Fluid Film Lubrication with Nonlinear Four-Element Viscoelastic Fluids*

(Flow Characteristics, a Modified Reynolds Equation and its Application to Parallel Circular Squeeze Bearings)

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The objective of this study is to establish an analytical procedure for viscoelastic fluid film lubrication based on a nonlinear four-element model, which should apply to lubricating oils containing a large quantity of high molecular-weight polymers. It is made clear from the flow characteristic test that the viscoelastic behavior of such lubricants is represented by a rheology equation based on a four-element model which consists of two nonlinear dashpots and two nonlinear springs. A modified Reynolds equation for finite-width bearings is derived by employing the nonlinear rheology equation. The effects of nonlinear viscoelastic characteristics on the film pressure are then studied with a parallel circular squeeze bearing, and it is shown that the experimental results agree well with the theoretical predictions.

Key Words: Lubrication Theory, Lubricant, Viscoelastic Fluid, Four-Element Model, Flow Characteristics, Nonlinear Rheology Equation, Modified Reynolds Equation, Squeeze Film, Parallel Circular Squeeze Bearing, Sinusoidal Squeezing Motion

1. Introduction

In recent years, lubricating oils have contained a large quantity of high molecular-weight polymers as a viscosity index improver and a viscosity thickener. Such lubricants display non-Newtonian viscosity similar to pseudo-plastic fluids when the amount of additives is small. The authors have, in fact, published a series of papers^{(1)~(4)} on fluid film lubrication with non-Newtonian lubricants.

As the amount of additives is increased, polymer-thickened oils begin to show the characteristics of viscoelasticity and non-Newtonian viscosity. Studies have been done on fluid film lubrication employing several viscoelastic models such as the Maxwell model^{(5)~(9)}, Oldroyd model⁽¹⁰⁾⁽¹¹⁾, Rivlin-Ericksen model^{(12)~(14)}, complex viscosity⁽¹⁵⁾⁽¹⁶⁾, and so on. In most previous investigations, the theories based on linear viscoelastic models were developed for infinite-width bearings and were not verified by comparing the

analytical predictions with the experimental results.

To establish an analytical procedure for viscoelastic fluid film lubrication, a finite-width bearing theory based on nonlinear viscoelastic models is necessary. The authors^{(17)~(19)} derived a modified Reynolds equation for finite-width bearings by using a nonlinear Maxwell model, where both the viscosity and elastic modulus were represented by a polynomial expression of shear stress, and found that the theoretical bearing performance of both a parallel circular squeeze bearing and a step thrust bearing agrees well with the experimental results. Recently, some investigations^{(20)~(22)} based on a nonlinear Maxwell model were reported, in which nonlinearity was considered only in the viscous term which followed the Eyring law. However, the frequency dependence of viscoelastic behavior in such lubricants could not be included in these studies because they were based on the simple Maxwell model.

In this paper, in order to introduce not only the nonlinear characteristics for shear stress but also the frequency dependence, a rheology equation for the four-element viscoelastic model is derived, which consists of two nonlinear dashpots and two nonlinear

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springs. Further, it is made clear that the rheology equation represents the nonlinear viscoelastic behavior of actual lubricating oils containing high molecular-weight polymers. A modified Reynolds equation for finite-width bearings is derived by employing the nonlinear rheology equation. The effect of the viscoelastic behavior of lubricants on the performance of parallel circular squeeze bearings is then clarified and compared with experimental results. Good agreement is seen between the theoretical results based on nonlinear four-element viscoelastic fluids and the experimental ones.

2. Rheology Equation

By applying the linear rheology equation proposed by Jeffreys⁽²³⁾ to a four-element model shown in Fig. 1 (a), and by introducing the nonlinearity for the shear stress, a nonlinear rheology equation is expressed as follows.

$$\tau_{ij} + a_1(\tau) \frac{\partial \tau_{ij}}{\partial t} + a_2(\tau) \frac{\partial^2 \tau_{ij}}{\partial t^2} = \mu_0(\tau) \left\{ \dot{\gamma}_{ij} + b_1(\tau) \frac{\partial \dot{\gamma}_{ij}}{\partial t} \right\} \quad (1)$$

where $a_1(\tau)$, $a_2(\tau)$, $b_1(\tau)$ and $\mu_0(\tau)$ are apparent material constants and (τ) indicates they are functions of the second invariant τ of the stress-deviation tensor. Using the apparent viscosities and the apparent elastic moduli of the four-element model, the apparent material constants are

$$a_1(\tau) = \frac{\mu_{a1}(\tau)}{G_{a1}(\tau)} + \frac{\mu_{a1}(\tau) + \mu_{a2}(\tau)}{G_{a2}(\tau)} \quad (2)$$

Fig. 1 Viscoelastic fluid model

$$a_2(\tau) = \frac{\mu_{a1}(\tau) \cdot \mu_{a2}(\tau)}{G_{a1}(\tau) \cdot G_{a2}(\tau)} \quad (3)$$

$$b_1(\tau) = \frac{\mu_{a2}(\tau)}{G_{a2}(\tau)} \quad (4)$$

$$\mu_0(\tau) = \mu_{a1}(\tau) \quad (5)$$

The nonlinearity of these apparent viscosities and elastic moduli are assumed as follows.

$$\mu_{ak}(\tau) = \frac{\mu_k}{1 + \delta_k \tau^2} \quad (k=1, 2) \quad (6)$$

$$G_{ak}(\tau) = \frac{G_k}{1 + \alpha_k \tau^2} \quad (k=1, 2) \quad (7)$$

where μ_k and G_k are called "initial viscosities" and "initial elastic moduli", respectively, and δ_k and α_k are called "nonlinear factors of viscosities" and "nonlinear factors of elastic moduli", respectively.

The above rheology equation can represent any of the other models in Fig. 1 in special cases.

3. Flow Characteristics

Flow characteristics are determined experimentally, and it is made clear that the nonlinear viscoelastic behavior of lubricants is represented by the above rheology equations.

3.1 Experimental apparatus and procedure

The experimental apparatus is shown schematically in Fig. 2. The oil fills the clearance between inner and outer co-axial cylinders. The outer cylinder is steadily rotated; the inner cylinder is supported by a leaf spring, which is sinusoidally vibrated, and the response of the inner cylinder is measured.

In the flow characteristics test, the amplitude of vibration is made sufficiently small so that the shear stress due to vibration is far smaller than that due to rotation. Therefore, the shear stress is regarded as τ

Fig. 2 Rheometer

$\dot{\gamma} = \tau_{r\theta}$. If the steady rotation of the outer cylinder is considered, the rheology equations in the rotational and vibrational directions, respectively, are as follows.

$$\tau_{r\theta} = \mu_0(\tau_{r\theta}) r \frac{\partial \omega(r)}{\partial r} \quad (8)$$

$$\begin{aligned} \tau_{rz} + a_1(\tau_{r\theta}) \frac{\partial \tau_{rz}}{\partial t} + a_2(\tau_{r\theta}) \frac{\partial^2 \tau_{rz}}{\partial t^2} \\ = \mu_0(\tau_{r\theta}) \left\{ \frac{\partial v_z}{\partial r} + b_1(\tau_{r\theta}) \frac{\partial}{\partial t} \left(\frac{\partial v_z}{\partial r} \right) \right\} \end{aligned} \quad (9)$$

Since the clearance in the radial direction between the inner and outer cylinders is small, Eqs. (8) and (9) can be analyzed separately by regarding the apparent material constants $a_1(\tau_{r\theta})$, $a_2(\tau_{r\theta})$, $b_1(\tau_{r\theta})$ and $\mu_0(\tau_{r\theta})$ as constants.

Application of the previous analysis⁽²⁴⁾ to Eq. (9) gives the relationship among the material constants, amplitude ratios and phase lags between the exciting vibration and the responding vibration. Therefore, if amplitude ratios and phase lags are measured for various exciting frequencies while the rotational speed of the outer cylinder is kept constant, a_1 , a_2 , b_1 and μ_0 can be obtained by the method of least squares. Using these experimental results, the apparent viscosities

and the apparent elastic moduli are calculated from Eqs. (2)–(5).

On the other hand, the shear stress on the inner cylinder is evaluated from Eq. (8). Therefore, when the experiments are performed for various rotational speeds of the outer cylinder, the viscoelastic nonlinearity for the shear stress is obtained by applying the method of least squares to Eqs. (6) and (7).

3.2 Experimental results and discussions

A spindle oil thickened with 4% polyisobutylene, by weight, is used as the typical viscoelastic fluid, and its flow characteristics are determined.

In order to compare the models shown in Fig. 1, the frequency characteristics of the real part of both the complex viscosity and complex elastic modulus⁽²⁴⁾⁽²⁵⁾ are shown in Figs. 3 and 4, in which circles are the experimental results and curves represent the empirical formulas of viscoelastic fluids on the basis of two-, three- and four-element models. From these figures, it is found that the rheology equation of the four-element model is able to represent well the viscoelastic behavior of the lubricants. Further, the experimental results on the nonlinear characteristics

Fig. 3 Frequency characteristics of real part of complex viscosity

Fig. 4 Frequency characteristics of real part of complex elastic modulus

Fig. 5 Nonlinear characteristics of apparent viscosity for shear stress

Fig. 6 Nonlinear characteristics of apparent elastic modulus for shear stress

Fig. 7 Geometry of sliding surface in orthogonal coordinates

are in good agreement with the empirical formula, as shown in Figs. 5 and 6, which shows they can be approximated by Eqs. (6) and (7).

4. Modified Reynolds equation

A modified Reynolds equation for finite-width bearings is derived by employing the above rheology equation of the nonlinear four-element model.

4.1 Rheology equation

Let us consider the fluid film shown in Fig. 7. The nonlinear rheology equations are

$$\tau_{yx} + a_1(\tau) \frac{\partial \tau_{yx}}{\partial t} + a_2(\tau) \frac{\partial^2 \tau_{yx}}{\partial t^2} = \mu_0(\tau) \left\{ \frac{\partial u}{\partial y} + b_1(\tau) \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial y} \right) \right\} \quad (10)$$

$$\tau_{yz} + a_1(\tau) \frac{\partial \tau_{yz}}{\partial t} + a_2(\tau) \frac{\partial^2 \tau_{yz}}{\partial t^2} = \mu_0(\tau) \left\{ \frac{\partial w}{\partial y} + b_1(\tau) \frac{\partial}{\partial t} \left(\frac{\partial w}{\partial y} \right) \right\} \quad (11)$$

where

$$\tau = \sqrt{\tau_{yx}^2 + \tau_{yz}^2} \quad (12)$$

A finite difference method of higher order with respect to time and a numerical integration method across the film are introduced in order to derive a modified Reynolds equation which is suitable for numerical analysis by a digital computer.

Assuming that $\partial \tau_{yx}/\partial t$ and $\partial(\partial u/\partial y)/\partial t$ vary linearly from time t_{m-2} to time t_m , the following equations are obtained.

$$\left(\frac{\partial^2 \tau_{yx}}{\partial t^2} \right)_m = \frac{2}{\Delta t} \left[\frac{1}{\Delta t} \{ (\tau_{yx})_m - (\tau_{yx})_{m-1} \} - \left(\frac{\partial \tau_{yx}}{\partial t} \right)_{m-1} \right] \quad (13)$$

$$\left(\frac{\partial \tau_{yx}}{\partial t} \right)_m = \frac{2}{\Delta t} \{ (\tau_{yx})_m - (\tau_{yx})_{m-1} \} - \left(\frac{\partial \tau_{yx}}{\partial t} \right)_{m-1} \quad (14)$$

$$\left\{ \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial y} \right) \right\}_m = \frac{2}{\Delta t} \left\{ \left(\frac{\partial u}{\partial y} \right)_m - \left(\frac{\partial u}{\partial y} \right)_{m-1} \right\} - \left\{ \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial y} \right) \right\}_{m-1} \quad (15)$$

Substitution of these equations into Eq. (10) gives the velocity gradient $(\partial u/\partial y)_m$

$$\left(\frac{\partial u}{\partial y} \right)_m = g_x(\tau_{yxm}, \tau_{yzm}) \quad (16)$$

Similarly, the velocity gradient $(\partial w/\partial y)_m$ is

$$\left(\frac{\partial w}{\partial y} \right)_m = g_z(\tau_{yxm}, \tau_{yzm}) \quad (17)$$

The functions g_x and g_z in Eqs. (16) and (17) are given by the following equations.

$$\begin{aligned} g_x(\tau_{yxm}, \tau_{yzm}) &= \frac{1}{\mu_0(\tau_m) \{ 1 + (2/\Delta t) b_1(\tau_m) \}} \\ &\times \left[\left\{ 1 + \frac{2}{\Delta t} a_1(\tau_m) + \frac{2}{(\Delta t)^2} a_2(\tau_m) \right\} (\tau_{yx})_m \right. \\ &- \left\{ \frac{2}{\Delta t} a_1(\tau_m) + \frac{2}{(\Delta t)^2} a_2(\tau_m) \right\} (\tau_{yx})_{m-1} \\ &- \left\{ a_1(\tau_m) + \frac{2}{\Delta t} a_2(\tau_m) \right\} \left(\frac{\partial \tau_{yx}}{\partial t} \right)_{m-1} \\ &\left. + \mu_0(\tau_m) b_1(\tau_m) \left\{ \frac{2}{\Delta t} (g_x)_{m-1} + \left(\frac{\partial g_x}{\partial t} \right)_{m-1} \right\} \right] \end{aligned} \quad (18)$$

$$\begin{aligned} g_z(\tau_{yxm}, \tau_{yzm}) &= \frac{1}{\mu_0(\tau_m) \{ 1 + (2/\Delta t) b_1(\tau_m) \}} \\ &\times \left[\left\{ 1 + \frac{2}{\Delta t} a_1(\tau_m) + \frac{2}{(\Delta t)^2} a_2(\tau_m) \right\} (\tau_{yz})_m \right. \\ &- \left\{ \frac{2}{\Delta t} a_1(\tau_m) + \frac{2}{(\Delta t)^2} a_2(\tau_m) \right\} (\tau_{yz})_{m-1} \\ &- \left\{ a_1(\tau_m) + \frac{2}{\Delta t} a_2(\tau_m) \right\} \left(\frac{\partial \tau_{yz}}{\partial t} \right)_{m-1} \\ &\left. + \mu_0(\tau_m) b_1(\tau_m) \left\{ \frac{2}{\Delta t} (g_z)_{m-1} + \left(\frac{\partial g_z}{\partial t} \right)_{m-1} \right\} \right] \end{aligned} \quad (19)$$

where

$$\tau_m = \sqrt{(\tau_{yx})_m^2 + (\tau_{yz})_m^2} \quad (20)$$

The apparent material constants $a_1(\tau_m)$, $a_2(\tau_m)$, $b_1(\tau_m)$ and $\mu_0(\tau_m)$ in Eqs. (18) and (19) are obtained by putting $\tau = \tau_m$ in Eqs. (2)–(7).

Since the quantities at time t_{m-1} are given in an iteration analysis, the fluid film lubrication theory can be developed by using Eqs. (16) and (17), which indicate the velocity gradients at time t_m .

4.2 Velocities

By integrating the equilibrium equations between the shear stress and the film pressure with respect to y , we have

$$\tau_{yxm} = \frac{\partial p_m}{\partial x} y + C_{xm} \quad (21)$$

$$\tau_{yzm} = \frac{\partial p_m}{\partial z} y + C_{zm} \quad (22)$$

where C_{xm} and C_{zm} are the integral constants.

By substituting Eqs. (21) and (22) into Eqs. (16) and (17), and integrating across the film with the following boundary conditions

$$u(y=0) = U_{1m}, \quad u(y=h_m) = U_{2m} \quad (23)$$

$$w(y=0) = 0, \quad w(y=h_m) = 0 \quad (24)$$

the velocities in the x - and z -directions become

$$u_m = \frac{1}{2} \left\{ \int_0^y g_x dy - \int_y^{h_m} g_x dy + U_{1m} + U_{2m} \right\} \quad (25)$$

$$w_m = \frac{1}{2} \left\{ \int_0^y g_z dy - \int_y^{h_m} g_z dy \right\} \quad (26)$$

The integral constants are evaluated by solving numerically the following simultaneous equations, which are obtained by applying Eqs.(23) and (24) to Eqs.(25) and (26), respectively.

$$\int_0^{h_m} g_x \left(\frac{\partial p_m}{\partial x} y + C_{xm}, \frac{\partial p_m}{\partial z} y + C_{zm} \right) dy = U_{2m} - U_{1m} \quad (27)$$

$$\int_0^{h_m} g_z \left(\frac{\partial p_m}{\partial x} y + C_{xm}, \frac{\partial p_m}{\partial z} y + C_{zm} \right) dy = 0 \quad (28)$$

4.3 Modified Reynolds equation

The continuity equation is given as follows.

$$\left(\frac{\partial u}{\partial x} \right)_m + \left(\frac{\partial v}{\partial y} \right)_m + \left(\frac{\partial w}{\partial z} \right)_m = 0 \quad (29)$$

Integrating Eq.(29) with respect to y with the boundary conditions

$$v(y=0) = V_{1m}, \quad v(y=h_m) = V_{2m} \quad (30)$$

gives

$$\begin{aligned} \frac{\partial}{\partial x} f_x \left(\frac{\partial p_m}{\partial x}, \frac{\partial p_m}{\partial z} \right) + \frac{\partial}{\partial z} f_z \left(\frac{\partial p_m}{\partial x}, \frac{\partial p_m}{\partial z} \right) \\ = \frac{U_{1m} - U_{2m}}{2} \frac{\partial h_m}{\partial x} + \frac{h_m}{2} \frac{\partial}{\partial x} (U_{1m} + U_{2m}) \\ + (V_{2m} - V_{1m}) \end{aligned} \quad (31)$$

where

$$f_x \left(\frac{\partial p_m}{\partial x}, \frac{\partial p_m}{\partial z} \right) = -\frac{1}{2} \int_0^{h_m} \left\{ \int_0^y g_x dy - \int_y^{h_m} g_x dy \right\} dy \quad (32)$$

$$f_z \left(\frac{\partial p_m}{\partial x}, \frac{\partial p_m}{\partial z} \right) = -\frac{1}{2} \int_0^{h_m} \left\{ \int_0^y g_z dy - \int_y^{h_m} g_z dy \right\} dy \quad (33)$$

Equation (31) is a modified Reynolds equation for nonlinear four-element viscoelastic fluids at time t_m and corresponds to the Reynolds equation for Newtonian fluids.

5. Application to Parallel Circular Squeeze Bearings

By applying the modified Reynolds equation to a parallel circular squeeze bearing, the bearing performance is calculated theoretically. The effects of nonlinear viscoelastic characteristics of lubricants on film pressure are clarified by comparing the theories with the experiments. In this section, variables with $\sim$ mean dimensional quantities; non-dimensional quantities are used as shown in Table 1.

5.1 Theory

Let us consider a parallel circular squeeze bearing shown in Fig. 8. A disk is sinusoidally oscillated in the y -direction without rotation against a stationary plane. Therefore, the non-dimensional film thickness

is

$$h_m = 1 + \varepsilon \cos t_m \quad (34)$$

where ε is the ratio of exciting amplitude to average film thickness $\hat{h}_{av}$.

The modified Reynolds equation in the cylindrical coordinates at time t_m becomes

$$\frac{\partial}{\partial r} r f_r \left(\frac{\partial p_m}{\partial r} \right) = r \left(\frac{\partial h}{\partial t} \right)_m \quad (35)$$

which is rewritten as follows, if no flow rate in the radial-direction at the center of the bearings is taken into account.

$$f_r \left(\frac{\partial p_m}{\partial r} \right) = \frac{r}{2} \left(\frac{\partial h}{\partial t} \right)_m \quad (36)$$

The integral constant C_{rm} in the above equation is determined to satisfy the following equation.

$$\int_0^{h_m} g_r(C_{rm}) dy = 0 \quad (37)$$

Table 1 Nondimensional quantities

$r = \hat{r}/\hat{R}$
$y = \hat{y}/\hat{h}_{av}$
$h = \hat{h}/\hat{h}_{av}$
$t = \hat{\omega}_f \hat{t}$
$p = \hat{p} \hat{h}_{av}^2 / (\hat{\mu}_1 \hat{R}^2 \hat{\omega}_f)$
$\tau = \hat{\tau} \hat{h}_{av} / (\hat{\mu}_1 \hat{R} \hat{\omega}_f)$
$a_1 = \hat{a}_1 \hat{\omega}_f$
$a_2 = \hat{a}_2 \hat{\omega}_f^2$
$b_1 = \hat{b}_1 \hat{\omega}_f$
$\mu_0 = \hat{\mu}_0 / \hat{\mu}_1$
$\mu_{ak} = \hat{\mu}_{ak} / \hat{\mu}_1$
$G_{ak} = \hat{G}_{ak} / (\hat{\mu}_1 \hat{\omega}_f)$
$\mu_k = \hat{\mu}_k / \hat{\mu}_1$
$G_k = \hat{G}_k / (\hat{\mu}_1 \hat{\omega}_f)$
$\delta_k = \hat{\delta}_k (\hat{\mu}_1 \hat{R} \hat{\omega}_f / \hat{h}_{av})^2$
$\alpha_k = \hat{\alpha}_k (\hat{\mu}_1 \hat{R} \hat{\omega}_f / \hat{h}_{av})^2$

where $k=1, 2$

$\sim$: Dimensional quantity

Fig. 8 Parallel circular squeeze bearing

Functions f_r and g_r in Eqs.(36) and (37) are evaluated by iteration analysis because they are functions of material constants, integral constant C_{rm} and pressure gradient $\partial p_m / \partial r$. Since these functions include values at time t_{m-1} , an initial condition is necessary. However, it is found from calculations for various initial values that a steady state solution of pressure, which is independent of the initial values, is obtained after an interval corresponding to two or three cycles of the exciting vibration. Therefore, a simple initial condition such as a pressure of zero at $t = 0$ and a boundary condition that the pressure is zero at the circumference of the bearings are used in the following numerical analysis. The steady state solutions are shown in the following figures.

Figure 9 shows a comparison of the pressures of Newtonian fluids, pseudo-plastic fluids and viscoelastic fluids. As the viscoelastic fluids, nonlinear Maxwell fluids whose viscosity depends on the shear stress are used. Figure 9 indicates that magnitudes of the pressure become smaller in the order of Newtonian fluids, pseudo-plastic fluids, and viscoelas-

tic fluids. When the squeezing velocity is zero, that is, $t=0, \pi$ and 2π , the pressure of Newtonian fluids and pseudo-plastic fluids is zero. On the other hand, the pressure of viscoelastic fluids is delayed in becoming zero, because the film pressure of viscoelastic fluids can be developed by the previous conditions even in the absence of squeezing action. In this paper, this delay is called "a phase lag of film pressure" of viscoelastic fluids.

As G_1 decreases, the film pressure decreases and the phase lag of the film pressure increases, as shown in Fig. 10. As G_2 decreases, the film pressure decreases, while the phase lag of the film pressure decreases when G_2 is small, Fig. 11(a), but increases when G_2 is large, Fig. 11(b). With the increase in μ_2 , the phase lag of the film pressure decreases; the film pressure decreases for small μ_2 , but increases for large μ_2 , as in Fig. 12. The increase in δ_1 causes the film pressure to decrease and the phase lag of the film pressure to decrease somewhat, as shown in Fig. 13. However, the effect of δ_2 is negligibly small, as shown in Fig. 14. The nonlinear factors of elastic moduli α_1 and α_2 have little

Fig. 9 Comparison of Newtonian fluids, pseudo-plastic fluids and viscoelastic fluids

Fig. 10 Effect of initial elastic modulus B_1

(a) In the case of small G_2

(b) In the case of large G_2

Fig. 11 Effect of initial elastic modulus G_2

(a) In the case of small μ_2 (b) In the case of large μ_2 Fig. 12 Effect of initial viscosity μ_2 Fig. 14 Effect of nonlinear factor of viscosity δ_2 Fig. 15 Effect of nonlinear factor of elastic modulus α_1 Fig. 13 Effect of nonlinear factor of viscosity δ_1 Fig. 16 Effect of nonlinear factor of elastic modulus α_2

effect on the film pressure, as shown in Figs. 15 and 16.

5.2 Experiment

The apparatus is illustrated schematically in Fig. 17, in which the lower plate is sinusoidally oscillated against the stationary upper plate by an electric magnet vibrator. The radius of these plates is 20 mm, and the average film thickness is 1 mm. The spindle oil thickened with 4% polyisobutylene, by weight, is used, of which the flow characteristics were clarified in section 3. The oil temperature is controlled at $20 \pm 0.5^\circ\text{C}$.

In Fig. 18, the experimental results are compared with the analytical results based on two-, three- and four-element nonlinear viscoelastic models, for various exciting frequencies, where the phase lag of the film pressure is evident. Further, though the magnitude of the film pressure is proportional to the exciting frequency with Newtonian fluids, if other conditions are kept constant, this is not the case with the oil thickened with polyisobutylene. Its behavior can be accurately represented by the theory with the four-element model, while the differences between the experimental results and theoretical ones based on two- or three- element models increase with increasing exciting frequency.

6. Conclusions

(1) A rheology equation based on a four-element model can represent viscoelastic behavior of lubricat-

Fig. 17 Experimental squeeze bearing apparatus

ing oils containing a large quantity of high molecular-weight polymers in describing their frequency dependence and nonlinear characteristics for shear stress.

(2) A modified Reynolds equation suitable for numerical analysis is derived for finite-width bearings, by employing the rheology equation, by introducing the finite difference method of higher order with respect to time, and by using the numerical integra-

Fig. 18 Effect of exciting frequency

tion method across the film thickness.

(3) The modified Reynolds equation is applied to a parallel circular squeeze bearing, and by comparing the analysis with experimental results, it is shown that one of the outstanding characteristics of viscoelastic lubricants is the phase lag of the film pressure. As G_1 of the four-element viscoelastic fluid decreases, the viscoelastic effects increase. The influence of G_2 and μ_2 is dependent on their magnitudes. Nonlinear factor δ_1 mainly affects the magnitude of film pressure. The other nonlinear factors have little effect on the film pressure in the squeeze bearings. The theoretical results for nonlinear four-element viscoelastic fluids are in good agreement with the experimental ones.

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