Experimental Study on the Dynamic Buckling of Cylindrical Tanks*
(Comparison Between Static Buckling and Dynamic Buckling)

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Buckling failures are often observed in tanks subjected to seismic load. In this paper, dynamic buckling is studied, using small, plastic cylindrical tanks made of polyester sheets. Static buckling tests on the fluid-filled tanks subjected to lateral load, and dynamic buckling tests under harmonic excitation using a shaking table are carried out, and the buckling modes are compared. The stress resulting from the frequency response (the circumferential wave numbers \(n=1\) for horizontal excitation and \(n=0\) for vertical excitation) is calculated using an axi-symmetric shell finite element program, which includes the effect of fluid structure interaction. Static buckling stress criteria are applied to the above calculations in order to predict the occurrence of buckling. The predictions are compared with the test results and they prove to be fairly satisfactory in the restricted region the natural frequency of the fundamental mode.

**Key Words**: Experimental Stress Analysis, Dynamic Buckling, Lateral Load, Horizontal Excitation, Vertical Excitation, Shaking Table, Fluid-filled Cylindrical Tank

1. Introduction

Buckling is a type of damage caused by earthquakes which affects cylindrical containers for liquids, such as storage tanks for wine, petroleum, or LNG and pressure vessels.

The usual, proposed buckling criterion is static in nature. The present method for converting seismic loads to static loads can be classified into three types: namely, lateral load buckling tests\(^{(1)}\), in which a horizontal force is applied on the tank tops; tilt buckling tests, in which buckling is caused by injecting liquids into cylinders in an inclined state\(^{(2)-(3)}\); and centrifugal force buckling tests, which permit liquid-containing cylinders to rotate or swing around. Also, other types of dynamic buckling tests\(^{(4)-(5)}\) are conducted using shaking tables. Niwa et al.\(^{(6)}\) conducted tests on wine storage tanks with simulated earthquake accelerations, and the buckling patterns observed were similar to those observed in actual tanks. On the basis of the experimental results, the following experimental Eq. (1) was obtained to evaluate the axial compression membrane stress \(\sigma_{cr}\), which causes diamond-shaped buckling in tanks, making them free to rise at the base of the tank wall.

\[
\sigma_{cr} = 0.373 \frac{Eh}{R}
\]

where \(E\) denotes the modulus of elasticity, \(R\) the radius, and \(h\) the shell thickness.

In another study, Shih et al.\(^{(7)}\) fabricated small, thin-walled cylinders with a polyester sheet based on the rule of similarity. Water was poured into these cylinders and sinusoidal wave vibrations were applied, using a shaking table. Then, the relations between the frequencies and the acceleration levels where buckling is actually caused (hereafter referred to as buckling acceleration) were clarified. Further, by assuming \(n = 1\) for circumferential wave numbers, the relations between frequency and buckling acceleration were calculated. The results were compared with actual
Experimental results. Through this study, it was clarified that the following buckling stress Eq. (2), can be applied to cylindrical tanks with fixed bases:

$$\sigma_{cr} = \sigma_{cr} = \frac{1}{\sqrt{3(1-\nu^2)}} \frac{Eh}{R}$$  (2)

where, $\sigma_{cr}$ represents the buckling stress under stress under static, axial compression loads. Also, $\nu$ denotes the Poisson ratio, while $E$, $R$, and $h$ are the modulus of elasticity, the radius, and the shell thickness, respectively.

Further, the present author, Nagashima et al. conducted horizontal excitation tests on a fluid-filled, thin-walled cylinder made of aluminum (radius of 500 mm, length of 2,000 mm, and thickness of 0.8 mm). Two factors were clarified through these tests: one was that two types of buckling modes were created; local buckling and diamond-shaped buckling. Local buckling occurred at areas slightly below the water surface, and was caused by negative pressure under coupled vibration. The diamond-shaped buckling occurred in the region of the fixed cylinder end, and was caused by bending. The other factor discovered related to the clarification of the generating mechanism of local buckling. This was achieved by conducting buckling analysis which considered initial stress.

In this study, small, thin-walled polyester cylinders, similar to those used by Shih et al., were employed as test cylinders. First, variations in buckling modes created by changes in liquid levels through conducting buckling tests under static, lateral loads were checked. Next, using a shaking table, the cylinders were vibrated horizontally and vertically. Through this test, relations between frequencies and buckling accelerations, and between excitation directions and buckling modes were studied. By such studies, we were able to investigate the applicability of buckling evaluation equations under static loads to evaluate dynamic buckling.

2. Material Constants and Test Tank Dimensions

The cylindrical tanks adopted for our tests were fabricated using commercially available polyester sheets (Product name: Mitsubishi Diafoil) to wrap the cylinders. Tensile test specimens shown in Fig. 1 were prepared, to facilitate the measurement of the material constants of these polyester sheets. Two types of specimens were made per sheet: one in the rolling direction and one perpendicular to the roll direction (the transverse direction), so that the anisotropy of the test pieces could be properly taken into account. The pieces were made in three thicknesses. Both ends of the test pieces were reinforced with acrylic plate 3 mm thick. On the sides of both reference lines, the 5 mm-wide band-form sections were coated with black paint as targets areas for a non-contact displacement meter. A tensile load $F$ was applied to these test pieces, and the elongation $\delta$ between the reference lines was measured with two units of an optical-type, noncontact displacement meter $\beta$, as shown in Fig. 2. The modulus of elasticity $E$ was calculated on the basis of the relations with the tensile load $F$ measured by the load cell $\delta$. Poisson ratios were not obtained through the measurements; instead, the conventionally used value of $\nu=0.3$ was employed. Measurement results are shown in Table 1.

As can be seen in Fig. 3, test cylinders were fabricated by applying precision machining to thin-walled steel pipes with a radius of 80 mm and a length of 325 mm. This was followed by wrapping with a polyester sheet and attaching and bonding the same 5 mm-wide sheet on both ends from the exterior. On the cylinder's upper end, a 3 mm-thick acrylic end plate was bonded with an epoxy adhesive. The lower end of the cylinder was fixed using a method similar to Ref. (5); namely, by fitting a low-melting-temperature alloy (Cerroloow-136) to grooves prepared on steel disk plates, as shown in Fig. 3. The properties and dimensions of the test cylinders are listed in Table 2. The modulus of elasticity $E$ appearing in this indicates the average value between the rolling direction and the transverse direction shown in Table 1. Also, $Z$

![Fig. 1 Tensile test specimen](image1)

![Fig. 2 Measurement block diagram of tensile tests](image2)

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**Note:** The image references have been replaced with placeholders as actual images are not provided. The text references are placeholders for the actual figures and equations mentioned.
represents the general length-range parameter for cylinders; that is, \( Z = \sqrt{1 - \nu^2} L^2 / R h \).

3. Lateral Load Static Buckling Tests

3.1 Experimental setup and method

A simplified, structural diagram of the experimental setup is shown in Fig. 4. The item indicated by ① is a test cylinder, ② is a support that can adjust the load device height, and ③ is a handle for moving the upper end of the test cylinder by a displacement \( \delta \). Further, ④ is a ball spline bearing to prevent axis rotation, ⑤ is a load cell for measuring lateral load \( Q \), ⑥ is a link-ball joint, and ⑦ is a rod-end joint. Item ⑧ is a strain-gauge-type displacement meter for measuring lateral displacement \( \delta \), and ⑨ is a dynamic strain meter. Items ⑧ and ⑨ are provided to prevent bending or twisting caused by the misalignment between the center of the cylinder and the load device while installing the test cylinder.

Water was used as the test liquid. In each test cylinder, the water level was increased by 10 mm per series, and variations in the relations between the lateral load \( Q \) and the lateral displacement \( \delta \) were recorded with an \( X-Y \) recorder ⑩.

3.2 Experimental results and discussions

As an example, the relation between the lateral

Table 1 Tensile test results

<table>
<thead>
<tr>
<th>Test piece No.</th>
<th>TP-1-1</th>
<th>TP-1-2</th>
<th>TP-1-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (mm)</td>
<td>200.3</td>
<td>200.7</td>
<td>200.0</td>
</tr>
<tr>
<td>b (mm)</td>
<td>15.03</td>
<td>14.98</td>
<td>14.91</td>
</tr>
<tr>
<td>h (mm)</td>
<td>0.051</td>
<td>0.098</td>
<td>0.240</td>
</tr>
<tr>
<td>E (GPa)</td>
<td>5.37</td>
<td>5.40</td>
<td>5.30</td>
</tr>
<tr>
<td>[kgf/mm²]</td>
<td>[548]</td>
<td>[551]</td>
<td>[540]</td>
</tr>
<tr>
<td>e (mm)</td>
<td>200.4</td>
<td>200.4</td>
<td>200.5</td>
</tr>
<tr>
<td>b (mm)</td>
<td>14.99</td>
<td>14.98</td>
<td>15.09</td>
</tr>
<tr>
<td>h (mm)</td>
<td>0.049</td>
<td>0.099</td>
<td>0.243</td>
</tr>
<tr>
<td>E (GPa)</td>
<td>5.65</td>
<td>5.68</td>
<td>5.36</td>
</tr>
<tr>
<td>[kgf/mm²]</td>
<td>[576]</td>
<td>[579]</td>
<td>[547]</td>
</tr>
</tbody>
</table>

Table 2 Material properties and tank dimensions

<table>
<thead>
<tr>
<th>Test Tank No.</th>
<th>TC-005</th>
<th>TC-010</th>
<th>TC-025</th>
</tr>
</thead>
<tbody>
<tr>
<td>R (mm)</td>
<td>80.0</td>
<td>80.0</td>
<td>80.0</td>
</tr>
<tr>
<td>L (mm)</td>
<td>320.0</td>
<td>320.0</td>
<td>320.0</td>
</tr>
<tr>
<td>h (mm)</td>
<td>0.050</td>
<td>0.098</td>
<td>0.241</td>
</tr>
<tr>
<td>E (GPa)</td>
<td>5.51</td>
<td>5.54</td>
<td>5.33</td>
</tr>
<tr>
<td>[kgf/mm²]</td>
<td>[562]</td>
<td>[565]</td>
<td>[544]</td>
</tr>
<tr>
<td>ν</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Z</td>
<td>24421</td>
<td>12210</td>
<td>4884</td>
</tr>
</tbody>
</table>

Fig. 4 Experimental setup for lateral load static buckling tests

Fig. 5 Relations between lateral load \( Q \) and lateral displacement \( \delta \) under various values of liquid level ratio \( l \).
load $Q$ and the lateral displacement $\delta$ is shown in Fig. 5. This figure also shows the experimental results concerning cylinder TC-025. In cylinder TC-025, while the liquid level ratio $l_h$ (water level/full cylinder length) remained 0 and 0.63, shear buckle in the slant direction on one side of the cylinder began to occur at point "s". This deformation extended further with an increase of the lateral displacement $\delta$. However, when this shear buckle reached the upper-end plate, further extension of the deformation was prevented, resulting in increased cylinder rigidity. Further, when the lateral displacement $\delta$ was increased, similar shear buckle occurred at point "s" on the other side of the cylinder. Subsequently, at point "b", diamond-shaped buckle (hereafter referred to as bending buckle) occurred on the compressed side of the cylinder. As the water level rose, the static hydraulic pressure caused the elevation of the positions where both the shear buckle and bending buckle occurred. When the liquid level ratio reached $l_h=0.97$, shear buckle no longer appeared, and bending buckle started developing immediately around the fixed end.

In noticed. However, in cylinder TC-005, whose shell thickness was very thin, symmetrical buckling failed to appear when the water level was low.

The relations between the lateral load $Q$ and the water level $l_h$, which triggered development of the shear buckle and the bending buckle are shown in Fig. 6. On the basis of this figure, it is evident that both shear buckling and bending buckling, along with the elevation of the water level $l_h$, dramatically intensified the buckling resistance. However, when the water level $l_h$ was low, or when bending buckle occurred immediately around the fixed end under high water conditions, the intensification of the buckling resistance resulting from static hydraulic pressure was limited. This situation was created when the water level was low, and shear buckle occurrence positions were higher than the water levels, as a consequence of being free from the influence of static hydraulic pressure. In the case where shear buckling did not occur under high water level conditions, it is conceivable that the influences caused by the static hydraulic pressure enhancing the bending buckling possibilities were not sufficiently strong.

With regard to cylinder TC-010, Figs. 7 and 8 show photographs of the deformed status after buckling occurred under respective liquid level ratios of $l_h=0.63$ and $l_h=0.94$. A photo showing the shear buckle and the bending buckle that developed on the upper cylinder side is shown in Fig. 7. The elevation of the buckling positions caused by static hydraulic pressure is clearly observable. On the other hand, the deformation status caused by bending buckling that occurred from the start, as a result of static hydraulic pressure, prevented shear buckling from occurring, as shown in Fig. 8. The bending buckle wavelength, shown in Fig.
8, is shorter than the bending buckle wavelength indicated in Fig. 7. Figures 7 and 8 suggest that when the water level exceeds a certain limit, shear buckling can be prevented by the effects of circumferential tension membrane stress. This stress is created by static hydraulic pressure, thus causing bending buckling to occur immediately.

4. Harmonic Excitation Buckling Tests

4.1 Measuring method and results on damping ratio

Cylinders TC-010 and TC-005 were used and water was employed as the test liquid. Under liquid level ratios of \( l_0 = 0.9, 0.8, \) and 0.7, the damping ratios \( \xi \) were measured using the free vibration method. Measured results are shown in Table 3. The “Up” appearing in the table, indicates the status where the shaking table floats as a result of the hydraulic-oil pressure. The “Down” denotes the condition where the shaking table is still. By comparing the values of \( \xi \) for TD-1 and TD-2 of the table, we are able to discern that the values differ from each other in the shaking table’s “Up” and “Down” conditions. Therefore, in the calculations explained hereafter, we shall adopt the values for the “Up” status, which more closely represent the conditions of harmonic excitation.

4.2 Experimental setup and method

A block diagram of the experimental setup is shown in Fig. 9, where ① indicates a test cylinder, ② a shaking table, and ③ a hydraulic oil actuator, ④ is a control panel, ⑤ is a function generator, ⑥ is a vortex-current-type noncontact displacement meter sensor, ⑦ denotes an aluminum piece used as a displacement meter target, ⑧ is a displacement meter unit, ⑨ a strain-gauge-type acceleration pickup for measuring shaking table acceleration, and ⑩ is a dynamic strain meter. The displacement and acceleration measured by these instruments was recorded in the data recorder ⑪, and output was produced, as required, with the paper oscillograph ⑫.

Sinusoidal waves were employed as the excitation waveform, and relations between the excitation frequencies and accelerations were sought. With regard to both the horizontal and vertical directions as the excitation direction, buckling occurrence was evaluated by the visually observed deformation status and the generated noise levels.

4.3 Frequency response analysis

By assuming that the employed fluid is an incompressible, nonviscous liquid, the dominant equation for generated pressure distribution becomes as follows:

\[ \rho \frac{\partial^2 p}{\partial t^2} = 0 \]  \hspace{1cm} (3)

where, \( \rho \) denotes the Laplacian.

The following equation can then be obtained by discretizing Eq. (3) in the finite element manner:

\[ (p) = -([H]^{-1}[S] \frac{\partial^2}{\partial t^2} (\delta)) \]  \hspace{1cm} (4)

where, \( p \) and \( \delta \) denote the pressure vector and displacement vector of the node, respectively, \([H]\) is the fluid stiffness matrix, and \([S]\) represents the matrix that constitutes the relation between the force and the displacement on the boundary surface between the fluid and structure.

As for the structure, the following equation can be obtained by applying the same discretization process and excluding the damping and external force terms.

\[ ([K] + [K_c])(\delta) + ([M] \frac{\partial^2}{\partial t^2} (\delta)) = 0 \]  \hspace{1cm} (5)

where, \([K]\) denotes the structure stiffness matrix, \([K_c]\) the geometric stiffness (initial stress) matrix, \([M]\) the structure mass matrix, and \( \rho \) the fluid mass.

Next, by substituting Eq. (4) into Eq. (5), the following equation can be obtained:

\[ ([K] + [K_c])(\delta) + ([M] + [M_a]) \frac{\partial^2}{\partial t^2} (\delta) = 0 \]  \hspace{1cm} (6a)

where,

\[ [M_a] = -\frac{1}{\rho} [S]^T [H]^{-1} [S] \]  \hspace{1cm} (6b)

and \([M_a]\) is the reduced mass matrix, generally ter-

Table 3 Measurement results of test tank damping ratio

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Shake table</th>
<th>Test tank No.</th>
<th>( l_0 )</th>
<th>( \xi ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD-1</td>
<td>Down</td>
<td>TC-010</td>
<td>0.9</td>
<td>0.72</td>
</tr>
<tr>
<td>TD-2</td>
<td></td>
<td></td>
<td>0.9</td>
<td>0.88</td>
</tr>
<tr>
<td>TD-3</td>
<td></td>
<td></td>
<td>0.8</td>
<td>1.2</td>
</tr>
<tr>
<td>TD-4</td>
<td></td>
<td></td>
<td>0.7</td>
<td>0.69</td>
</tr>
<tr>
<td>TD-5</td>
<td></td>
<td></td>
<td>0.9</td>
<td>1.5</td>
</tr>
<tr>
<td>TD-6</td>
<td></td>
<td>TC-005</td>
<td>0.8</td>
<td>1.7</td>
</tr>
<tr>
<td>TD-7</td>
<td></td>
<td></td>
<td>0.7</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Fig. 9 Diagram of experimental setup for harmonic excitation dynamic buckling tests

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med an added mass matrix.

Consequently, the characteristic frequency equation of Eq. (6) becomes as follows:

$$\det([K] + [K_c]) - \omega^2([M] + [M_c]) = 0$$  \hspace{1cm} (7)

By solving Eq. (7) using the subspace method, the characteristic frequency $\omega$ and the eigen vector $\{\phi_i\}$ $(i=1, \cdots, n)$ are obtained. Next, assuming that the damping matrix $[C]$ satisfies the relations expressed by the following equation, the frequency response can be obtained by applying the mode superposition method while considering the external force term as the sinusoidal-wave function:

$$\{\phi_i\}^T[C]\{\phi_i\} = 2\omega_i^2 \xi \delta_{ii}$$  \hspace{1cm} (8)

where $\xi$ is the mode damping parameter, and $\delta_{ii}$ is Kronecker's delta.

The employed finite element is an axi-symmetric element, as shown in Fig. 10. The circumferential displacement distribution is formed as $\sin n\theta$ or $\cos n\theta$, in terms of angle $\theta$. Here, $n$ represents the number of circumferential waves. The fluid element is the isoparametric axi-symmetric shell element with 4 nodes, while pressure variations on the element boundary are linear. Further, the shell element interpolates the in-plane displacement with a zero-order Hermite multinomial expression, and the out-of-plane displacement with a first-order Hermite multinomial expression. Meanwhile, Novozhilov's expressions are used as the relations between the displacements and strains.

An example of the calculation results of the characteristic frequency is shown in Fig. 11(a). This figure shows the axial displacement distribution, the pressure distribution, and the axial membrane stress distribution under the conditions of circumferential wave number $n=1$ and axial mode number $m=1$. Also shown in Fig. 11(b) is the distribution of displacements, pressure, and axial membrane stress under the condition of frequency $f=0$; namely, when constant acceleration was applied. All these values are normalized to nondimensional numbers in terms of their maximum values.

An example of the calculation results on frequency response is shown in Fig. 12. This example was obtained under the following conditions: shell thickness of $h=0.05$ mm, liquid level ratio of $L_0=0.9$, and damping ratio of $\zeta=1.5\%$. The results pertain to the axial membrane stress $\sigma$ at the lower end of the cylinder. In this case, the circumferential wave number $n=1$ was adopted. In this figure, the lateral axis indicates the excitation frequency $f$, while the longitudinal axis represents the absolute value $|\sigma|$ of the axial membrane stress generated at the lower end of the cylinder when a horizontal acceleration of 1 Gal was applied. Under the first-order characteristic frequency of $f_c=26.0$ Hz, the distribution, as shown in Fig. 11(a), was created. Under the frequency of $f=0$ Hz, the distribution illustrated in Fig. 11(b) was demonstrated. Within the intermediate frequency region, the distribution was shown to be in linear combination.

4.4 Buckling evaluation method

In our study, it was assumed that a static buckling evaluation equation can be applied to predict the buckling caused by excitation. On the basis of this assumption, we considered that buckling develops when stress values obtained by frequency response analysis become equal to those of the stress determined by the static buckling evaluation equation. Acceleration resulting from this equalization was employed to denote buckling acceleration. The values of this buckling acceleration were compared with the actual buckling acceleration obtained through experiments.

4.5 Experimental results and discussions

![Fig. 11 An example of displacement distribution, pressure and axial membrane stress](image-url)
When excited in the horizontal direction, two types of buckling phenomena were observed. In the region of the cylinder’s lower end, diamond-shaped buckling (termed buckling mode 1) was observed, having been caused by axial compression membrane stress. On the other hand, around the entire circumference of the cylinder’s upper side, buckling that was apparently caused by circumferential compression membrane stress occurred. This latter type of buckling (termed buckling mode 2) was difficult to distinguish from the vibration mode. An example of a photograph showing buckling deformation under mode 1 is shown in Fig. 13, while, the photograph in Fig. 14 reveals buckling deformation under mode 2 conditions. However, no shear buckle, which was observed in the lateral load buckling tests, appeared in these experiments.

The distribution of bending moment $M/M_{\text{max}}$ and shear force $S/S_{\text{max}}$ is illustrated in Fig. 15. The vibration mode conditions were assumed to be circumferential wave number $n=1$ and axial vibration mode number $m=1$. Also indicated in this figure are the calculation results under the liquid level ratios of $l_0=0.7$ and 0.9. No meaningful differences were evident between the shell thicknesses of $h=0.050$ mm and $h=0.098$ mm. For the sake of contrast, the distribution occurring when concentrated lateral loads were applied on the cylinder’s upper end is also indicated. On the basis of this figure, it is conceivable that when horizontal excitation was applied, static hydraulic pressure was strong at the cylinder’s lower end whereas shear force was large, resulting in the prevention of shear buckling. On the other hand, both the static hydraulic pressure and the shear force were small at the cylinder’s upper end, and this must have prevented shear buckling. Thus, in the horizontal excitation buckling tests, shear buckling did not occur; however, it did appear in the lateral load buckling tests.

An example of the relation between the excitation frequency $f$ and the buckling acceleration $a_x$ is shown in Fig. 16. Specifically, it shows experimental results concerning test cylinder TC-005 under the liquid level ratio of $l_0=0.9$. The data in this figure can be studied by dividing it into three regions:

- The area where the frequency was low, but was influenced by intense sloshing (hereafter referred to as the low-frequency region).
- The area where the frequency was lower than the

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Fig. 12 A theoretical result of frequency response analysis

Fig. 13 Bending moment distribution $M/M_{\text{max}}$ and shear force distribution $S/S_{\text{max}}$ of test tank

Fig. 14 Mode 1 buckle (TC-010, $f=35$ Hz)

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first-order characteristic frequency and was a little influenced by sloshing influences (hereafter referred to as the medium-frequency region).

- The area where the frequency was higher than the first-order characteristic frequency (hereafter referred to as the high-frequency region).

In the low-frequency region, it is considered that surface motion was intense, and the sloshing influence was more significant than the cylinder vibration itself. Mode 1 (marked with an open circle in the figure) occurred first, followed by mode 2 (marked with a closed circle in the figure) as acceleration was increased. In the medium-frequency region, modes 1 and 2 occurred almost simultaneously under the conditions indicated in Fig. 15. This resulted in excessive local deformation and a large amount of noise generation. On the other hand, as the cylinder wall became thicker, or as the water level fell, buckling mode 2 occurred earlier than buckling mode 1. In the high-frequency region, buckling mode 1 no longer occurred; thus, only buckling mode 2 was observed.

The solid lines in Fig. 16 represent the results of buckling acceleration calculated from the frequency response indicated in Fig. 12. This denotes the acceleration levels required to enable axial compression membrane stress at the cylinder's lower end to equal the buckling stress $\sigma_{cr}$ calculated in Eq. (9).

$$\sigma_{cr} = 1.3\sigma_{ct} = \frac{1.3}{\sqrt{3(1-\nu^2)}} \frac{Eh}{R}$$

(9)

where $\sigma_{ct}$ denotes the classical buckling stress for axial compression, and the coefficient 1.3 indicates an increase in buckling stress caused by nonuniform stress distribution.

Consequently, in our study, a value of 1.3 times the buckling stress calculated from Eq. (2) was considered as the actual buckling stress. Also, although the consideration of static hydraulic pressure influences was not incorporated into Eq. (9), we observed no occurrence of shear buckling in the lateral load buckling tests. Further, we confirmed that static hydraulic pressure influences on buckling resistance are minimal when bending buckling occurs immediately. For these reasons, the adoption of the Eq. (9) is appropriate. That the analytical values and the experimental values are in close accord is clarified in Fig. 16. However, when the frequency is high, the assumed buckling modes are different from the actual buckling modes. Hence, further studies on this point are believed to be essential.

We believe it has been clarified that within the medium-frequency region, Eq. (9), the static buckling evaluation equation can be adopted to evaluate the buckling phenomena caused by excitation.

In the calculations pertaining to Fig. 16, damping ratios measured by the free vibration method were employed. The calculated values of buckling acceleration under the assumption of damping ratios of $\zeta = 0.5\%$, $1.0\%$, and $2.0\%$ are shown in Fig. 17. From this figure, it can be clarified that buckling acceleration differs only slightly in a highly limited range in the first-order characteristic frequency region. Consequently, under the conditions selected for our experiments, the damping ratio caused only a negligible influence on the buckling acceleration.

Our study included checks on the applicability of a similar method on vertical excitation. A portion of the experimental results of this study is shown in Fig. 18. The buckling deformation in this case can be considered as being entirely buckling mode 2. By conducting a frequency response $(n=0)$ analysis, the following assumptions could be established; that is, buckling occurs when differences between generated
circumferential compression membrane stress and circumferential tension membrane stress caused by static hydraulic pressure become equal to the buckling stress $\sigma_{cr}$ (10) caused by external pressure, as calculated by the following equation:

$$\sigma_{cr} = \frac{K_0 \pi^2 E}{12(1-\nu^2)} \left( \frac{L}{h} \right)^2$$

where,

$$K_0 = \begin{cases} 158 & (h=0.050 \text{ mm}) \\ 114 & (h=0.098 \text{ mm}) \end{cases}$$

(10)

The solid lines in Fig. 18 represent calculation results obtained by assuming the damping ratio to be $\zeta = 1.0\%$. The fact that even by adopting a similar assumption based on the damping ratio of $\zeta = 2.0\%$, the difference between the two cases is negligible, is clearly indicated in Fig. 19. Similarly to horizontally applied excitation, the assumption in Eq. (10) holds true in the frequency region below the first-order characteristic frequency.

5. Conclusions

By employing small, polyester test cylinders containing water, we conducted buckling tests under static, lateral loads and under harmonic excitation generated by a shaking table. The experimental results can be summarized as follows:

(1) From the lateral load buckling tests, it has been clarified that shear buckling first occurs in the test cylinders; but beyond certain water levels, bending buckling occurs first. Both shear buckling and
bending buckling are restricted from developing as the water level rises, concomitant with the elevation of potential buckling positions. However, when the water level is low or when bending buckling occurs first under high water-level conditions, static hydraulic pressure exerts little influence on buckling resistance.

(2) From the harmonic excitation buckling tests using the shaking table, it has been confirmed that the static buckling evaluation equation is applicable to excitation in both the horizontal and vertical directions within the frequency region below first-order characteristic frequencies. The static buckling stress equation of cylinders under pure bending can be utilized for excitation in the horizontal direction, while the static buckling stress equation of cylinders under external pressure can be employed in vertical direction excitation. Further, shear buckling as observed in lateral-load tests did not occur. As an explanation of this phenomenon, we have arrived at the conclusion that under horizontal excitation, shear buckling is prevented because the static hydraulic pressure is strong at the cylinder's lower end, where shear force is also large. At the cylinder's upper end, on the other hand, the static hydraulic pressure is not large but shear force is equally small, again acting as factors to preventing shear buckling.

References

(10) Gerard, G. and Becker, H., p. 46 of Ref. (9).