On the Occurrence of Regenerative Chatter Vibrations

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Cutting forces and deflections of a workpiece are measured from the start of cutting operations and one of the important physical causes of regenerative chatter is found to be a small change of the cutting force that necessarily exists, even in a stable cutting condition. In a stable cutting condition, the workpiece is slightly disturbed by a hammer blow, and the changes of the cutting forces and the workpiece deflections are simultaneously measured. Vibratory parameters of a workpiece, that is, spring constant, damping coefficient and so on are precisely measured, and more suitable equations of motion for chatter vibration are proposed. These proposed differential equations of motion are calculated by a digital computer for two initial conditions: first, the workpiece suffers a small disturbance of displacement; second, the workpiece is disturbed by a hammer blow. Results of numerical calculations are compared with experimental results, and a good quantitative coincidence is derived.

Key Words: Cutting, Regenerative Chatter Vibration, Vibratory Properties of Workpiece, Proposed Differential Equation of Motion

1. Introduction

In lathe operations, when a regenerative chatter vibration is caused by a small disturbance, it is impossible to continue cutting a workpiece. Hence, for a long time, many investigators have studied the stability limit of regenerative chatter. Doi and Kato⁽¹⁾ and Ota and Kono⁽²⁾ determined the stability limit using their experimental results, where the fluctuation of cutting forces lagged behind the depth variations, and Tobias and Fishwick⁽³⁾ determined it by considering the penetration effect. Recently some reports⁽⁴⁾⁽⁵⁾ concerning the regenerative effect on a chatter vibration were published. Kondo et al.⁽⁴⁾ showed that the regenerative effect suppressed chatter vibration when an amplitude of a workpiece deflection was so large that the workpiece partly left a tool. Also, Kaneko et al.⁽⁵⁾ proposed differential equations of motion with two

degrees of freedom after investigating experimental data of the cutting force and the workpiece deflection.

However, it has not yet been determined whether a small waviness with constant amplitude exists or not on a workpiece surface, as suggested by theoretical analysis in the case of the stability limit, and how a large disturbance causes regenerative chatter. With exception of Hahn⁽⁶⁾, it has not been investigated how chatter vibration grows under the influence of a regenerative effect because of the many difficulties in measuring cutting forces and workpiece deflections in the initial stage.

One of the purposes of this study was to determine the influence of a small change of a cutting force on regenerative chatter, the influence of regenerative effect on chatter vibration, and the relation between the cutting force and the workpiece deflection. For this purpose, the authors used the micro-computer. Another purpose was to propose more suitable differential equations of motion and to verify the propriety of these equations by comparing these results with the experimental results derived by two different cases: first, where the workpiece suffered a small disturbance of displacement, and second, where

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the workpiece was disturbed by a hammer blow. The authors measured the characteristics of two components of the cutting force, and numerically solved the differential equations using these experimental values.

2. Differential Equation of Motion

For a vibratory system with two degrees of freedom in which the stiffness of a workpiece is considerably lower than that of a tool and only a workpiece is assumed to deflect, the orthogonal coordinates system ($O-\xi\eta$) is not generally considered in accordance with the orthogonal coordinates $O-xy$. Here, $O-\xi\eta$ consists of two coordinate axes of natural vibration and $O-xy$ consists of two coordinate axes, horizontal axis x and vertical axis y , and the origin O is the equilibrium point of the workpiece in no operation of cutting.

Now, with regards to the coordinate system $O-\xi\eta$, the authors introduce spring constants k_ξ, k_η and damping coefficients c_ξ, c_η as follows:

$$\begin{aligned} k_\xi &= (1 + \Delta_k)k, & k_\eta &= (1 - \Delta_k)k \\ c_\xi &= (1 + \Delta_c)c, & c_\eta &= (1 - \Delta_c)c \end{aligned} \quad (1)$$

where k is the mean spring constant and c is the mean damping coefficient. With no operation of cutting, the differential equations of motion of the workpiece regarding the coordinate system $O-\xi\eta$, are as follows:

$$\begin{aligned} m\ddot{\xi} + c_\xi\dot{\xi} + k_\xi\xi &= 0 \\ m\ddot{\eta} + c_\eta\dot{\eta} + k_\eta\eta &= 0 \end{aligned} \quad (2)$$

Equation (2) is then transformed into the equations expressed in the coordinate system $O-xy$ by Eq. (3), and two components of the cutting force, i.e. the horizontal cutting force F_x and the vertical cutting force F_y are added to the right-hand side.

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \ddot{\xi} \\ \ddot{\eta} \end{pmatrix} + \begin{pmatrix} F_x \\ F_y \end{pmatrix} \quad (3)$$

Here, the following parameters are introduced; $p = \sqrt{k/m}$, $n = c/(2m)$

m : equivalent mass of the workpiece (assumed to be concentrated on the A-A plane shown in Fig.1)

α : the angle $\angle xO\xi$ between the coordinate axis ξ and the coordinate axis x . Counter-clockwise direction is taken as positive.

d_0 : feed (mm/rev)

d : instantaneous depth of cut. General expression is as follows: ⁽⁴⁾

$$\begin{aligned} d &= \min \{ d_0 + x(t-T) - x(t), \\ &2d_0 + x(t-2T) - x(t), \\ &3d_0 + x(t-3T) - x(t), \dots \} \end{aligned} \quad (4)$$

where T is the time during one rotation of the workpiece.

However, if d is calculated to have a negative value in Eq.(4), then for the purpose of further calculation, is taken as $d=0$.

Further, the authors use the following dimensionless quantities⁽⁵⁾ and then obtain dimensionless differential equations of motion (6) concerning the coordinate system $O-xy$. Hereafter, the prime, representing dimensionless quantities, is omitted.

$$\begin{aligned} x' &= x/d_0, & y' &= y/d_0, & t' &= pt \\ f_x &= F_x/(kd_0), & f_y &= F_y/(kd_0) \end{aligned} \quad (5)$$

$$\left. \begin{aligned} \ddot{x} &+ (2n/p)(1 + \Delta_c \cos 2\alpha) \dot{x} \\ &+ (2n/p)\Delta_c(\sin 2\alpha) \dot{y} \\ &+ (1 + \Delta_k \cos 2\alpha)x + \Delta_k(\sin 2\alpha)y \\ &= f_x(d) \\ \ddot{y} &+ (2n/p)\Delta_c(\sin 2\alpha) \dot{x} \\ &+ (2n/p)(1 - \Delta_c \cos 2\alpha) \dot{y} \\ &+ \Delta_k(\sin 2\alpha)x + (1 - \Delta_k \cos 2\alpha)y \\ &= f_y(d) \end{aligned} \right\} \quad (6)$$

3. Experimental Apparatus and Measuring Procedure

In experiments, the authors measured the following quantities: two components of a deflection of the workpiece, i.e. the horizontal deflection x and the vertical deflection y ; a relative displacement between the workpiece and the tool; two components of the cutting force, F_x and F_y ; revolution marks of the workpiece. Figure 1 shows a schematic view of the experimental apparatus. A workpiece was cut in the orthogonal cutting condition with the lathe (Okuma, LS540 $\times$ 800 type). The steel shaft ①, diameter 60 mm, length 270 mm (S45C), was held by a four-jaw 12 inch chuck in order to reduce the effect of the chuck itself. By use of holder ②, this shaft carried a workpiece ③ (Brass, JIS H3201 BsP3) of width 2 or 3 mm and diameter nearly 150 mm. The tool ④ consisted of the holder (Tungaloy Turning System, E25L-33HU) and a throw-away tip (Mitubishi Carbide, TNPL332) which was changed in every cutting operation. The dynamometer ⑤ for measuring two components of the cutting force was a piezo-electric transducer

Fig. 1 Experimental apparatus

(Kistler, 9257A type) which was designed to give a very rigid measuring rig with high natural frequency. Three displacement pick-ups of eddy current type, ⑥a, ⑥b and ⑥c (AEC-1553T) were used: ⑥a was set to measure a relative displacement between the tool and the workpiece and a depth of cut, and ⑥b and ⑥c was set to measure the horizontal deflection x and the vertical deflection y . The authors changed the cutting speed continuously by the use of a stepless speed changer, and then controlled it by measuring the revolution marks generated by the photo-interrupter ⑦. These signals were supplied to a data recorder (TEAC, MR-30) and then fed into a micro-computer (North Star Horizon, Black-Box), which recorded them in a period of 2.5 seconds in digital form at 4000 data points per second per channel. The authors investigated these data by utilizing the central computer (FACOM M382). This procedure is shown in Fig. 2. For our purpose, the following special apparatus were conceived: for measuring these data from the start of the cutting operation, the authors installed a trigger pulse generator in the feed device of the lathe; for detecting the instant when a hammer touched the shaft carrying a workpiece, the device was designed to generate a pulse signal at that instant; for high accuracy, the material ⑧ facing the displacement pick-ups ⑥a, ⑥b, ⑥c was made of brass; for installing a dynamometer and displacement pick-ups on the lathe with high rigidity, the authors used the block ⑨ etc. made of cast iron.

The authors defined the positive direction of these measured quantities as follows (see also the coordi-

nate system $O-xy$ shown in Fig. 1): upward for a vertical cutting force F_y , to the right for a horizontal cutting force F_x , counter-clockwise for a rotating angle. Cutting conditions were taken as shown in Table 1.

4. Experimental and Calculated Results

4.1 Vibratory properties of workpiece

In preliminary operations, the authors measured the vibratory properties of the workpiece, i.e. the natural frequency and damping coefficient, by disturbing the workpiece by a hammer blow without cutting. Figure 3 shows typical wave forms of deflections x and y , where the upper figure shows the wave form derived by a disturbance of ξ -direction and the lower figure by a disturbance of η -direction. Though a small non-linearity of the damping coefficient exists in Fig. 3, i.e. the smaller an amplitude, the smaller a damping coefficient, this non-linearity was neglected and the viscous damping coefficient was estimated from a wave form at an amplitude of about 0.05 mm. The authors evaluated the spring constant by a ratio of a cutting force to the deflection caused by it. The difference Δk of the two spring constants, k_ξ and k_η was in good agreement with the difference calculated from natural frequencies in both the ξ and η directions. Table 2 shows these vibratory parameters.

4.2 Characteristics of cutting force

Under stable cutting conditions, the authors measured cutting forces and deflections of the workpiece from the start of the cutting operation and obtained relations between the cutting forces F_x and F_y and the instantaneous depth of cut in a short time. Figures 4

Table 1 Cutting condition

Width b (mm)	Chip thickness d_o (mm/rev)	Cutting speed V_o (m/min)
2.0	0.05 ~ 0.25	100 ~ 200
3.0	0.025 ~ 0.25	75 ~ 200

Fig. 2 Block diagram of measuring method

Fig. 3 Natural vibrations of workpiece

Table 2 Vibratory properties of workpiece

Mean value of natural frequency	$\rho/2\pi$ (Hz)	170
Spring constant	k (MN/m)	6.8
	Δ_k	-0.06
Damping coefficient	n/p	0.073
	Δ_c	0.35
Equivalent mass	m (kg)	5.96
Angle	α (°)	-10

(a) $b=2\text{mm}$ (b) $b=3\text{mm}$

Fig. 4 Effect of depth of cut on cutting forces

show the typical relations for various feed rates, d_0 , and for workpieces of 2 or 3 mm in width at a constant cutting speed of 150 m/min. In Fig. 4, cutting forces are related only to the instantaneous depth of cut and not to the feed rates. As pointed out by Albrecht⁽⁸⁾ the cutting force F_x , and also F_y , approaches a straight line. Fig. 5 shows a plot of the cutting force versus cutting speed at 100, 150 and 200 m/min for workpieces of widths 2 and 3 mm at a

Fig. 5 Effect of cutting speed on cutting forces

constant feed rate of 0.125 mm/rev. In this figure, both components of the cutting force decrease by 20% with a 100% increase of the cutting speed from 100 to 200 m/min. Indeed, Fig. 5 shows that the cutting forces changed only by 2% for a change of cutting speed by 10%, which was the maximum change of cutting speed in the experiments; hence, the authors neglected the effects of vertical velocity $\dot{y}$ on the cutting force, and applied the relation between the cutting forces and the depth of cut (Fig. 4) to Eq. (6).

4.3 Growth of chatter vibration

Figures 6 show the typical experimental results of the horizontal deflection of the workpiece. In these figures, the abscissa is the rotating angle. When $\theta=0$, the cutting operation starts and when θ is in a range of $2\pi N \leq \theta < 2\pi(N+1)$, θ is defined as the N th rotation, where $N=0, 1, 2, \dots, 7$. The ordinate is the horizontal deflection x . As integer N increases, x is displaced downward by Nd_0 . Figures 6 (a) and (b) show examples with chatter, and Fig. 6 (c) without chatter. In Fig. 6 (b), small disturbances of amplitude of about $2\mu\text{m}$, which appear at points A1, A2 and A3 but disappear in a short time, increase its amplitude with rotation by rotation, due to a regenerative effect. But these small disturbances also exist even in a stable cutting condition (see points B1, B2, B3 in Fig. 6 (c)). In previous researches on regenerative chatter, many researchers paid attention only to the characteristics of chatter vibration when it had grown up, due to the difficulty of measurement. From these experimental facts, the authors concluded that regenerative chatter grows in the following fashion: first, small changes of the cutting force, which may be caused by inclusions in a material or the lack of uniformity of the material and exist even in stable cutting conditions, cause small deflections of the workpiece which disappear in a short time and leave short wavinesses on the workpiece surface; second, in an unstable

(a) $b=2\text{ mm}$, $d_o=0.05\text{ mm/rev}$ (b) $b=3\text{ mm}$, $d_o=0.05\text{ mm/rev}$ (c) $b=2\text{ mm}$, $d_o=0.125\text{ mm/rev}$

Fig. 6 Growth of horizontal deflection of workpiece
(Experimental results where $V_0=150\text{ /min}$)

(a) $b=2\text{ mm}$ (b) $b=3\text{ mm}$

(Calculated with the vibratory parameters shown in Table 2 and the following initial conditions:

$$x(0)=x_0+2\text{ }\mu\text{m}, y(0)=y_0, \dot{x}(0)=\dot{y}(0)=0\text{ m/s}$$

Fig. 7 Growth of horizontal deflections of workpiece
(Calculated results where $d_o=0.05\text{ mm/rev}$ and $V_0=150\text{ m/min}$)

cutting condition, these small wavinesses increase their amplitude and spread over the workpiece surface, rotation by rotation, due to a regenerative effect. By now, the assumption that waviness of a small amplitude existed uniformly on the whole surface of a workpiece at a stability limit and increased its amplitude in unstable cutting conditions was accepted. But above experimental results show that it is not a good assumption in the usual cutting condition where rigidity, the damping coefficient, and the natural frequency of a workpiece are all similarly high as this experiments.

Next, the authors compared these experimental results with the calculated results for a case where the workpiece suffered a small disturbance. Figures 7 (a) and (b) show the results calculated numerically using the Gear method from Eq.(6) under the following conditions: (a) vibratory parameters were as shown in Table 2; (b) cutting conditions were the same as

those for Fig.6(a) and (b); (c) initial conditions were: $x(0)=x_0+2\ \mu\text{m}$, $y(0)=y_0$, $\dot{x}(0)=\dot{y}(0)=0\ \text{m/s}$, where x_0, y_0 were steady state deflections of the workpiece for $d=d_0$. The similarities between Fig.6(a) and Fig.7(a), and also between Fig.6(b) and Fig.7(b), confirmed that Eq.(6) represented well the actual system shown in Fig.1, and that a small disturbance which existed even in stable cutting conditions could cause regenerative chatter. But there remained small differences between experimental and calculated results. It is thought that these differences were caused by the fact that the authors neglected the effects of the radius of the cutting edge and the friction at the cutting edge and that they didn't use the dynamic value of the vibratory characteristics of the workpiece.

Figure 8 shows how deflections of the workpiece x and y , the cutting forces F_x and F_y and the energies of vibratory system E and E_s change during a period of 50 ms. In this figure, an instant ($t=0$) corresponds to a rotation angle $\theta/2\pi=6.75$, as in Fig.6(b). Here, the total energy E is the input energy due to cutting forces subtracted by dissipation energy, and the energy of the vibratory system E_s is a sum of the elastic energy and the kinetic energy of the workpiece, letting E and E_s be equal at $t=0$. The calculations and comparisons of E and E_s had the following meanings: first, the authors could relate deflections of the workpiece or cutting forces to the change of total energy E . Hence, if E had a different value after one rotation, then E made the amplitude of deflection increase or decrease; second, it was a good criteria in order to estimate the accuracy of the experiments and the propriety of vibratory parameters of the workpiece. Because E was calculated with values of F_x , F_y , x , y , k , Δ_k , etc. and E_s with m , k , Δ_k , $\dot{x}$, $\dot{y}$, etc., if these values were not of good accuracy, E may have

(Experimental results where $b=2\ \text{mm}$, $d_0=0.05\ \text{mm/rev}$, $V_0=150\ \text{m/min}$)

Fig. 8 Changes of workpiece deflections, cutting forces and total energies

had quite a different value from E_s under a certain condition.

Figure 8 shows a small difference between E and E_s , hence, it shows the accuracy and good reasonability of vibratory parameters shown in Table 2. Further, Fig.9 shows the following relations between deflections and cutting forces: (1) the workpiece moves in the clockwise direction along an oblique line with a positive slope [upper left figure]; (2) the cutting force changes along an oblique line with a positive slope in the counter-clockwise direction [lower left figure]; (3) the right two figures, both upper and lower, show loci in the clockwise direction along an oblique line with a negative slope. This means that cutting forces increase the energy of the vibratory system.

Though these characteristics have been pointed out by many researchers, the regenerative effect on the chatter vibration and the relations between the cutting forces and the deflections of the workpiece were not sufficiently determined.

4.4 Results with disturbance by a hammer blow

Under stable cutting conditions, the workpiece was disturbed by a hammer blow. How this disturbance affected the cutting operation during subsequent rotations was then measured. Figure 10(a) shows typical experimental results under the condition that $b=2\ \text{mm}$, $d_0=0.125\ \text{mm/rev}$, $V_0=150\ \text{m/min}$ and the workpiece was disturbed at $\theta=0$. Similarly, Fig.10(b) shows a calculated result under the same

(Experimental results where $b=2\ \text{mm}$, $d_0=0.05\ \text{mm/rev}$, $V_0=150\ \text{m/min}$)

Fig. 9 Relations between workpiece deflections and cutting forces

(a) Experimental result

(b) Calculated result

(Calculated with the vibratory parameters shown in Table 2 and the following initial conditions:

$$x(0)=x_0, y(0)=y_0, \dot{x}(0)=0.067 \text{ m/s}, \dot{y}(0)=0 \text{ m/s}$$

Fig. 10 Growth of horizontal deflection of workpiece

$$(b=2 \text{ mm}, d_0=0.125 \text{ mm/rev}, V_0=150 \text{ m/min})$$

(a) Experimental results

(b) Calculated results

$$(b=2 \text{ mm}, d_0=0.125 \text{ mm/rev}, V_0=150 \text{ m/min})$$

Fig. 11 Changes of workpiece deflections, cutting forces, and energies

cutting conditions, with initial conditions such that: $x(0)=x_0, y(0)=y_0, \dot{x}(0)=0.067 \text{ m/s}$ and $\dot{y}(0)=0 \text{ m/s}$, where x_0, y_0 have the same definition as in Fig.7. Figure 10(b) shows that a chatter mark left by an initial disturbance on a workpiece surface spread over the workpiece surface, rotation by rotation, due to a regenerative effect similar to that in Fig.10(a), and it gives us a key for investigating chatter vibrations analytically. Figure 11(a) shows, in more detail, the characteristics of regenerative chatter represented in Fig.10(a), i.e., how deflections of the workpiece x and y , the cutting forces F_x and F_y and energies of the system E and E_s change during 50 ms. At a glance, it was found that a good agreement between E and E_s . It was then verified that the experiment was also in good accuracy under the condition where a workpiece suffered a disturbance by a hammer blow. In Fig.11

(a) and (b), an instant ($t=0$) corresponds to the rotation angle $\theta/2\pi=1$, as in Fig.10(a) and (b). Figures 12(a) and (b) show the relation between the deflections x and y and the cutting forces F_x and F_y represented in Fig.11(a) and (b). However, it has not yet been determined whether the workpiece moves in a clockwise or counterclockwise direction. But now it could be explained through Eq.(6), why this contradiction took place. First, let $\alpha=0^\circ$ in Eq.(6) and consider the situation where no elastic coupling exists between two axes x and y . Equation (6) tells one that the horizontal cutting force acts as a kind of spring force, increasing the horizontal natural frequency. However, the vertical cutting force acts as an external force, thus causing no change in the natural frequency in the vertical direction. Further, Eq.(6) indicates that if, under a cutting operation, Δ_k has a

($b=2$ mm, $d_0=0.125$ mm/rev, $V_0=150$ m/min)

Fig. 12 Relations between workpiece deflections and cutting forces

negative value sufficient to keep the vertical natural frequency higher than the horizontal one, which makes a phase lag of the vertical displacement behind the cutting force smaller than that of the horizontal displacement and then makes the workpiece move in the clockwise direction. Further, the authors estimated the effect of Δ_k and α on chatter vibration by evaluating Eq.(6) numerically for various values, and verified the effect of Δ_k and found a secondary effect of α on chatter vibration.

Finally, the upper right and lower right figures in Fig.12(a) and (b) show cutting forces increase their value initially and then make the workpiece move in clockwise direction along a line with negative slope. In this case, deflections lag 130° in phase behind the cutting forces (see Fig.11(a),(b)). Now a good agreement was shown between experimental results and results calculated numerically from Eq.(6) with the cutting force characteristics shown in Fig.4 to give a key to the investigation of the transient behavior of regenerative chatter vibration. However, the dynamics of the cutting forces shown in Fig.12(b) are somewhat different from that in Fig.11(b) because the authors considered only the static characteristics of cutting force.

5. Conclusions

In this paper, the following results were present-

ed:

(1) The relation between the cutting force and the depth of cut was recognized by measuring these values from the start of the cutting operation. For this purpose, it took only two seconds.

(2) A workpiece was found to vibrate as a result of small disturbances of the cutting force, which exist even in stable cutting conditions. A workpiece commenced chatter when these vibrations were magnified by the regenerative effect.

(3) Relations between deflections x and y and cutting forces F_x and F_y were recognized under a condition such that chatter vibration increased its magnitude by the regenerative effect. For this purpose, the authors disturbed the workpiece by a hammer blow and measured these values.

(4) Total energy E , the input energy due to the cutting force subtracted by the dissipation energy, and the energy of vibratory system E_s , which was the sum of elastic and kinetic energy of the workpiece, were in good agreement when chatter vibration increased its magnitude. This fact confirmed the propriety of the vibratory parameters and the good accuracy of the experiments.

(5) Calculated results, especially concerning the relations between the deflection of the workpiece and the cutting force and the succession for deflection of the workpiece to increase its amplitude, were in good

agreement quantitatively with experimental results. Here, calculations were performed for two cases: first, a workpiece suffered a small disturbance of displacement; second, a workpiece was disturbed by a hammer blow.

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