Transient Vibration of a Rotating Shaft with Nonlinear Spring Characteristics during Acceleration through a Major Critical Speed*

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This paper deals with the transient vibration of a rotating shaft with nonlinear spring characteristics during constant acceleration passing through a major critical speed. In a theoretical analysis, we solve the equations of motion numerically and discuss this phenomenon, paying attention to the nonlinear components represented by polar coordinates. As a result, it is clarified that when the angular acceleration λ increases, the maximum amplitude of the shaft decreases gradually for small values of λ and to a large extent at a certain value of $\lambda = \lambda_0$. This characteristic is different from that of a linear system, where the maximum amplitude decreases markedly in the range of small angular accelerations. We verified these phenomena in experiments. Generally speaking, it is more difficult to pass through a critical speed in a nonlinear system than in a linear system.

Key Words: Vibration of Rotating Body, Acceleration, Transient Vibration, Major Critical Speed, Nonlinear Spring Characteristics

1. Introduction

Since Lewis⁽¹⁾ investigated the vibration of a rotating shaft passing through the major critical speed, many papers discussing this phenomenon have been published. One of the authors of this paper has also reported on this subject⁽²⁾. Most of these reports deal with linear systems, and are divided into two groups. One group deals with a phenomenon during constant acceleration, and the other treats the phenomenon in a system in which there exists an interaction between the power source and the rotor. For example, reports by Yanabe et al.^{(3)~(6)} were of the former, and those by Matsuura^{(7)~(10)} were of the latter.

In contrast to such researches, few reports have dealt with the phenomena in a rotating shaft system

with nonlinear spring characteristics. As a similar problem, some researchers have investigated the transient phenomena in rectilinear systems with nonlinear spring characteristics. Meuser - Weibel et al.⁽¹¹⁾ and Mitropol'skii⁽¹²⁾ studied a spring-mass system with one degree of freedom. Backova⁽¹³⁾ discussed a beam system.

The authors of this paper have previously investigated a rotating shaft system with nonlinear spring characteristics and have clarified some of the properties of various steady-state nonlinear forced oscillations. In these reports⁽¹⁴⁾⁽¹⁵⁾, it was pointed out that the features of oscillations, such as the probability of their occurrence, can be clarified easily and completely if we express the nonlinear spring characteristics in polar coordinates.

In this paper, a transient phenomenon occurring during constant acceleration through a major critical speed is discussed for a rotating shaft system with nonlinear spring characteristics. In the experimental apparatus, the nonlinearity is caused by ball bearings. A theoretical analysis is performed in a two-degree-of-freedom rotating shaft system, paying attention to the nonlinear components represented by polar coor-

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ordinates. The results of this theoretical analysis are compared with the experimental results. In addition, the difference between the phenomena occurring in a nonlinear system and a linear system is clarified.

2. Equations of motion

The experimental apparatus to be described later is a four-degree-of-freedom system whose deflection and inclination of the rotor couple each other. However, in the theoretical analysis, for simplicity, we deal with only a deflection oscillation in the case where the rotor is mounted in the middle of the shaft and the deflection and the inclination do not couple each other. As shown in Fig.1, we consider a stationary rectangular coordinate system $O-xy$ in a plane which is perpendicular to the bearing center line, that is, the line which connects the centers of the upper and lower bearings. The origin O is located on this bearing center line. We designate the mass of the rotor by m , the polar moment of inertia by I_p , the spring constant of the shaft by k , the damping coefficient by c , and the driving torque by T . The eccentricity $e=\overline{CG}$ is the distance between the center of gravity $G(x_G, y_G)$ and the geometrical center $C(x, y)$ of the rotor. The angle of rotation of the rotor is expressed by the angle ψ between $\overline{CG}$ and the x -axis. We also consider a rotating rectangular coordinate system $O-uv$ which rotates with the rotor and whose u -axis is located in the direction of CG . The nonlinear terms are expressed as the sum of the terms up to the third power of the coordinates. The coefficients of the unsymmetrical nonlinear terms (the second power terms) are denoted by $\varepsilon_{ij}(i+j=3)$, and those of the symmetrical nonlinear terms (the third power terms) are denoted by $\beta_{ij}(i+j=4)$. By adopting the representative length $e_{st} = mg/k$ (g : the acceleration of gravity), we define the following dimensionless quantities:

$$\left. \begin{aligned} x' &= x/e_{st}, y' = y/e_{st}, e' = e/e_{st} \\ t' &= t\sqrt{k/m}, c' = c/\sqrt{mk}, \varepsilon'_{ij} = \varepsilon_{ij}e_{st}/k \\ \beta'_{ij} &= \beta_{ij}e_{st}^2/k, T' = mT/(kI_p), K = me_{st}/I_p \end{aligned} \right\} \quad (1)$$

Using these dimensionless quantities, we obtain the equations of motion as follows:

$$\left. \begin{aligned} \ddot{x} + c\dot{x} + x + N_x &= e\dot{\psi}^2 \cos \psi + e\ddot{\psi} \sin \psi \\ \ddot{y} + c\dot{y} + y + N_y &= e\dot{\psi}^2 \sin \psi - e\ddot{\psi} \cos \psi \\ \ddot{\psi} &= T - K e(x \sin \psi - y \cos \psi) \end{aligned} \right\} \quad (2)$$

where the primes of the dimensionless quantities are omitted. The symbols N_x and N_y represent nonlinear terms and their complete forms are derived from the potential $V = V_0 + V_N$ (V_0 and V_N are the components corresponding to the linear and the nonlinear terms in the restoring forces).

Fig. 1 Coordinate systems

$$V_N(x, y) = \left\{ \sum_{\substack{i,j=0 \\ (i+j=3)}}^3 \varepsilon_{ij} x^i y^j + \sum_{\substack{i,j=0 \\ (i+j=4)}}^4 \beta_{ij} x^i y^j \right\} \quad (3)$$

$$N_x = \partial V_N / \partial x, N_y = \partial V_N / \partial y$$

In Eqs. (2) and (3), the quantities $c, e, \varepsilon_{ij}, \beta_{ij}$ are supposed to be the same order as the small ε . We represent this order by the symbol $O(\varepsilon)$.

We express the nonlinear terms by polar coordinates in the same way as in the previous papers⁽¹⁴⁾⁽¹⁵⁾. By substituting the transformation

$$x = r \cos \varphi, y = r \sin \varphi \quad (4)$$

into Eq. (3), we obtain the following expression.

$$\begin{aligned} V_N(r, \varphi) &= (\varepsilon_c^{(1)} \cos \varphi + \varepsilon_s^{(1)} \sin \varphi + \varepsilon_c^{(3)} \cos 3\varphi \\ &\quad + \varepsilon_s^{(3)} \sin 3\varphi) r^3 + (\beta_c^{(0)} + \beta_c^{(2)} \cos 2\varphi + \beta_s^{(2)} \sin 2\varphi \\ &\quad + \beta_c^{(4)} \cos 4\varphi + \beta_s^{(4)} \sin 4\varphi) r^4 \end{aligned} \quad (5)$$

The superscripted numbers n ($n=1 \sim 4$) of the coefficient represent the number of times of periodic variation in the magnitude of the term while the angle φ changes by 2π . We denote the nonlinear component, which is expressed by such terms, by the symbol $N(n)$. The relationships between the coefficients $\varepsilon_{ij}, \beta_{ij}$ and $\varepsilon_c^{(1)}, \beta_c^{(0)}$, etc. were shown in the previous paper⁽¹⁵⁾.

3. Steady-state solutions and resonance curves

In this chapter, we investigate the amplitude when the rotating speed $\dot{\psi}(=\omega)$ is kept constant. Denoting a natural frequency by p , we obtain the frequency equation as follows:

$$1 - p^2 = 0 \quad (6)$$

This equation has roots $p = \pm 1$. The root $p=1$ represents a forward and $p=-1$ a backward precessional motion. When the shaft rotating speed ω nearly equals the natural frequency $p=1$, a resonance phenomenon occurs. The steady-state solution has the following form with an accuracy of $O(\varepsilon^0)$.

$$\left. \begin{aligned} x &= R_0 \cos(\omega t + \beta_0) \\ y &= R_0 \sin(\omega t + \beta_0) \end{aligned} \right\} \quad (7)$$

The resonance curves are given by the following equations with an accuracy of $O(\varepsilon)$.

$$\left. \begin{aligned} [(-\sigma + 4\beta^{(0)}R_0^2) + c^2\omega^2]R_0^2 &= e^2\omega^2 \\ \tan \beta_0 &= \frac{-c\omega}{-\sigma + 4\beta^{(0)}R_0^2} \end{aligned} \right\} \quad (8)$$

where $\sigma = \omega^2 - 1$. The stability criteria can be obtained in the same manner as in the previous report⁽¹⁶⁾ and are given as follows.

$$(\sigma + 12\beta^{(0)}R_0^2)(\sigma + 4\beta^{(0)}R_0^2) + c^2\omega^2 > 0 \quad (9)$$

As only $\beta^{(0)}$ among the various coefficients in Eq. (5) is contained in Eqs. (8) and (9), we find that only the component $N(0)$ has an influence on this steady-state oscillation with an accuracy of $O(\varepsilon)$. Figure shows an influence of component $N(0)$ (the coefficient $\beta^{(0)}$) on the resonance curve. We see from this figure that the coefficient $\beta^{(0)}$ determines the inclination of the resonance curve. It is also seen that the maximum amplitude of the resonance curve increases as $\beta^{(0)}$ becomes large. Therefore, even if the magnitude of the unbalance e is constant, it becomes dangerous to operate a rotating machine when component $N(0)$ starts to increase. In addition, it is imagined that passing through the critical speed in a nonlinear system is more difficult than in a linear system.

The symbol $\circ$ in the figure represents an amplitude obtained from the numerical simulation performed to check the accuracy of the approximate solution (8). In these simulations, the coefficients of nonlinear terms, except $\beta^{(0)}$, are set at zero. The results agree well with the approximate solutions.

In the approximate solution (8), obtained with an accuracy of $O(\varepsilon)$, only $\beta^{(0)}$ is contained. However, the other components $N(1) \sim N(4)$ influence the phenomena with an accuracy of $O(\varepsilon^2)$. These components are derived from the directional difference of the potential distribution⁽¹⁵⁾. Therefore, the orbit of the whirling motion in the neighborhood of the major critical speed deviates slightly from a circle. As an example, in Fig. 3, we show an orbit and the fluctuation in the amplitude (the maximum and the minimum shaft deflection) in the case where the coefficients $\beta_c^{(2)}$

and $\beta_s^{(2)}$, belonging to component $N(2)$, coexist with $N(0)$. In this case, the orbit is an ellipse and the deflection r changes twice periodically while the shaft whirls once.

4. The case where the rotor passes through the critical speed with a constant acceleration

4.1 Transformation of the equations of motion

When a rotor passes through a resonance point with a constant acceleration λ and with an initial conditions that $\dot{\phi} = \omega_0, \phi = \phi_0$ at $t = 0$, the angle ϕ is given as

$$\phi = \frac{1}{2}\lambda t^2 + \omega_0 t + \phi_0 \quad (10)$$

For this case, the first and second equations in the equations of motion (2) can be solved independently of the third equation. For convenience's sake, we transform the first and second equations in Eq. (2) into polar coordinates by Eq. (4). Then, we obtain the following equation.

$$\left. \begin{aligned} \dot{r} &= -(1 - \dot{\phi}^2)r - c\dot{r} - (N_x \cos \varphi + N_y \sin \varphi) \\ &\quad + e\dot{\phi}^2 \cos \theta + e\ddot{\phi} \sin \theta \\ \ddot{\phi} &= -c\dot{\phi} - \{2\dot{r}\dot{\phi} + (-N_x \sin \varphi + N_y \cos \varphi) \\ &\quad - e\dot{\phi}^2 \sin \theta + e\ddot{\phi} \cos \theta\}/r \end{aligned} \right\} \quad (11)$$

where $\theta = \phi - \varphi$. The complete form of the nonlinear terms are given as follows:

$$\left. \begin{aligned} N_x C + N_y S &= 3r^2\{(\varepsilon_c^{(1)} + \varepsilon_c^{(3)})C^3 \\ &\quad + (\varepsilon_s^{(1)} + 3\varepsilon_c^{(3)})C^2S + (\varepsilon_c^{(1)} - 3\varepsilon_c^{(3)})CS^2 \\ &\quad + (\varepsilon_s^{(1)} - \varepsilon_s^{(3)})S^3\} + 4r^3\{(\beta^{(0)} + \beta_c^{(2)} + \beta_c^{(4)})C^4 \\ &\quad + 2(\beta_s^{(2)} + 2\beta_s^{(4)})C^3S + 2(\beta^{(0)} - 3\beta_c^{(4)})C^2S^2 \\ &\quad + 2(\beta_s^{(2)} - 2\beta_s^{(4)})CS^3 + (\beta^{(0)} - \beta_c^{(2)} + \beta_c^{(4)})S^4\} \\ &\quad - N_x S + N_y C = r^2\{(\varepsilon_s^{(1)} + 3\varepsilon_s^{(3)})C^3 \\ &\quad - (\varepsilon_c^{(1)} + 9\varepsilon_c^{(3)})C^2S + (\varepsilon_s^{(1)} - 9\varepsilon_s^{(3)})CS^2 \\ &\quad - (\varepsilon_c^{(1)} - 3\varepsilon_c^{(3)})S^3\} + 2r^3\{(\beta_s^{(2)} + 2\beta_s^{(4)})C^4 \\ &\quad - 2(\beta_c^{(2)} + 4\beta_c^{(4)})C^3S - 12\beta_s^{(4)}C^2S^2 \\ &\quad - 2(\beta_c^{(2)} - 4\beta_c^{(4)})CS^3 - (\beta_s^{(2)} - 2\beta_s^{(4)})S^4\} \end{aligned} \right\} \quad (12)$$

Here, the abbreviated forms $C = \cos \varphi$ and $S = \sin \varphi$ are used.

4.2 Results of numerical calculations

We investigate Eq. (11) numerically by the Runge-

Fig. 2 Influence of component $N(0)$ on the resonance curve

Fig. 3 Influence of component $N(2)$ on the resonance curve

about 1.75 in Fig.4(d)), the sign of $-\dot{\theta}$ does not change, although its magnitude varies. This situation corresponds to the range from ① to ② in Fig.4(b). In this case, the degree of variation of the angular velocity during one whirling motion depends on the degree of deviation of the center of the spiral from point O . In Fig.4(d), this variation is not so large when $\dot{\psi}=1.54$ but it becomes very large when $\dot{\psi}=1.74$. When $\dot{\psi}=1.77$, for example, the origin O is located at the right side and outside of the spiral. Therefore, in the vicinity of this rotating speed, the direction of rotation of the position vector $\vec{OC}$ is different on the right and left sides of the trajectories. Namely, the sign of the quantity $-\dot{\theta}$ changes during one cycle of whirling motion. The rapid change from ② to ③ in Fig.4(b) corresponds to the change which is observed when the origin O shifts from the inside to the outside of the trajectory in the uv -plane at about $\dot{\psi}=1.75$ in Fig.4(d).

4.2.2 Influences of angular acceleration λ

In Fig.5, we show an influence of the angular acceleration λ on the transient oscillation. The magnitude of unbalance in Fig.5(a) is different from that of Fig.5(b). We discuss Fig.5(b) as an example where the peak of the resonance curve expands too much to be observed experimentally. When λ is small, in Fig.5(a), the amplitude slowly, traces the resonance curve and then jumps to the resonance curve of smaller

(a) The case when unbalance e is small

(b) The case when unbalance e is large

Fig. 5 Influence of angular acceleration λ on the change in amplitude r

amplitude at about $\dot{\psi}=1.58$, where the resonance curve of larger amplitude disappears. Although the feature that the maximum amplitude $r_{\max}$ decreases as λ increases is similar to that of a linear system, the rate of decrement is considerably smaller than in the case of a linear system (see Figs.6 and 7). The difference between the linear system and the nonlinear system appears markedly when the maximum amplitude of the resonance curve is large. In Fig.5(b), the amplitude r increases along the resonance curve without jumping to that of smaller amplitude when the angular acceleration is smaller than 0.0028. As λ increases, the maximum amplitude $r_{\max}$ decreases abruptly at about $\lambda=0.0028 \sim 0.0029$. The existence of such a critical acceleration $\lambda=\lambda_0$ is one of the features of the nonlinear system. Therefore, in Fig.5(b), an acceleration larger than this critical value λ_0 is necessary to pass safely through the critical speed with a small amplitude. In the nonlinear system, whether or not it is possible to pass through the critical speed depends on the magnitude of this critical value λ_0 .

In Fig.6, the maximum amplitude $r_{\max}$ is shown as a function of the angular acceleration λ for various magnitudes of unbalance. When $\lambda=0$, $r_{\max}$ corresponds to the maximum amplitude $R_{\max}$ of the resonance curve of steady-state oscillation. The curve $r_{\max}$ shows a characteristic change. In the case where $R_{\max}$ is small, as in the case of $e=0.020$, $r_{\max}$ first decreases linearly, then decreases abruptly at a certain value, and becomes almost constant as the angular acceleration λ increases. In the case where $R_{\max}$ is large, as in the case of $e=0.0028$, $r_{\max}$ decreases abruptly at $\lambda=\lambda_0$ and then the rate of decrement becomes small as λ increases. The change in the latter case is clearly shown in Fig.5(b).

For comparison, we show a similar figure for a linear system in Fig.7. In such a case, $r_{\max}$ decreases rapidly at small values of λ and then decreases gradually. There exists no critical value corresponding to λ_0 .

Fig. 6 Influence of the magnitude of the unbalance e on the $r_{\max}-\lambda$ curve (nonlinear system)

Fig. 7 Influence of the magnitude of the unbalance e on the $r_{\max}$ - λ curve (linear system)

Fig. 8 Influence of the coefficient of nonlinear term $\beta^{(0)}$ on the $r_{\max}$ - λ curve

Fig. 9 Influence of nonlinear component $N(2)$ on the wave of the transient oscillation

From the comparison of Figs.6 and 7, it is concluded that it is easier to pass through the critical speed in a linear system than in a nonlinear system.

4.2.3 Influences of the nonlinear component $N(0)$

In Fig.8, we show λ_0 for various values of $\beta^{(0)}$. The value λ_0 increases with $\beta^{(0)}$ and it becomes more difficult to pass through the critical speed as $N(0)$ increases.

From Figs.2 and 8, the following is concluded. As the component $N(0)$ increases, (a) the resonance curve inclines more intensely, (b) the maximum amplitude $R_{\max}$ of the resonance curve increases, (c) the minimum angular acceleration λ_0 necessary to separate from the resonance curve with larger amplitude becomes larger, and then (d) it becomes more difficult to pass through the critical speed.

4.2.4 Influences of the other nonlinear components $N(n)$

Similar to the case of a steady-state oscillation, the shaft deflection r changes due to component $N(n)$ ($n=1\sim 4$) and the transient waves representing the amplitude change shown in Figs.4 and 5 have vibratory components of higher order. For example, in the case where $N(0)$ and $N(2)$ coexist, as in Fig.3, the change in amplitude becomes that shown in Fig.9. These components of higher order are caused by the

Fig. 10 Experimental apparatus

distortion of the spiral trajectory shown in Fig.4 to the elliptic shape due to $N(2)$.

5. Experimental apparatus and experimental method

An experimental apparatus is shown in Fig.10. A disc R was mounted at the position $a:b=1:4$ on an elastic shaft S with a circular cross section. The upper end of the shaft was simply supported by a self-aligning double-row ball bearing (#1200). The lower end was simply supported by a self-aligning double-row ball bearing (#1204) or fixed-supported by a single-row angular contact ball bearing (#6204). However, in the case of the latter, the restoring force of the shaft had nonlinear spring characteristics due to the angular clearance of the bearing. The dimensions of the rotor were as follows: the diameter is 481.3 mm, the thickness is 5.55 mm, the polar moment of inertia $I_p=0.228 \text{ kg}\cdot\text{m}^2$, and the diametral moment of inertia $I=0.114 \text{ kg}\cdot\text{m}^2$. The dimensions of the shaft were as follows: the length $l=700 \text{ mm}$ and the diame-

ter $d=12$ mm.

A small disc was mounted at the middle of the shaft and its deflections, x and y , were measured in two directions perpendicular to each other. The mass of this disc was negligible. By processing the detected signals, x and y , in an electronic circuit, we obtained the value $r=\sqrt{x^2+y^2}$. This signal r was sent to a personal computer. The rotating speed was detected by a rotary encoder attached to the shaft. The pulse from the rotary encoder, whose frequency was proportional to the rotating speed, was transformed (D/A-transformation) into a voltage signal and then it

was sent to the personal computer. In the personal computer, the voltages, which were proportional to the amplitude r , and the rotating speed were sampled. On the CRT of the personal computer, the amplitude change was shown in coordinates whose ordinate and abscissa represented the amplitude r and rotating speed ψ , respectively. The result was recorded by an X-Y plotter. In the following figures, the ordinate represents the shaft deflection (mm) which was obtained after calibration.

6. Experimental results

6.1 Angular acceleration

In Fig.11, changes in angular velocity $\dot{\psi}$ for various angular accelerations are shown. It is ascertained that the rotor was accelerated with a constant angular acceleration.

6.2 Experimental results in a nonlinear system

We performed experiments in a nonlinear system with a single-row deep-groove ball bearing. The amplitude change is shown in Fig.12. The sampling frequency f_s shown in the figure was decided from the angular acceleration and the memory size of the personal computer. In every experiment, the acceleration was started from a rotating speed below 800 rpm and the deceleration from that over 2000 rpm. The sampling of the data was begun at a different rotating speed. The edge of the resonance curve represents the starting point of data sampling. A detail of the resonance curve is shown in Fig.12(b). From this enlarged figure, we see that these curves have a component whose amplitude changes twice during one cycle of whirling motion. The following possible reasons are given for the occurrence of this component: the existence of the above-mentioned component $N(2)$, the directional difference of fit between a bearing pedestal and a bearing, the difference between output voltages from sensors in the x - and y -directions due to small

Fig. 11 Changes in angular velocity $\dot{\psi}$

(a)

(b)

(c)

Fig. 12 Changes in amplitudes during the passing through the major critical speed (nonlinear systems)

Fig. 13 Changes in the maximum amplitude $r_{\max}$ for angular acceleration λ

Fig. 14 Changes in amplitudes during the passing through the major critical speed (a case that the unbalance e is comparatively large, nonlinear system)

differences in installation conditions, and so on. In assembling this experimental apparatus, great care was taken to ensure that the upper and lower bearing centers coincided. Therefore, the isotropic nonlinear component $N(0)$ should be large and the other components $N(1) \sim N(4)$ small⁽¹⁴⁾.

Figure 12(a) shows the amplitude change in the case of acceleration $\lambda = \pm 3.91$ rpm/s. These are small accelerations and the amplitude curve almost coincides with the resonance curve of the steady-state oscillation except when jumpings occurred.

As shown in Fig. 12, a hysteresis phenomenon due to the inclination of the resonance curve is observed. Although the angular accelerations are very different among each case in Fig. 12, the amplitude changes along the resonance curve and the maximum amplitudes during the acceleration are almost the same.

The variation of the maximum amplitude $r_{\max}$ is shown as a function of angular acceleration $\lambda (>0)$ in Fig. 13. For comparison, the results in a linear system with self-aligning double-row ball bearings are shown. There exists a remarkable difference between these two systems. Namely, $r_{\max}$ decreases rapidly in the range of small angular acceleration in the linear system, but it does not change in the experimental range in the nonlinear system. In this experimental apparatus, because the magnitude of the nonlinear component $N(0)$ could not be measured quantitatively, the angular acceleration λ_0 shown in Fig. 6 is undefined. However, it is thought that the angular acceleration range of Fig. 13 lies below such a critical value. The angular acceleration $\lambda = 200$ rpm/s corresponds to $\lambda \approx 0.0013$ in a dimensionless form.

Figure 14 shows a result for the case in which the unbalance was large and the maximum amplitude of the resonance curve exceeded the measurable range. In such cases, it is almost impossible to pass through the critical speed within the acceleration shown in Fig. 13. In Fig. 14, the experiment was stopped when the shaft touched the guard ring prepared to limit the amplitude for safety.

Fig. 15 Changes in amplitudes during the passing through the major critical speed (linear system)

6.3 Results in a linear system

Experimental results in a linear system are shown in Fig. 15. When the angular acceleration is changed from $\lambda = 3.91$ to 77.4 rpm/s, the maximum amplitude $r_{\max}$ in accelerating the rotating speed decreases considerably. For the same absolute value of acceleration, the maximum amplitude obtained in the process of increasing the rotating speed is larger than that in the process of decreasing. The change of $r_{\max}$ with λ is shown in Fig. 13. The magnitude of $r_{\max}$ decreases rapidly in the small range of angular acceleration. This behavior is qualitatively similar to that of the numerical result shown in Fig. 13. It is also concluded from Fig. 13 that the amplitude decreases more easily in the linear system than in the nonlinear system, and that it is easier to pass through the critical speed in the linear system than in the nonlinear system.

7. Conclusions

(1) In the nonlinear system, due to the change of the resonance curve of a steady-state oscillation into the hard or soft spring type, a hysteresis phenomenon appears in a transient oscillation.

(2) In the nonlinear system, the maximum amplitude $r_{\max}$ shows a characteristic change when the angular acceleration λ increases. In the case of increasing the rotating speed in a system with a resonance curve of a hard spring type, the maximum amplitude shows the following changes. When the maximum amplitude of the steady-state oscillation is comparatively small, the maximum amplitude decreases abruptly at a certain value of λ after previously decreasing linearly in the range of small angular acceleration, and then it gradually begins to decrease further. When the maximum amplitude of the steady-state oscillation is considerably large, the rotor cannot pass through the critical speed with an angular acceleration smaller than a critical value λ_0 . In this case, $r_{\max}$ decreases abruptly at this value λ_0 , and then decreases gradually. This critical value λ_0 depends on the magnitude of the nonlinear component $N(0)$.

(3) As the nonlinear component $N(0)$ increases,

the inclination of the resonance curve of the steady-state oscillation becomes stronger and, at the same time, its maximum amplitude increases. A larger angular acceleration is necessary to depress the maximum amplitude in the transition.

(4) When a nonlinear component $N(n)$ ($n=1\sim 4$) exists, the radius of the trajectory changes n times while the rotor whirls once in the transition of the critical speed.

(5) In the experiments on the nonlinear system, where the shaft was supported by a single-row deep groove ball bearing, the maximum amplitude $r_{\max}$ was almost constant for various values of acceleration. However, in the experiments on the linear system, where the shaft was supported by self-aligning double-row ball bearings, the maximum amplitude decreases rapidly in the range of small angular acceleration and then became almost constant.

(6) In general, it is more difficult to pass through the critical speed with a small amplitude in the nonlinear system than in the linear system.

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