presented as in Fig.2. A vortex was recognized, which increased in strength as the tip clearance became larger.

3.3 Influence of casing boundary layer

In the casing boundary layer, the inlet flow angle relative to the rotor blade is different from that of the main flow, and leakage flow interferes with the secondary flow between blades. Wagner et al.⁽²⁵⁾⁽²⁶⁾ made extensive experiments varying the thickness of the casing boundary layer. Smith⁽²⁷⁾ simplified the problem by assuming that the displacement thickness of the casing boundary layer was increased by the thickness of the tip clearance. He successfully predicted the performances of multistage compressors, based on the assumption that the displacement thickness increases the non-effective length of the blade near the blade tip.

The experimental data⁽²⁸⁾ with different values of inlet boundary layer thickness were correctly predicted by assuming that the work of impeller blades was influenced as the simple sum of the casing boundary layer effect and the tip clearance effect. The efficiency drop at the blade tip zone varied in proportion to the tip clearance, where the tip clearance varied from 0.43% to 4.3% of the blade pitch, and the authors concluded that the optimum tip clearance must be less than 1.0% of the blade pitch, if it exists at all.

Wennerstrom⁽²⁹⁾ tested an axial transonic compressor with three values of the tip clearance, 0.415%, 0.675% and 0.935% of the chord by shaving off 0.25mm each time at the blade tip, and found that the clearance 0.675% demonstrated the highest efficiency, about 0.7% higher than the other two cases, and also the highest pressure ratio. Apparently, the optimum clearance varies, case by case. According to Lakshminarayana⁽¹³⁾ it is 1 to 1.5% of the chord for impellers, while it is 3 to 5% for cascades.

Regarding the dimensionless presentation of the tip clearance, it is divided by the blade chord or by the blade pitch when the flow pattern and the performance of the blade near the tip are discussed. It should be divided by the blade height when the performance of the impeller or the stage as a whole is discussed.

If the motion of the shroud casing relative to the blades influences the flow in the tip clearance zone, the surface roughness⁽³⁰⁾ on the shroud casing is not negligible, and an additional parameter may be included.

3.4 Surge limit

Since the tip clearance increases the displacement

thickness of the casing boundary layer, the characteristic curves as a whole are shifted toward the left, or lower flow rate. If the stall of the blades in the tip zone is not responsible for surge, the surge flow rate should be reduced by the tip clearance. However, in reality the critical flow rate is little changed or slightly increased⁽³¹⁾ with the tip clearance.

Smith⁽²⁷⁾ deduced from many experimental data that the impeller stalled when the displacement thickness of the casing boundary layer exceeded the value

$$\delta^*/g|_{\text{stall}} = 0.20 + \varepsilon/g \quad (5)$$

where ε is the clearance while g is the passage width between the blades rather than the pitch. He also mentioned that the tip clearance increased the displacement thickness by the magnitude of the clearance at the peak pressure ratio. Combining these two assumptions, it may be concluded that the stall limit is hardly influenced by the tip clearance. However, there is a report⁽³²⁾ where the critical flow rate for rotating stall was considerably influenced by the tip clearance.

4. Prediction of flow field effected by tip clearance

4.1 Computational analysis of flow

Parallel to the advancement of the computer, the inviscid flow analysis for turbomachinery has progressed from the two-dimensional to the quasi-three dimensional to finally the three-dimensional. These days, it is not impossible to perform three-dimensional viscous flow analysis, but the computing time is not short, and it is not practical to use the analysis for the design of turbomachines. To save computer time, it is desirable to reduce the number of grid points in each cross-section. However, a flow analysis between blades with tip clearance requires many grid points, because the grid must be considerably smaller than the clearance while the leakage flow rolls up, forming a vortex of a diameter about ten times the tip clearance.

Moore et al.⁽³³⁾ developed a partially parabolic calculation procedure to calculate three-dimensional viscous flow in a centrifugal impeller. The three-dimensional pressure field within the impeller was obtained by first performing a three-dimensional inviscid flow calculation, and then adding a viscosity model and a viscous-wall boundary condition to allow calculation of the three-dimensional viscous flow. The mixing-length model was used as the turbulent viscosity. The calculated wake development in Fig.3 and pressure distributions at the exit were compared⁽³⁴⁾ with measurements⁽⁴⁾. The results were satisfactory, predicting the impeller efficiency within 1%. One of the reasons why the steep gradient of velocity was not predicted in the analysis in Fig.3 was the very coarse mesh of about 100 grid points per section used.

Fig. 3 Comparison of predicted and experimental velocity distributions at the exit of a centrifugal impeller⁽³⁴⁾.

Regarding the flow in axial flow turbomachines with tip clearance and casing boundary layer, Hah solved the Reynolds-averaged Navier-Stokes equation in elliptic form for a turbine⁽³⁵⁾ and a compressor⁽³⁶⁾, and compared the predictions with the experimental data for the clearances 1% and 5.1% of the chord. As shown in Fig. 4, the spanwise distributions of the flow angle and the axial velocity component agree well at the two flow rates compared.

Pouagare⁽³⁷⁾ analyzed turbulent flow through a linear decelerating cascade with tip clearance, using a multisweep space-marching method. Good agreement with experiments was obtained for a few sizes of tip clearance at a few levels of blade loading. Dawes⁽³⁸⁾ also made predictions for a transonic axial compressor and compared with experiments.

By collecting many numerical examples at different values of blade loading and boundary layer parameters, it may be possible to understand how leakage flow interferes with the secondary flow in the casing boundary layer, and how the flow patterns are related to the loss of total pressure.

4.2 Prediction of velocity distribution

In cases of axial impellers, it is not difficult to twist the blade along the span so that the blade nicely fits to the local flow direction. Therefore, it is important to predict the velocity distribution at the impeller exit for the design of the following stator blades.

Lakshminarayana⁽¹³⁾ assumed that the leakage flow through a blade tip clearance rolls up, forming a vortex, which is located at a distance a from the line connecting the tips of adjacent blades, and he proposed an empirical equation for the distance a . Furthermore, he calculated the distribution of flow angle between blades, assuming that vortices are uniformly distributed within a circle of radius a .

In order to examine the flow pattern through a cascade where a tip clearance exists between an end wall and the tip of the blades, the cascade was represented as one with a gap at the middle of the span so that the wall was substituted by the mirror image of the cascade. Experiments were performed for three

Fig. 4 Spanwise distributions of pitch-mean flow angle and axial velocity-component behind an axial impeller. Circular and square symbols are measured data where tip clearance was 1% and 5% of chord respectively, while full lines are predicted values⁽³⁶⁾.

conditions: uniform inlet flow, distorted inlet velocity distribution representing a wall boundary layer without wall and with a stationary wall at the middle of the gap. According to the author, the distributions of flow angle behind the cascade were almost equal to each other and they were close to his predictions, disregarding the inlet boundary layer. When the wall was attached to the blades without clearance, flow separation was observed near the wall and the thickness of the underturning flow layer was about three times the thickness of the underturning flow layer observed in the cascade with tip clearance⁽¹³⁾.

In this case, separation of the wall boundary layer was cured by leakage, or leakage contributed to improving the performance a certain amount. In such cases, there must be an optimum value for the tip clearance.

Roberts⁽³⁹⁾ measured the distributions of flow angle behind a rotor and a stator of a multistage axial compressor, and the differences from the two-dimen-

sional case were related to the boundary layer thickness and the tip clearance.

5. Physics of pressure loss due to tip clearance

The conditions of flow at the tip region of impeller blades are quite complicated, due to a combination of three factors: 1) the incidence angle to the blade varies spanwise due to the circumferential motion of blades relative to the boundary layer flow, 2) secondary flow exists in the casing boundary layer from the pressure side of a blade to the suction side of the adjacent blade, and 3) the leakage flow through the blade tip clearance interferes with the secondary flow.

The blade loading varies mutually with the flow pattern. The mutual relations are so complicated that the only way to quantitatively evaluate the relationship is by computational analysis, as mentioned in the last chapter.

In order to understand the physical aspects of pressure loss due to tip clearance, it is intended to propose flow models consisting of only a few parameters which are of primary importance. If the pressure loss estimated by the flow model is close to the experimental data, and it can correctly predict how the pressure loss varies at different operating conditions, the flow model is reasonable and useful for understanding the phenomena. It may be improved further by empirical coefficients which represent the neglected minor parameters in the proposed model of pressure losses.

5.1 Simplification of blade forces

The leakage flow through the blade tip clearance rolls up, forming a vortex core which induces secondary flow between blades and modifies the loading of the blade. In general, the blade loading is light at the edge of the blade, while near the edge, the loading can be larger than the two-dimensional value⁽⁴⁰⁾, and separation sometimes occurs near the blade tip and trailing edge zone.

In this chapter, such complicated phenomena are disregarded for the sake of simplicity, and it is assumed that the blades work as an ideal two-dimensional cascade. Therefore, reduction of blade loading due to tip clearance may be handled as a reduction of the effective length of blade and a minor reduction of the effective flow area between blades. Such a correction does not change the basic relations of the pressure loss to the tip clearance, and in this chapter, discussion is made without including such minor correction factors.

5.2 Leakage loss through tip clearance

It is usually believed that the pressure loss due to tip clearance is nothing but the loss of kinetic energy of the leakage flow through the tip clearance of the

blade. It is applicable to turbine cascades as well as to compressor cascades. According to Ref.(41), Rains derived the following equation for the energy loss of leakage flow, assuming that the blade loading varies linearly from the leading edge to the trailing edge.

$$\Delta E = (2\sqrt{2}/5)k\lambda^3\rho\delta cw_\infty^3 C_L^{3/2} \quad (6)$$

where k is the contraction coefficient of leakage flow, δ is the clearance, c is the chord, C_L is the lift coefficient, λ is the velocity factor of leakage and W_∞ is the vector mean velocity of inlet and exit velocities. The equivalent drag coefficient C_{Dt} is derived from Eq. (6), such that

$$C_{Dt} = \frac{4\sqrt{2}}{5}k\lambda^3\frac{\delta}{h}C_L^{3/2}$$

where h is the blade height. There is a remark that although $k=0.5$ and $\lambda=0.8$ if the tip clearance is a fraction of the blade thickness, the pressure loss becomes considerably larger than the above prediction due to a separation of flow from the suction side of blades if the clearance is the size of the blade thickness.

Regarding the balance of forces in the direction of main flow, the pressure loss must be balanced with a drag force or a momentum change in the main flow direction. The pressure difference across the blade induces only the velocity component normal to the main flow direction; therefore, the leakage flow does not appear to be a cause of pressure loss. Senoo answered the contradiction as follows⁽¹⁴⁾: the fluid on the pressure side of the clearance is accelerated normal to the blade but the velocity component parallel to the blade w_p remains unchanged when it arrives at the suction side of the blade, where the main flow velocity is w_s . Therefore, the product of the leakage flow rate and the velocity difference $w_s - w_p$ is the momentum defect, or the resistance to the main flow. The energy loss due to the resistance is the product of the mean velocity $(w_s + w_p)/2$ and the resistance, which is identical to the product of the leakage flow rate and the pressure difference across the blade.

5.3 Pressure loss due to induced drag

If a blade with a finite span is flown in air, vortex filaments representing the circulation around the blade are shed downstream and induce drag to the airfoil.

In cases of an impeller rotating in a casing or a cascade with a gap at the end of the blades, the leakage flow through the tip clearance rolls up, forming a vortex tube that flows downstream. Therefore, it is natural to assume that the energy loss may be handled in the same way as in the case of a flying wing, but it is very difficult to estimate the strength of the trailing vortex in the case of leakage flow.

In the case of a flying wing, a downwash velocity

w is induced by the trailing vortices. Therefore, the direction of the relative velocity to the airfoil inclines by $\tan^{-1} w/u$ from the direction of flight, where u is the flight speed. Since the lift force L of the wing is normal to the direction of relative velocity, the lift force has the component Lw/u against the direction of flight. Incidentally, both L and w are proportional to the intensity of circulation, which is proportional to the lift coefficient; therefore, the induced drag is proportional to the square of the lift coefficient C_L .

For cases of leakage through blade tip clearance, the following expression⁽⁴¹⁾ was derived as the induced drag coefficient by Bety for Kaplan turbines, and by Mehldahl for axial compressors.

$$C_{Di} = \frac{1}{4} C_L^2 \frac{\delta}{h} \frac{\sigma}{\cos \beta_2} \quad (8)$$

where δ is the clearance, h is the blade length, σ is the solidity and β_2 is the exit flow angle measured from the axial direction.

Lakshminarayana⁽¹³⁾ proposed that

$$C_{Di} = 0.7 C_L^2 \sigma \delta / h \quad (9)$$

and that the pressure loss coefficient due to the induced drag is

$$\zeta_i = \frac{-\Delta p}{\rho v_i^2 / 2} = 0.7 C_L^2 \frac{\delta}{h} \sigma^2 \frac{\cos^2 \beta_1}{\cos^2 \beta_m} \quad (10)$$

According to him, the tip clearance changes the spanwise distribution of blade pressure, and a secondary flow is created in the boundary layer along the blade. The energy loss must then be added to the induced drag as the energy loss due to the tip clearance. Using a work coefficient ψ and a flow coefficient ϕ , which are related to C_L such that

$$C_L = (\psi / \phi \sigma) \cos \beta_m$$

the efficiency drop is expressed as

$$\Delta \eta = \frac{0.74 \phi}{\cos \beta_m} \frac{\delta}{h} \left(1 + 10 \sqrt{\frac{\phi}{c} \frac{\delta}{\cos \beta_m}} \right)$$

where c is the chord and β_m is the mean flow angle. The coefficient of 10 in the second term was chosen so that the equation agrees with experimental data, and there was no discussion whether the second term is a reasonable value for the energy loss of the secondary flow along the blades. He applied this equation to many compressors, pumps and to a turbine, and good agreement with experimental data was demonstrated⁽¹³⁾.

The concept that the induced drag coefficient is proportional to the square of the lift coefficient was derived for a wing flying in open air; that is, for the case of extremely large tip clearance. On the other hand, the expressions (8) ~ (11) are applicable only for very small values of δ/h . Otherwise the induced drag becomes unreasonably large for cases of large tip clearance. Therefore, it is dubious to apply the relationship that C_{Di} is proportional to C_L^2 for cases

of narrow tip clearance.

5.4 Balance of forces in cases of decelerating cascade

In cases of decelerating cascade, the flow in the annulus of tip clearance must withstand the axial pressure gradient without blade force. Usually the dynamic pressure of the axial component of the inlet velocity is less than the pressure difference across the rotor; therefore, there is very little flow passing through the annulus.

It is assumed that the flow is uniform downstream, where the pressure is higher than the inlet pressure by p_c . The balance of forces is

$$p_c (\pi D_o \delta + A) = X \quad (12)$$

where A is the annular area covered by blades, D_o is the impeller diameter, and X is the axial force acting on the blades.

In the case of no tip clearance, an impeller which has an axial thrust X_o on the blades can achieve a pressure rise $p_o = X_o/A$. Because there is practically no flow passing through the annulus of tip clearance, it is assumed that the blades with tip clearance work identically to the case without tip clearance, that is, $X = X_o$. Under such conditions, Eq. (12) becomes

$$(p_o - p_c) A = \pi D_o \delta p_c \quad (13)$$

$$p_o - p_c = \frac{\delta}{h} \frac{2 D_o}{D_o + D_h} p_c \quad (14)$$

where D_h is the hub diameter and h is the blade length. In other words, the pressure loss due to the tip clearance is proportional to the tip clearance/blade height ratio and to the pressure rise across the impeller. It means that the pressure loss coefficient is proportional to the lift coefficient of the blades. This relation is different from the relations in the literature, such as where it is proportional to C_L^2 , based on the induced drag, or where it is proportional to $C_L^{1.5}$, based on the loss of kinetic energy of leakage.

As was mentioned in Sec.3.3, Smith was successful for predicting the performance of multistage axial compressors, where he assumed that the tip clearance increased the displacement thickness of the wall boundary layer and, as a result, the virtual annulus area where the blades did not work was also increased. The concept is essentially identical to the present method to estimate the pressure loss.

The above concept can be handled in a different way which closely relates to the induced drag. A uniform flow is assumed upstream, where the axial velocity component is v_1 and the tangential component is u_1 . Near a decelerating cascade with a tip clearance, there is no axial velocity in the annulus of tip clearance and the axial velocity through the cascade is increased to v_2 , while u_1 remains unchanged before it enters the cascade. Immediately behind the cascade,

the tangential velocity component is changed from u_1 to u_2 , while v_2 remains unchanged. Further downstream, the flow is uniform, filling up the annulus of tip clearance, and the velocity components are u_2 and v_1 .

The lift force L is perpendicular to the vector mean velocity through the cascade, which makes an angle $\beta = \tan^{-1}[(u_1 + u_2)/2v_2]$ in the axial direction. For an ideal cascade without tip clearance which changes the inlet velocity components u_1, v_1 to the exit velocity components u_2, v_1 , the direction of vector mean velocity passing through the cascade is $\beta_o = \tan^{-1}[(u_1 + u_2)/2v_1]$. That is, the direction of the lift force acting on the blades with tip clearance is different from that without tip clearance by an angle $\beta_o - \beta = (\sin \beta \cos \beta_o)(v_2 - v_1)/v_1$.

Both cascades create an identical flow field downstream; nevertheless, the blade forces acting on the cascades are different. As the latter is the ideal cascade, the former is not ideal, and the difference in the direction of forces is the source of pressure loss. That is, $L(\sin \beta \cos \beta_o)(v_2 - v_1)/v_1$ of the former is the drag force due to the tip clearance. As $(v_2 - v_1)/v_1$ is equal to δ/h , the induced drag coefficient C_{Di} due to tip clearance is $C_{Di} = C_L(\delta/h) \sin \beta \cos \beta_o$.

5.5 Tip clearance loss and leakage loss

The words "leakage loss" and "tip clearance loss" are often used with the same meaning. However, in the last section, it was demonstrated that a pressure loss is induced by a combination of a tip clearance and an axial pressure gradient, regardless of the leakage through the tip clearance of the blades. In cases of impulse stages, where no axial pressure difference exists across a rotor, there is no pressure loss based on the concept in Sec. 5.4, but there is obviously a pressure loss due to leakage through the tip clearance of the blades.

Senoo⁽¹⁴⁾ claimed that the tip clearance loss was the sum of the leakage loss in Sec.5.2 and the pressure loss due to axial pressure difference in Sec.5.4, and he derived theoretical equations which were applicable to centrifugal as well as axial impellers, and he demonstrated satisfactory results comparing experimental data in the literature⁽¹²⁾⁽¹³⁾.

As was mentioned in Sec.5.2, the leakage loss is induced by the drag force acting on the main flow due to leakage fluid, which is slower than the main flow regarding the velocity component in the main flow direction. The momentum of the leakage flow perpendicular to the main flow is not included in the cause of the pressure loss. The momentum is equal to the lift force acting on the imaginary blades, which fills up the tip clearance of the blades in the ideal case. In order to dissipate the momentum and to balance the forces, some kind of shear force such as mixing with the main

flow is required, and the energy loss is equal to the one based on the shear force needed to support the annulus of tip clearance against the pressure difference without blades. That is, the pressure loss due to leakage in Sec. 5.2 and the pressure loss in Sec. 5.4 are not the same energy loss explained in two different ways, but are two different kinds of pressure loss to be added up for completeness.

5.6 Tip clearance loss of centrifugal impellers

The tip clearance loss of centrifugal impellers is not different in character from the tip clearance loss of axial impellers, but in cases of centrifugal impellers, there are a few additional factors to be considered. The radius varies along the shroud, and the centrifugal force acting on the fluid in the annulus of tip clearance supports the fluid against the pressure gradient in the meridional plane. For an impeller with z blades, blades are mounted at an angle of $2\pi/z$ with each other; therefore, the directions of leakages flowing through the tip clearances of two adjacent blades are different from each other, and the difference of the momentums of leakages creates an outward force along the blades on a control volume of one blade pitch.

By these additional factors, Senoo⁽¹⁴⁾ derived equations to evaluate the tip clearance loss, which consists of the two kinds of losses in Secs.5.2 and 5.4, and a small additional loss based on the increment of blockage in the blade channel due to the clearance. In cases of centrifugal impellers of 6 : 1 pressure ratio, the predicted tip clearance effects agreed well with experimental data in the range of 50% to 100% of the shaft speed and the entire tested flow range⁽¹²⁾.

6. Conclusions

The tip clearance effects on the performances of turbomachines are quite complicated. The clearance effects vary, depending upon the geometry of cascade, operating conditions and the velocity distribution of the inlet boundary layer. Furthermore, the performance of stator blades behind the impeller is considerably influenced by the distorted velocity distribution induced by the tip clearance of impeller blades. Therefore, conditions differ from one experiment to another, and it is very difficult to secure consistent and accurate experimental data from them. In addition, the physics of pressure loss due to tip clearance was not clarified and experimental data were not handled in a proper way.

Regarding the application of computational fluid mechanics to the problems of tip clearance, leakage flow through a narrow tip clearance rolls up, forming a large vortex between blades; therefore, a grid of small mesh must cover a large area, or the number of

grid points is large and the computational time is too long.

Recently, there has been much interest in problems relating to tip clearance effects, and many experimental and theoretical researches have been reported in the literature. It is hoped that such efforts will contribute to the better design of compressors by minimizing the ill effects of tip clearance.

Some people are concerned about the optimum value of tip clearance. There are qualitative reasons to support the concept, but the reduction in the pressure loss by the leakage through the tip clearance must be closely related to the original flow pattern and the pressure loss at an extremely small tip clearance. Therefore, it is dubious whether there is a unique quality applicable to all cases. In some case, it is said to be about 1 % of the blade pitch, but it is safe to assume that the optimum value is smaller than the practical value for manufacture, until such time as many numerical analyses demonstrate the optimum clearances for practical designs of compressors.

The pressure loss of an impeller near the casing is induced by a complex combination of a distorted flow field due to the wall boundary layer, and further distortion due to the tip clearance. It is not correct to discuss the effect of tip clearance without knowing the variation of blade forces due to the distorted flow field. Nevertheless, in this paper, emphasis was placed on the tip clearance effects and many reports on the behaviour of impeller blades in the casing boundary layer were not included due to the length of this study.

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