The Boundary-layer Transition on Rotating Cones in Axial Flow with Free-stream Turbulence*

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This paper is concerned with the laminar-turbulent transition of three-dimensional boundary layers on cones rotating in external axial flow. The experiment was carried out for three cone angles of 15° , 30° and 60° in a range of free-stream turbulence (0.04% to 3.5%) using a hot-wire anemometer and a flow-visualization technique. The transition region was determined with the critical Reynolds number and the transition Reynolds number in relation to the rotational speed ratio. The spiral vortices were found to fix relative to the rotating surface. The relation of the direction of the spiral vortices to the rotational speed ratio remains unchanged as the cone angle becomes larger, while the number of vortices increases with increased cone angle. These experimental results were compared with the present numerical results based on the linear stability theory. The free-stream turbulence has no effect on the transition Reynolds number, while the effect becomes more sensitive to the critical Reynolds number as the cone angle is increased.

Key Words: Boundary Layer, Turbulence, Rotating Body, Cone, Transition, Instability, Spiral Vortex, Taylor Vortex, Free-stream Turbulence

1. Introduction

The detailed study of the mechanism of the laminar-turbulent transition of the boundary layer on a rotating axisymmetric body is important for understanding the basic characteristics of transition phenomena for more complex three-dimensional boundary layers of turbomachines and other rotating bodies. We previously presented a linear stability theory for the boundary layer on a rotating cone in external axial flow⁽¹⁾ and an experiment for the laminar-turbulent transition on a 30° cone rotating in axial flow⁽²⁾, in which we carried out an experimental verification of the stability theory and some measurements of the flow structure of the transition region. This study was further extended to include the boundary layers over cones rotating in a still fluid⁽³⁾, where the total angle of the test cones was in the range of 30° to 180° (circular

disk).

The purpose of the present paper is to investigate the transition phenomenon of the three-dimensional boundary layer on a cone rotating in an external axial flow for different total included angles of the cone, and to make clear the effect of the free-stream turbulence on the laminar-turbulent transition. The measurements are compared with the theoretical analysis obtained on the basis of the linear stability theory⁽¹⁾.

2. Experimental apparatus and procedure

2.1 Test cones and procedure

We used a low-turbulence wind tunnel⁽⁴⁾⁽⁵⁾ at the Laboratory for Air-flow Measurements in the Institute of High Speed Mechanics, Tōhoku University. The test section was of the open-jet type, 1.42 m long. The shape of the contraction exit was a regular octagon of sublength 810 mm. The experiment was conducted for three test cones having total angles of $2\theta = 15^\circ$, 30° and 60° . The generation-line length L of the test cones was 172.7 mm for $2\theta = 30^\circ$ and 200 mm for $2\theta = 15^\circ$ and 60° . The cones shown in Fig.1 were made of aluminium alloy, and their surfaces were carefully finished and smooth. Each cone was

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mounted horizontally in the test section of the wind tunnel and was driven by a DC motor through a V-belt. The rotating speed could be controlled continuously up to 3 900 rpm. The external axial flow velocity U_∞ was under 15 m/s to create conditions under which the laminar-turbulent transition appeared on the surface of the test cones. The turbulence intensity $\sqrt{u'^2}/U_\infty$ in the test section was 0.04%. We were careful to remove the eccentricity of the rotating cone when the test cone was mounted on the rotation axis. A preliminary experiment showed that an increase in the eccentricity from 10 μm to 50 μm had no influence on the critical Reynolds number or the transition Reynolds number.

A hot-wire anemometer and a frequency analyzer were used to measure the velocity field. Flow patterns in the transition region were visualized by using smoke which was produced by spreading titanium tetrachloride on the surface of other black-painted cones having generation-line lengths of $L=380$ mm for $2\theta=30^\circ$ and $L=400$ mm for $2\theta=60^\circ$. The flow visualizations were made in another 600 mm $\times$ 600 mm open-circuit wind tunnel.

2.2 Turbulence grids

In order to vary the turbulence intensity in the test section, turbulence grids were used. The turbulence grid was set between the main contraction and the second contraction, located 640 mm upstream from the exit of the second contraction. The shape of the tunnel section, where the turbulence grid was set, was a regular octagon of sublength 1.01 m. The distance from the turbulence grid to the cone apex was in the range of 955 mm to 1 020 mm. Six turbulence grids of stainless steel were available for use in the wind tunnel. We used four of the turbulence grids, from No.3 to No.6, as shown in Fig.2 and Table 1. The open-area ratio $(1-d/h)^2$ of the grids was in the range of 0.572 to 0.661, which satisfied the critical condition $(1-d/h)^2 > 0.57$ for avoiding instability of the flow through the grid pores⁽⁶⁾. Grid No.3 is different in structure from the others, as shown in Fig.2. Grid No.

3 has flat top rods, while the others are of the spot-welding type. The turbulence intensity $\sqrt{u'^2}/U_\infty$ measured at a position 203 mm downstream from the exit of the second contraction was varied from 0.04% to 3.5%, as shown in Table 1. We had confirmed that the region, in which the mean flow velocity distribution and the turbulence intensity distribution are uniform in the test cross section, was large enough compared with the cross sectional area of the test cones, and that the velocity fluctuations of the free-stream flow had no excellent frequency components at the test section. According to Dryden et al.⁽⁷⁾, the wake of the turbulence grid reaches homogeneous turbulence more than $16h$ downstream from the turbulence grid. Corrsin⁽⁸⁾ showed that the range of isotropic turbulence is more than $40h$ downstream from the grid. Thus, the turbulence grids used in the present experiment are consistent with these conditions.

3. Experimental results

3.1 Flow velocity at outer edge of boundary layer

Figure 3 shows distributions of the flow velocity U_e at the outer edge of the boundary layer along the surface of the test cones. The measurement of U_e was made directly by the hot-wire anemometer under non-rotating conditions. We had confirmed that the U_e value remains unchanged for conditions of both rotation and no-rotation. The broken lines in Fig.3 are theoretical results indicating potential flow around the cone. The surface velocity along the generation-line is

Table 1 Turbulence grids and turbulence intensities at test section

Turbulence grid	Dimensions		Open-area ratio $(1-d/h)^2$	Turbulence intensity $\sqrt{u'^2}/U_\infty(\%)$
	$d(\text{mm})$	$h(\text{mm})$		
No grid	—	—	—	0.04
No.3	3.5	18.7	0.661	0.50
No.4	6.0	32.0	0.660	2.4
No.5	8.9	45.3	0.646	2.8
No.6	12.0	49.3	0.572	3.5

Fig. 1 Rotating cone

Fig. 2 Structures of turbulence grids

Fig. 3 Flow velocity at outer edge of boundary layer

Fig. 4 Flow visualization of boundary layer on a rotating cone ($2\theta=15^\circ$, clockwise rotation)

given as $U_e = a\tilde{x}^m$, where a is a constant and m is a function of the total cone angle 2θ ; that is, $m=0.0079$ for $2\theta=15^\circ$, 0.0314 for $2\theta=30^\circ$ and 0.1156 for $2\theta=60^\circ$ ⁽⁹⁾. In Fig.3, the constant a was determined by the condition that the theoretical value of U_e/U_∞ becomes equal to the experimental value at the location of $\tilde{x}/L=0.4$. It is clear that the experimental results are in good agreement with the theoretical values, except for the range close to the cone base.

3.2 Flow visualization of transition region

The photographs in Fig.4 show the flow patterns of the boundary layer on the rotating cone of $2\theta=15^\circ$, where a uniform flow of the velocity U_∞ is given from the left and the rotation vector of the cone is in the right direction of the rotation axis, as shown in Fig. 1. The photograph (a) in Fig.4 represents the flow condition in which the cone surface is covered wholly with the laminar boundary layer. In Fig.4(b), one can

(a) $2\theta = 30^\circ$, $U_\infty = 1.7$ m/s, $N = 670$ rpm

(b) $2\theta = 60^\circ$, $U_\infty = 4.3$ m/s, $N = 3650$ rpm

Fig. 5 Flow visualization of boundary layer on rotating cones with different cone angles (clockwise rotation)

see a transition region between the laminar and turbulent boundary layers, in which regular spiral vortices appear. The location of the transition shifts in the direction of the cone apex as the ratio of the rotational speed to the external axial flow velocity is increased. With increasing rotational speed ratio, the direction ε of the vortex axis in the transition region decreases and the number n of the spiral vortices becomes less. Figure 4(c) shows the flow pattern of the boundary layer on a rotating cone in a still fluid where the spiral vortices become toroidal that is, $\varepsilon=0$. Figure 5 shows the flow patterns for $2\theta=30^\circ$ and 60° . All the spiral vortices for $2\theta=15^\circ$, 30° and 60° show similar patterns. They are, however, somewhat different from each other in structure. Figure 6 gives the cross-sectional flows of the spiral vortices, which were made visible by a sheet of light along the meridian of the rotating cones for $2\theta=30^\circ$ ⁽²⁾ and 60° ⁽¹⁰⁾. It can be clearly stated that the spiral vortices for $2\theta=30^\circ$ are made of pairs of counter-rotating vortices (Taylor-vortex type), while the spiral vortices for the 60° cone are simply made of co-rotating vortices (co-rotating-vortex type). The latter is the same as the one observed for a rotating disk⁽¹¹⁾⁽¹²⁾ and a sphere⁽¹³⁾ in a still fluid. The flow visualizations in the previous paper⁽¹¹⁾ showed that the spiral-vortices

on the rotating cones in a still fluid are of the Taylor-vortex type for $2\theta < 30^\circ$, the co-rotating-vortex type for $90^\circ < 2\theta < 180^\circ$, and in a mixed state of the two types for $2\theta = 60^\circ$. It is, however, clear that the spiral vortices on the rotating 60° -cone in the external axial flow are of the co-rotating-vortex type. It can be, therefore, seen that the structure of the spiral vortices on the rotating cone changes from the Taylor-vortex type into the co-rotating-vortex type, depending on the magnitude of the rotational speed ratio for the same total angle.

3.3 Critical and transition Reynolds numbers

The critical and transition Reynolds numbers are

(a) $2\theta = 30^\circ$, $U_\infty = 1.7$ m/s,
 $N = 670$ rpm

(b) $2\theta = 60^\circ$, $U_\infty = 1.7$ m/s,
 $N = 500$ rpm

Fig. 6 Cross-sectional structure of spiral vortices

defined as $Re_{lc} = \bar{x}_c U_e / \nu$ and $Re_{lt} = \bar{x}_t U_e / \nu$, respectively, where $\bar{x}_c$ denotes the upstream end (critical point) of the transition region, $\bar{x}_t$ is its downstream end (transition point) and U_e is the velocity at the outer edge of the boundary layer on the local position $\bar{x}_c$ or $\bar{x}_t$. The critical point $\bar{x}_c$ was measured as the point where periodical signals indicating the spiral vortices were first observed by frequency analysis. The hot-wire probe was set about 0.5 mm distant from the cone surface for finding the critical point $\bar{x}_c$. The transition point $\bar{x}_t$ was defined as the state where the velocity fluctuations gave a frequency spectrum of the turbulent boundary layer.

Figure 7 shows the critical and transition points ($\bar{x}_c, \bar{x}_t$) measured for the cone of $2\theta = 60^\circ$. The figure indicates that both points of the transition region moved downstream as the external flow velocity U_∞ was increased, while they moved upstream with increasing rotational speed N . Figure 8 expresses such

Fig. 7 Critical and transition points ($\bar{x}_c, \bar{x}_t$) of transition region ($2\theta = 60^\circ$, no grid)

Fig. 8 Relation of critical Reynolds number Re_{lc} and transition Reynolds number Re_{lt} to rotational speed ratio S (no grid)

results as the relation between the Reynolds numbers (Re_{lc}, Re_{lt}) and the rotational speed ratio S for three cones of $2\theta=15^\circ, 30^\circ$ and 60° . The rotational speed ratio is defined as

$$S = \omega R / U_e \quad (1)$$

where ωR is the local circumferential velocity of the cone surface, ω denotes the angular speed of the cone and R the local radius of the cone. It is seen from Fig. 8 that the critical and transition points can be expressed in the relation of Re_{lc} and Re_{lt} to S for a fixed cone angle. The decrease of the Reynolds numbers Re_{lc} and Re_{lt} with increasing S comes from the increase of the instability caused by the centrifugal force. When the total angle of the cone becomes larger from $2\theta=15^\circ$ to 30° or 60° , Re_{lc} and Re_{lt} move to larger values of S .

Figure 8 includes some experimental results for another large 30° -cone of $L=380$ mm. It shows that the relations of Re_{lc} and Re_{lt} to S are independent of L .

Salzberg and Kezios⁽¹⁴⁾ measured the transition point $\bar{x}_t$ by experiments on local mass transfer from a rotating 30° naphthalene cone, and gave the results as the relation of the transition Reynolds number Re_{lt} to the rotational parameter $\omega R_b / U_\infty$, where R_b was the base diameter of the cone. The transition point $\bar{x}_t$ was determined from a deformation of the naphthalene surface due to mass transfer. If we rewrite our experimental results in their relation ($Re_{lt}, \omega R_b / U_\infty$), the results give different curves for different lengths L . Okamoto et al.⁽¹⁵⁾ obtained the transition condition with another parameter $\omega R / U_\infty$, which was about 0.90 for the 15° cone and 1.45 for the 30° cone, irrespective of the rotational speed N . In terms of our parameters, the transition condition would then be $S=0.93$ for the 15° cone and $S=1.58$ to 1.69 for the 30° cone and is independent of the Reynolds number Re_{lt} . This is qualitatively different from our present result. As

Fig. 9 Direction ε of spiral vortices in relation to rotational speed ratio S

shown in Fig. 8, the values of S at the critical and transition points vary widely, depending upon Re_{lt} . The linear stability theory⁽¹⁾ supports the present expressions. We, therefore, consider that the relation of the critical and transition Reynolds numbers (Re_{lc}, Re_{lt}) to the rotational speed ratio S is a better non-dimensional expression for the transition region appearing on a rotating cone in axial flow. Although theoretical results obtained by the linear stability theory were qualitatively in agreement with the present experimental results, they were considerably smaller values than those obtained in the experiment. As seen later, the theory and the experiment agree well on the number n and the direction ε of the spiral vortices.

3.4 Measurements of spiral vortices

3.4.1 Direction of spiral vortices

In order to determine the direction ε of the spiral vortices appearing in the transition region, we used a parallel probe made of two hot-wires separated by 3.0 mm, which could be rotated around an axis normal to the cone surface over a range of 180° . We decided the angle ε of the spiral vortices by measuring the direction in which periodical signals obtained from the two parallel hot-wires were in the same phase. It has been already confirmed in Ref. (2) that the values of ε obtained by this method agree well with those measured directly by the flow visualization technique. The results are shown in Fig. 9 with relation to S , where the value of S was obtained by using Eq. (1) for any position $\bar{x}$. Figure 9 shows that the direction ε decreases as the rotational speed ratio S is increased. The solid line in Fig. 9 shows theoretical results calculated by using the linear stability theory⁽¹⁾ for three cone angles. It shows that the relation between the direction ε and the rotational speed ratio S is independent of the total angle 2θ of the cone. The experimental values agree well with the theory. The measured values of ε for the 60° cone are somewhat less than those in the other cases. The reason for this is not yet clear. As already reported in detail in Ref. (16), the angle ε for $2\theta=15^\circ$ and 30° changes discontinuously from the finite values to zero at $S=6$ to 8, and the spiral vortices transfer to toroidal vortices. A limiting case of $S \rightarrow \infty$ for each cone angle is the flow over a cone rotating in a still fluid. Kobayashi and Izumi (1983)⁽³⁾ found that in such a case, the angle ε is almost zero for $2\theta < 30^\circ$, while it increases from zero to about 14° for $2\theta=30^\circ$ to 180° .

3.4.2 Phase velocity of spiral vortices

Two methods were used to observe the traveling velocity of the spiral vortices relative to the rotating cone surfaces. In the first method, we used a single hot-wire probe, which indicated velocity fluctuations of

Fig. 10 Examination of phase velocity of spiral vortices

Fig. 11 Number n of spiral vortices in relation to rotational speed ratio S

the spiral vortices having no simple sinusoidal profiles, but rather a unique profile for each vortex. Figure 10 shows typical traces of the velocity fluctuations starting at a definite circumferential location and recorded 7 times repeatedly. The fact that Fig. 10 shows no phase difference per rotation means that the spiral vortices fix on the surface of the rotating cone. Similar results were confirmed for the cones of $2\theta=15^\circ, 30^\circ$ and 60° over the experimental region S . In the second method, we visualized the spiral vortices. Observation by a stroboscope showed that the spiral vortices fix relative to the rotating cone surface. We can, therefore, conclude that the spiral vortices have no phase velocity and that they fix relative to the rotating cone surface. Similar results were found in the transition region on a rotating disk in a still fluid by Kohama (1984)⁽¹²⁾.

3.4.3 Number of spiral vortices

As seen in the photographs in Figs. 4 and 5, the number n of the spiral vortices appearing in a circumference varies depending upon the cone angle 2θ and the rotational speed ratio. Figure 11 gives the number n of the vortices against the rotation speed ratio S , which was measured from the frequency analysis of the velocity fluctuations. The number n of the spiral vortices decreases as the cone angle is decreased and as S is increased. The three lines drawn in Fig. 11 are the theoretical values calculated by the linear stability

(a) $2\theta = 15^\circ$ (b) $2\theta = 30^\circ$ Fig. 12 Effects of free-stream turbulence intensity on Re_{1c} and Re_{1t}

theory⁽¹⁾. They are in good agreement with the experiments for $2\theta=15^\circ$ and 30° , while, for $2\theta=60^\circ$, the experimental results are larger than the theoretical results. It could be said that the spiral vortices for $2\theta=60^\circ$ are of a different pattern from the other cones.

3.5 Effects of turbulence intensity

The low-turbulence wind tunnel used in the present experiment has turbulence intensities of 0.02% in the test section of the closed type and 0.04% for the open-jet type. These low levels of free-stream turbulence are very useful for detailed investigations on the laminar-turbulent transition process of boundary

layers on test bodies. It is, meanwhile, important from a practical point of view to determine how the magnitude of turbulence intensity in the external flow influences the present laminar-turbulent transition phenomenon.

Figures 12(a) to (c) show the critical Reynolds number Re_{lc} and the transition Reynolds number Re_{lt} over a wide range of turbulence intensities, from 0.04% to 3.5%. It can be first seen that there was no effect of turbulence intensity on the transition Reynolds number Re_{lt} for the three cones. On the other hand, the critical Reynolds number Re_{lc} was found to vary in high levels of turbulence intensity. For the case of the 15° cone in Fig.12(a), there was no effect of the turbulence intensities 0.5% (grid No. 3) and 2.4% (grid No.4) on Re_{lc} , where the spiral vortices appeared uniformly over the circumference of the cone at the critical position $\tilde{x}_c$; however at the higher turbulence intensities 2.8% (grid No.5) and 3.5% (grid No.6), few spiral vortices initially appeared on the circumference at the critical position $\tilde{x}_c$, but as the number of the vortices increased downstream, they finally appeared uniformly over the circumference of the cone. Since the critical position $\tilde{x}_c$ is located upstream compared with the case of the low turbulence level, the critical Reynolds number Re_{lc} becomes less.

Figure 12(b) gives the results for $2\theta=30^\circ$. The turbulence levels of 0.5% (grid No.3) and 2.4% (grid No.4) had no effect on the critical Reynolds number Re_{lc} , similar to the case of $2\theta=15^\circ$. A close observation finds that Re_{lc} becomes slightly larger as the turbulence level is increased from 0.5% to 2.4%. This comes from a slight change of the condition detecting the onset of the spiral vortices in the laminar boundary layer under the high level of the free-stream turbulence. The Re_{lc} value decreased as the turbulence intensity increased to 2.8% (grid No.5) and 3.5% (grid No.6). The case of $2\theta=60^\circ$ shown in Fig.12(c) is more sensitive to the turbulence intensity.

4. Conclusions

The results are summarized as follows,

(1) The location of the transition region on the rotating cone can be given in the relation of the critical Reynolds number Re_{lc} and the transition Reynolds number Re_{lt} to the rotational speed ratio S . The relation varies for differences in cone angle 2θ , as shown in Fig.8.

(2) The spiral vortices appearing in the transition region are fixed relative to the rotating surface.

(3) The direction ε of the spiral vortices and their number n generating on the rotating cone surface are given in Figs.9 and 11. The direction ε can be expressed in relation to the rotational speed ratio S , irrespective of differences of cone angle 2θ , while the number n tends to increase as the cone angle 2θ becomes larger.

(4) The free-stream turbulence has no effect on the transition Reynolds number. On the other hand, the critical Reynolds number decreases as the turbulence intensity is increased, and it becomes more sensitive to the turbulence level as the cone angle is increased.

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