A Study of Grippers with Multiple Degrees of Mobility*

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The number and dimensional syntheses of mechanical grippers with multiple degrees of mobility are presented systematically. Several types of contacts of fingers with a grasped body are considered as kinematic pairs, and number conditions for grasping a body are derived by applying Grübler's mobility equation to the gripper-body kinematic system. A number synthesis of a gripper with one input and three degrees of mobility, as an example, is carried out, and dimensions of parameters of such a gripper are determined by considering action forces and transmissibilities. Moreover, the graspable size of a circular cylinder by such a gripper is obtained, and is compared numerically to that with the use of a conventional parallel-two-finger-type gripper. Also, characteristics such as shapes, sizes, and masses of the bodies are obtained with respect to a circular cylinder, a regular square prism, and a hexagonal prism.

Key Words: Mechanism, Gripper, Number Synthesis, Dimensional Synthesis, Grasping Characteristics

1. Introduction

Industrial robots, which have been introduced to several manufacturing processes in recent years, usually have grippers with few degrees of mobility such as the parallel-two-finger-type, and therefore, large limitations exist in the range of graspable shapes and sizes of bodies without changing endeffectors. On the other hand, constraint conditions of bodies have been analyzed⁽¹⁾, and some trial linkage-type⁽²⁾⁽³⁾ or wire-driven-type grippers⁽⁴⁾⁻⁽⁶⁾ with multiple degrees of mobility based on analyses of human movements have been reported, and moreover, grasping forces of a gripper with flexible fingers⁽⁷⁾ or with three fingers⁽⁸⁾ have been analyzed. However, these reports are rather qualitative.

In this paper, a mobility equation of a gripper-

body kinematic system is derived and number and dimensional syntheses of a gripper composed of planar mechanisms with three degrees of freedom and one input are performed. Then, grasping forces of such a gripper for several shapes of bodies are obtained numerically.

2. Definition and classification of grasping

As a general case, when links of a gripper come in contact with a body, the gripper-body kinematic system constructs a spatial mechanism with some separable pairs. We call a group of links of a spatial mechanism, where displacements, velocities, forces, etc. are given, as input links, and these variables, inputs. We similarly call a pair of links including a body, as output links, and some of these variables, outputs.

When all variables of links in a general spatial mechanism are displacements, the minimum number of inputs which are necessary for constraining output links (number of independent components of displacements) is called the mobility of the mechanism, and when the variables of output links except outputs (independent components of displacements) and those

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of links except the input and output links can vary without influencing the inputs and outputs, the sum of the numbers of such variables is called degree of redundant freedom. In a gripper-body kinematic system, we distinguish the above redundant freedom by calling the former variables "redundant freedom of body", and the latter ones "redundant freedom of gripper". As a special case, the mobility of a system is considered to be zero at a change point, even though the mobility calculated theoretically is 1.

In this paper, we define grasping as a state when the degree of mobility of a gripper-body system is equal to or less than zero. It is called kinematic grasping when the system is not at a change point (Fig. 1 (a)), and kinetic stable (Fig. 1 (b)) or unstable (Fig. 1 (c)) grasping when it is at a change point and grasping can or cannot be maintained under an infinitesimal displacement of the body. Moreover, grasping is split into a spatial grasping (Fig. 2 (a)) and a planar one (Fig. 2 (b)).

3. Conditions for grasping and number syntheses of mechanisms

The state of contact of a link to a rigid body is either a point, a line, or a plane contact. Considering two extreme cases, where the frictional force at contact is zero (without friction) and infinity (with friction), and modeling the contact by a kinematic pair, we obtained a classification as shown in Table 1. Now, applying Grübler's mobility equation to a gripper-body system, we clarify the types of contacts and number conditions for grasping a body.

Fig. 1 Classification of grasps

Fig. 2 Spatial and planar grasps

The mobility equation of a gripper constructed only of pairs with 1 degree of freedom is

$$M_m = 6(N-1) - 5J \quad (1)$$

where

M_m : Degrees of mobility of a gripper

N : Number of links of a gripper

J : Number of pairs of a gripper

Since a body may be considered as a link which has n pairing elements, when it contacts a gripper at n points, the mobility equation of a gripper-body system is

$$M = 6N - 5J - \sum(6-f)I_f \quad (2)$$

where

M : Degrees of mobility of a system

f : Degrees of freedom of a pair at a contact

I_f : Number of pairs with degrees of freedom f

The right third term of Eq. (2), i.e.,

$$n_0 = \sum(6-f)I_f \quad (3)$$

is called the number of equivalent contact points. Let m be the number of inputs; the mobility condition for grasping is

$$M \leq m - 1 \quad (4)$$

The condition for the number of equivalent contact points is obtained from Eqs. (1)~(4) as follows.

$$n_0 \geq M_m - m - f_r + 7 \quad (5)$$

where f_r is the redundant freedom of a body. Moreover, considering some duplications of components of moments acting on a body and improving Eq. (3) such that the upper limit of the degrees of freedom constrained by the components of moments is 3, we have

$$n_0 = 4J_0 + 4J_1 + 3J_3^{(2)} + J_3^{(1)} + J_4 + J_5 + \min\{(J_4 + 2J_3^{(1)} + J_1 + 2J_0), 3\} \quad (6)$$

where

J_0 : Number of plane contacts with friction

J_1 : Number of line contacts with friction

$J_3^{(1)}$: Number of plane contacts without friction

$J_3^{(2)}$: Number of point contacts with friction

J_4 : Number of line contacts without friction

J_5 : Number of point contacts without friction

Similarly, for planar grasping, we have

Table 1 Kinematic classification of contact points

Condition of contact		Spatial grasp		Plane grasp	
		Degree of freedom	Name of pair	Degree of freedom	Name of pair
Without friction	Point contact	5	Sphere-plane pair	2	Square shank sphere-slotted cylinder pair
	Line contact	4	Cylinder-plane pair	1	Prismatic pair
	Plane contact	3	Planar pair	—	—
With friction	Point contact	3	Spheric pair	1	Revolute pair
	Line contact	1	Revolute pair	0	—
	Plane contact	0	—	—	—

$$n_0 \geq M_m - m - f_r + 4 \quad (7)$$

$$n_0 = 2J_0 + J_1^{(1)} + 2J_1^{(2)} + J_2 + \min\{(J_0 + J_1^{(1)}), 1\} \quad (8)$$

where

J_0 : Number of line contacts with friction

$J_1^{(1)}$: Number of line contacts without friction

$J_1^{(2)}$: Number of point contacts with friction

J_2 : Number of point contacts without friction

From Eq.(7), a planar gripper with one input and three degrees of mobility, as an example, should satisfy

For a circular cylinder

$$n_0 \geq 5$$

For a multi-angular prism

$$n_0 \geq 6$$

Using the above results, we carried out a number synthesis of planar mechanisms, as an example, grasping a circular cylinder at five contact points. The conditions for synthesis are given as follows.

- (1) All pairs should be the simplest revolute pairs with degree of mobility 1.
- (2) The mechanism should be symmetrical, similar to the body, and the positions of the input links of the right and left sides of the mechanism should also be symmetrical.

In order to satisfy condition (2), the degrees of mobility of each side of the mechanism should be 2. Further conditions for the synthesis of this mechanism are given, considering some simplifications of construction as follows.

- (3) The number of arms should be 2.
- (4) All links should be binary or ternary links.
- (5) The number of moving circuits should be 1 or 0.

As a result of a number synthesis⁽⁹⁾ performed under the above five conditions, we obtained one type of 5-link mechanism and five types of 7-link ones, as shown in Fig. 3.

4. Dimensional syntheses of mechanisms

Table 2 shows the relationships between the degrees of mobility and the grasping characteristics of a planar gripper using the results obtained in Section 3. As a general case, a gripper is required to fit several shapes, sizes, and masses of a body. It is desirable that, from a kinetic point of view, each grasping force should be small and averaged, i.e., the gripper should have many contact points and, from a geometrical point of view, the gripper should have few links and be compact, and moreover, the number of inputs should be small for control. A method for satisfying these requirements is to let the degrees of mobility be larger without changing the number of inputs as shown in

Table 2 Number of contact points in planar grasp

m	M_m	n_0	
		$f_r = 1$	$f_r = 2$
1	1	3	2
	2	4	—
	3	5	—
2	2	3	2
	3	4	—
	4	5	—
3	3	3	—
	4	4	—
	5	5	—

Five-Link Mechanism

Seven-Link Mechanism (A)

Seven-Link Mechanism (B)

Seven-Link Mechanism (C)

Seven-Link Mechanism (D)

Seven-Link Mechanism (E)

Fig. 3 Results of number synthesis

Table 2. The gripper with $m=1, M=3$, as an example, has 5/3 times as many contact points as the one with $m=1, M=3$, and therefore it is possible to reduce the grasping forces of the former to 3/5 as much as those of the latter for the same body. Such a gripper, whose degrees of mobility are larger than its number of inputs, can change its own shape according to the shapes and sizes of bodies, and has some flexibilities for grasping.

Then, we carried out a dimensional synthesis of the mechanism shown in Fig. 3 as a "Seven-Link Mechanism (B)", as an example, using the method shown in Fig. 4. Since the degrees of mobility of this mechanism are 2 under no grasping conditions, a linear coil spring with sufficiently small rigidity is attached to the revolute pair on the coupler links which contact the body, for the purpose of determining the shape of the mechanism and the order of contacts of the coupler links to the body. The equations of motion are given by

$$\left. \begin{aligned} F_{j1} &= C_{j1} T_{kj} + C_{j2} T_{ij} \\ F_{j2} &= C_{j3} T_{kj} + C_{j4} T_{ij} \\ F_3 &= \sum C_{j5} F_{j1} + C_{j6} F_{j2} \\ T_i &= T_{i1} + T_{i2} \quad (j=1, 2) \end{aligned} \right\} \quad (9)$$

Fig. 4 Flow chart for dimensional synthesis

where

- F_{j1}, F_{j2}, F_3 : Grasping force at each contact point
- T_{kj} : Moment resulting from the coil spring
- T_{ij} : Input torque on one side
- $C_{j1} \sim C_{j6}$: Constants determined by the shapes and displacements of links

The conditions for the dimensional synthesis are now given as follows.

- (1) The input torque of the mechanism should be less than or equal to $0.98\text{N}\cdot\text{m}$.
- (2) The body to be grasped should be a circular cylinder whose diameter D ranges from 35 mm to 70 mm.
- (3) The thickness h_0 of the links should be 8 mm considering the use of roller bearings for the revolute pairs.
- (4) The nondimensional graspable mass $M_{0\max}^*$, defined by the following equation, should be larger than or equal to 3.

where

- $M_{0\max}$: Graspable mass
- g : Acceleration of gravity
- μ : Maximum coefficient of static friction in the vertical direction at each contact point
- $F_{\max}$: Maximum grasping force at contact points (calculated from Eq.(9))

When a circular cylinder is grasped, for instance, by a parallel-two-finger-type gripper at two contact points, $M_{0\max}^*=2$. For the gripper studied in this paper, $M_{0\max}^*$ is a function of the diameter of the circular cylinder.

- (5) The transmission index TI should be larger than or equal to 0.5 during grasping motions.

Since the former transmissibility⁽¹⁰⁾ defined for mechanisms with degree of mobility 1 can not be applied directly to grippers with no load or with degrees of mobility 0 or

Table 3 Geometrical and kinetic parameters of gripper

Parameter	Length m	Angle deg	Mass m kg	Moment of inertia I kgm ²	Center of gravity	
					G m	y deg
a	a_0 a_1 a_2	5.0×10^{-2} 4.0×10^{-2} 1.5×10^{-2}				
$b_1(b_2)$	$b_{10}(b_{20})$	5.5×10^{-2}	51.2×10^{-3}	8.76×10^{-6}	18.6×10^{-3}	0.0
$c_1(c_2)$	$c_{10}(c_{20})$	5.0×10^{-2}	18.7×10^{-3}	5.86×10^{-6}	25.0×10^{-3}	0.0
$d_1(d_2)$	$d_{10}(d_{20})$	1.5×10^{-2}	30.0×10^{-3}	2.79×10^{-6}	13.2×10^{-3}	13.0
	$d_{11}(d_{21})$	3.0×10^{-2}				
	β_{d2}					
$e_1(e_2)$	$e_{10}(e_{20})$	4.0×10^{-2}	23.7×10^{-3}	3.53×10^{-6}	20.9×10^{-3}	0.0
$f_1(f_2)$	$f_{10}(f_{20})$	3.5×10^{-2}	19.0×10^{-3}	3.58×10^{-6}	7.55×10^{-3}	57.9
	$f_{11}(f_{21})$	1.6×10^{-2}				
	β_{f2}					
$g_1(g_2)$	$g_{10}(g_{20})$	6.0×10^{-2}	69.6×10^{-3}	15.60×10^{-6}	23.4×10^{-3}	4.7
	$g_{11}(g_{21})$	2.5×10^{-2}				
	β_{g2}					
	h_0	0.8×10^{-2}				
	h_1	20.0×10^{-2}				

() Left Side

larger than 2, we consider the transmissibility only of each mechanism with degree of mobility 1 when a pair of links g_{10} and g_{20} contacts the body. If this index is too small, the power flows from link g directly to the body and the body might be moved before being grasped.

- (6) The sum of the link lengths f_{10} and f_0 should be less than or equal to the sum of the minimum and maximum values of the diameter of the cylinder determined in term (2), i.e., 105 mm for the purpose of limiting the sizes of gripper.

The dimensional synthesis has been carried out under the above six conditions according to the procedure shown in Fig. 4. The results are shown in Fig. 5 and in the first and second columns of Table 3.

5. Experimental investigations of grasping forces

5.1 Experimental apparatus

The mechanism proposed in Section 4 was constructed for the purpose of investigating grasping force. The actuator is a D. C. servomotor with reduction gears (ratio : 1/30), the power of which is transmitted to both sides of a mechanism through other gears (ratio : 1/2). The spring constants of two coil springs attached to the pair, $J_{f_{g1}}$ and $J_{f_{g2}}$, are each $K = 3.638 \times 10^{-3} \text{ N}\cdot\text{m/rad}$. The geometrical and kinetic parameters of each link are shown in Table 3. The total weight of the gripper including an actuator is almost 1.5kg. The insides of links f_{10} and g_{10} are flexible for bending, and the grasping forces may be measured from these bendings by strain gauges.

Fig. 5 Gripper with one input and three degrees of mobility

5.2 Experimental results : Static grasp on a horizontal plane

The contact forces when the gripper grasps three kinds of bodies, i.e., a circular cylinder, a regular hexagonal prism, and a regular square prism, on a horizontal working plane are analyzed experimentally. The sizes of these bodies are expressed systematically by the diameters of their circumcircles.

(1) Circular cylinder

The grasping forces obtained are shown in Fig. 6 ($T_i = T_{i1} + T_{i2}$), which almost coincide with the theoretical values drawn by solid lines. When the force F_3 shown in Fig. 5 is less than zero, the gripper cannot hold the body. Figure 6 (b) shows that force F_{12} increases with respect to D , in other words, F_3 does not easily become negative. This means that the graspable range for circular cylinders by this gripper is very wide. The nondimensional graspable mass of this gripper is compared to that of a conventional parallel-two-finger-type gripper with the same actuator, as shown in Figure 7. The former graspable mass is a function of the diameter and has the maximum value of about 2.5 times that of the latter ($M_{0\max}^* = 2 : \text{Const.}$), because the former gripper has as many as five contact points with the cylinder.

Fig. 6 Grasping forces (circular cylinder)

Fig. 7 Graspable mass of circular cylinder

Fig. 8 Symmetrical grasping of hexagonal prism

Fig. 9 Symmetrical grasping of square prism

Fig. 10 Grasping forces (hexagonal prism)

(2) Regular multi-angular prisms

There are three types of symmetrical grasplings for a regular hexagonal prism (Fig.8) and two types for a square prism (Fig.9). The contact forces for the types of Fig. 8(a) and 9(a) are shown in Fig. 10 and

(a)

(b)

Fig. 11 Grasping forces (square prism)

11. Although the ranges of graspable sizes D in both Figures are considerably narrow compared to those of the cylinder, the experimental values approximately coincide with the theoretical ones except for the case when D is large in Fig. 11. These errors were caused by the measuring in accuracies which are influenced by the distances between the contact point and the fulcrum of the flexible link.

The ranges of the graspable sizes of the above-mentioned three kinds of bodies are shown in Fig. 12, in which (a), (b), and (c) correspond to those numbers of Fig.8 and 9. As a result, the gripper synthesized in this paper has flexibilities for the sizes, shapes, and orientations of bodies.

5.3 Experimental results : Grasping on a general plane and conveying

The characteristics are analyzed experimentally when the gripper grasps bodies in arbitrary orientations and conveys them on a horizontal plane with acceleration.

The orientations are defined by two angles θ_0, ϕ_0 , in Fig. 13 (a), and the frictional forces are considered to act only in the z -direction, vertical to the moving plane. The graspable masses measured for the input torque $T_i = 0.98 \text{ N}\cdot\text{m}$, the diameter of cylinder $D = 50$

Fig. 12 Graspable sizes

(a)

(b)

Fig. 13 Grasping of cylinders in arbitrary orientations

Fig. 14 Grasping characteristics moving with accelerations

mm, and the maximum coefficient of static friction $\mu = 0.2$ are shown in Fig. 13 (b), in which $M_{0\max}$ is determined kinematically in the domain to the left of the broken line or kinetically (by friction) in the right domain. Similar patterns have been obtained for other cylinder sizes, and most graspable masses have minimum values ($M_{0\max} = 0.31 \text{ kg}$ for $D = 50 \text{ mm}$) when $\theta_0 = \phi_0 = 0^\circ$ or $\phi_0 = -90^\circ$.

Figure 14 shows the relationships between the graspable mass M_0 and the direction θ_a and magnitude α of the acceleration of the gripper with a cylinder under the same conditions shown in Fig. 13. The graspable masses are determined by the contact force on the frame in the domain to the right of the broken line and by the input torque in the left domain. Similar results are obtained for other cylinder sizes, and they show that the acceleration at which the gripper can hold the body becomes a maximum when $\theta_a = 180^\circ$.

6. Conclusions

The results obtained in this paper are summarized as follows.

(1) A mobility equation for gripper-body systems has been suggested, and graspable conditions have been derived, using a number of equivalent contact points, for the purpose of carrying out a number synthesis of the gripper.

(2) Number and dimensional syntheses of a planar gripper with one input and three degrees of mobility, as an example, have been carried out by giving the sizes and shapes of the bodies, the grasping forces, and the transmissibility of the mechanism.

(3) The grasping characteristics of the above mechanism have been investigated theoretically and

experimentally, and the fact that the weights of graspable circular cylinders are 1.5~2.5 times those of a conventional parallel-two-finger-type gripper has been clarified.

(4) The high flexibilities in the sizes and shapes of bodies grasped by this gripper have been confirmed by experiments of grasping regular hexagonal prisms and square prisms.

(5) Optimum grasping forces and conveying speeds have been clarified by analyzing the grasping characteristics in arbitrary orientations and in conveying with accelerations.

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