Rheological Characteristics of Magnetic Fluids*

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Studies of the rheological characteristics of water-, hydrocarbon- and diester-based magnetic fluids were carried out with and without a magnetic field. Experimental studies were made by means of the concentric-cylinder-type rotating viscometer which is operated within a strong magnetic field to clarify the effects of magnetic field, temperature and shear rate on the rheological characteristics of the magnetic fluids. The magnetic fluids are Newtonian in the case of no magnetic field. In the case of applied magnetic field, the flow curve for hydrocarbon-based fluid is still Newtonian. On the other hand, the water-based and diester-based fluids show pseudoplastic behaviour. Empirical formulae are obtained on the basis of the theoretical consideration of the laminar flow between rotating concentric cylinders.

Key Words: Non-newtonian Fluid, Magnetic Fluid, Viscosity, Magnetic Field, Experimental Study

1. Introduction

Research into the engineering applications of magnetic fluids such as fluids with superparamagnetic properties has been advanced in various fields in connection with the development of magnetic fluid seals, levitation and energy conversion. For the development of these devices, it is important to clarify the flow characteristics of the fluids in the magnetic field region. Several fundamental studies of rheology(1)-(6) and pipe flow characteristics have been carried out. Thixotropic behaviour(7) in some magnetic fluids, and an extraordinary increase(8) in pipe flow resistance for a water-based fluid have been reported. However, the rheological characteristics in magnetic fields for wider operating conditions have not yet been adequately determined. It is therefore indispensable that more detailed work be done.

In the present paper, a study of the rheological characteristics of water-, hydrocarbon- and diester-based magnetic fluids is undertaken to clarify the effects of magnetic field, liquid temperature and shear rate on the rheological characteristics. Experimental work was done by means of the concentric-cylinder-type rotating viscometer improved to operate in a strong magnetic field. Furthermore, empirical formulae of the rheological characteristics are obtained on the basis of a theoretical consideration of the increase in apparent viscosity due to the magnetic field.

Nomenclatures

\[
\begin{align*}
\alpha & : \text{particle radius} \\
B & : \text{magnetic flux density} \\
D & : \text{shear rate} \\
H & : \text{magnetic field strength} \\
H_b & : \text{strength of imposed magnetic field} \\
H_e & : \text{effective strength of magnetic field} \\
i & : \text{supply current to electromagnet} \\
I & : \text{sum of moment of inertia of particles per unit volume} \\
k & : \text{Boltzmann constant} \\
L & : \text{Langevin function}
\end{align*}
\]

* Received 20th October, 1986. Paper No.86-0110A
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JSME International Journal

1987, Vol. 30, No. 263
\[ m \text{: magnetic moment of a particle} \]
\[ M \text{: magnetization} \]
\[ n \text{: revolutions of rotor} \]
\[ N \text{: number density of particles} \]
\[ N_e \text{: demagnetizing factor} \]
\[ p \text{: static pressure} \]
\[ (r, \theta, z) \text{: cylindrical coordinates} \]
\[ R_1 \text{: inner radius} \]
\[ R_0 \text{: outer radius} \]
\[ S \text{: internal angular momentum} \]
\[ T \text{: liquid temperature} \]
\[ T_a \text{: Tailor number} \]
\[ v \text{: flow velocity} \]
\[ V \text{: particle volume} \]
\[ \eta \text{: viscosity of magnetic fluid} \]
\[ \eta_0 \text{: viscosity of carrier liquid} \]
\[ \mu_0 \text{: permeability of space} \]
\[ \rho \text{: mass density of magnetic fluid} \]
\[ \rho_e \text{: mass density of particles} \]
\[ \tau \text{: shear stress} \]
\[ \tau_o \text{: relaxation time of particle rotation due to frictional resistance of fluid} \]
\[ \tau_b \text{: Brownian time of rotational diffusion} \]
\[ \phi \text{: volumetric density of particles} \]
\[ \omega \text{: angular velocity of rotating cylinder} \]
\[ \Omega \text{: local angular velocity of fluid} \]

Here, underlines of each quantity indicate vector.

2. Theoretical Consideration

2.1 Basic equations

To analyze the flow of a magnetic fluid inside a rotating viscometer, the following basic equations proposed by Shlimis\[s\] are used. Namely, equation of motion:

\[ (Dy/\lambda t) = - \nabla p + n\lambda_{\nu} + \mu_0 (M \cdot \nabla) H + (1/2\tau_o) \nabla \times (S - \Omega) \]

(1)

equation of internal angular momentum:

\[ (D\Omega/\lambda t) = \mu_0 (M \times H) + (1/\tau_o) (S - \Omega) \]

(2)

relaxation equation of magnetization \( M \):

\[ D M/\lambda t = (1/\lambda) (S \times M) - (1/\tau_a) (M - M_0 H^2)|H| \]

(3)

where

\[ \tau_a = 3 V_{\phi} / kT, \tau_o = d^2 \rho_e / 15 \eta_0 I = 8 \pi \rho / \rho_e N \]

(4)

Also, equilibrium magnetization \( M_0 \) would be expressed by the Langevin function:

\[ M_0 = N \mu L(\xi), L(\xi) = \coth \xi - \xi^{-1}, \xi = \mu_0 m H kT \]

(5)

2.2 Method of solution

The aforementioned basic equations are adapted to the laminar flow of the magnetic fluid between rotating concentric cylinders where magnetic field \( H \) is applied transversely to the axis of the cylinders, as shown in Fig. 1.

2.2.1 The case of \( \Omega r_o \ll 1 \)

According to a consideration similar to the steady flow in a circular pipe\[s\], it is shown that the flow becomes Newtonian with apparent viscosity \( \eta_a = \eta + \alpha r_0 \phi \) in the case of a small shear rate of \( \Omega r_o \ll 1 \).

Where

\[ \phi = (4/3) \pi d^3 n, A = \mu_0 \tau_0 \tau_e M_0 H \]

(6)

If \( v_0 = 0 \) at \( r = r_o \), as a boundary condition, the tangential velocity component \( v_0 \) is expressed in the following equation:

\[ v_0 = (r_0^3 / r^3 - 1) \omega R^2 / (R^2 - r_0^2) \]

(7)

2.2.2 The case of \( \Omega r_o \gg 1 \)

It is difficult to solve the basic equations analytically in the case of \( \Omega r_o \gg 1 \). Thus, the approximate procedure based on the same consideration as for steady flow in a pipe is adopted to solve the equations. When the ratio of the \( x \)-component of the magnetization \( M_x \) to the equilibrium magnetization \( M_0 \) is represented by \( \delta = M_x / M_0 \), the following equations are obtained:

\[ M_x = M_0 \delta \Omega r_o / (1 + A \delta) \]

(9)

\[ (\Omega r_o)^2 \delta = (1 - \delta) (1 + A \delta)^2 \]

(10)

Therefore, it is possible that the increase of the viscosity \( \eta_a \) be approximately regarded in terms of the following equations:

\[ \eta_a = (2/3) \phi_{\phi} A \delta / (1 + A \delta) \]

(11)

\[ \Rightarrow F(A \delta) = E(\mu_0 m H kT, \eta_0 D / \mu_0 MH) \]

(11)

3. Experimental Study

3.1 Experimental apparatus and magnetic fluids used

The effects of imposed magnetic field, shear rate and liquid temperature on the rheological characteristics were studied experimentally in the case of 3 kinds of magnetic fluids. To make clear these effects, the concentric-cylinder-type rotating viscometer was adopted, as schematically shown in Fig. 2. It was improved so as to be able to operate in a strong magnetic field region. The inner cylinder, i.e., the rotor, is made
of phenolic resin: the diameter is 40.10 mm and the height is 60 mm. The shaft which connects the rotor to the measuring drive unit, is made of acrylic resin. Furthermore, a magnetic shield was placed between the measuring drive unit and the electromagnet so that the influence of the magnetic field does not exert on the measuring system. Also, the cup for the sample liquid, i.e., the stator, is made of stainless steel. The liquid temperature inside it can be kept constant within a range of from 0°C up to 85°C with a constant temperature vessel and temperature regulating system. The liquid temperature during the test was always observed by the thermocouple which was incorporated into the bottom of the cup. The electromagnet can impose a magnetic field of up to 0.4 T, by adjusting the supply current.

The axial and tangential distribution of magnetic flux inside the rotating viscometer was measured beforehand by means of a Gaussmeter. The mean value within the effective operating area of the rotor was regarded as being the strength of the applied magnetic field \( H \). Also, the magnetic fluids used here were Ferricollloid W-35 (water-based fluid), HC-50 (hydrocarbon-based fluid) and DES-40 (diester-based fluid) produced by Tokin Co., and MARPOMAGNA FN-40 (hydrocarbon-based fluid) produced by Marumo Yushi-Seiyaku Co. Ltd.

### 3.2 Results and discussion

Measurements of the flow characteristics were made at liquid temperatures of 20, 40 and 60°C in the cases of hydrocarbon-based and diester-based fluids, and 20 and 40°C in the case of water-based fluid. Magnetic field was applied to the magnetic fluid inside the viscometer by adjusting the supply current to the electromagnet \( i \) over a range of from 3 up to 20 A. Shear rate was determined by setting the revolutions of the rotor to a specified value.

A flow curve at a liquid temperature of 20°C is shown in Fig. 3 as an example of the experimental results. It is clearly indicated that each magnetic fluid in the case of no magnetic field showed the flow characteristics of a Newtonian fluid: the shear stress increases linearly with the shear rate. Hydrocarbon-based fluids behaved as Newtonian fluid, even in the case of applied magnetic field. However, water-based and diester-based fluids in the case of applied magnetic field showed the characteristics of a pseudoplastic fluid: the increasing rate of apparent viscosity changes with apparent shear rate. In particular, it is clearly shown that the viscosity of the water-based fluid increased extraordinarily due to the application of magnetic field at the region of small apparent shear rate. This increase is thought to be related closely to an extraordinary pipe flow resistance in the case of

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**Fig. 2** Scheme of experimental apparatus

**Fig. 3** Flow curve (\( T = 20°C \))
the water-based magnetic fluid.

In relation to Tailor instability in the flow inside the viscometer, the Tailor number $T_a$ was estimated by using the values in the case of no magnetic field as the viscosity. The value was obtained as $T_a<40$, except for the special cases of hydrocarbon-based fluid with temperatures of 40 and 60°C, and revolutions of 400 rpm. Thus, it is within the stable region ($T_a<41.3$) that Tailor vortex is not induced. The revolutions of the rotor were changed continuously from the stationary condition to the specified values, so that the relation of the revolutions to the shear stress was determined. The influence due to the Tailor vortex was not observed, even in the case of hydrocarbon-based fluid at revolutions of 400 rpm, since $\Delta \tau/\Delta \theta$ is always constant from much lower revolutions to 400 rpm. Therefore, it can be concluded that the influence of Taylor instability on the measured values is negligible in this case.

From the experimental results, the role of each factor in the rheological characteristics in the above-mentioned theoretical consideration was examined. That is, the following equation based on Eq. (6) is used in the case of hydrocarbon-based fluids:

$$\Delta \eta/\eta = (3/2)\eta_0/\eta_1 \xi (\xi - 1)/[2 + \xi L(\xi)].$$

(12)

Also, the following empirical formula based on Eq. (11)* is provided in the cases of water-based and diester-based fluids:

$$\Delta \eta/\eta = C(\mu_0 mH_e/kT)^{\alpha}(\eta_0/\mu_0 mH_e)^{\beta}$$

(13)

Where $H_e = H_e - N_0 M$, and the demagnetizing factor $N_0$ was taken to be unity since the fluid layer is assumed to be an infinitely wide thin plate magnetized in the normal direction to the surface.

Figure 4 shows the result in the case of hydrocarbon-based fluid FN-40. It is indicated that data on the increasing rate of apparent viscosity agree well with those obtained from the theoretical analysis, where the viscosity of only solvent is used as $\eta_0$. The result in the case of HC-50 is shown in Fig. 5. The curves in this figure represent the predicted values based on the theoretical consideration. The solid line corresponds to the result obtained by using the viscosity of only solvent (kerosine) as $\eta_0$ and the chain line corresponds to the viscosity of kerosine involving oleic acid of 30% mass concentration in accordance with the preparation for HC-50. The agreement of the predicted curve to the experimental data is much better when the measured viscosity of kerosine involving oleic acid is used as $\eta_0$. This result is able to be explained by the fact that with the addition of oleic acid to the solvent, the viscosity of the liquid is considerably higher than that of only the solvent. On the other hand, the agreement of experimental data with the predicted curve estimated by using the viscosity of only the solvent was good in the case of FN-40. It may be explained by the fact that the viscosity of the hydrocarbon as the solvent for FN-40 is higher than for HC-50; thus, the effect of the surfactant contained in the fluid on the viscosity is considered to be rather small.

Figures 6 and 7 show the experimental results for the water-based and diester-based magnetic fluids, respectively. The results regarding the increasing rate of effective viscosity were expressed well by means of the experimental equation (13). The coefficients of Eq. (13) were obtained as follows: $C = 1.218 \times 10^{-5}$, $b_0 = -0.0265$, $b_1 = -0.6305$ in the case of water-based fluid, and $C = 8.247 \times 10^{-5}$, $b_0 = 0.2099$, $b_1 = -0.1153$ in the case of diester-based fluid. In addition, it is shown in Fig. 6 that the increased rate of viscosity due to the magnetic field was extremely high in the region of small shear rate. On the other hand, the predicted value based on the theoretical analysis of Eq. (11) is considerably small compared with the measured val-

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**Fig. 4** Increasing rate of apparent viscosity (FN-40)  
**Fig. 5** Increasing rate of apparent viscosity (HC-50)
Fig. 6 Increasing rate of apparent viscosity (W-35)

The difference in affinity of the surfactant for the solvent is regarded as being one of the reasons why the rheological characteristics in the magnetic field region are different according to the kind of based liquid. Thus, from the point of view of the composition, the consideration of the characteristics was carried out regarding hydrocarbon-based fluid (HC-50) and water-based fluid (W-35) whose compositions are well-known. HC-50 is composed of kerosine as the solvent and oleate ion as the surfactant which adsorbs magnetic particles, i.e. magnetite particles. On the other hand, in the case of W-35, the solvent is water and two kinds of surfactant are used: oleate ion adsorbing magnetic particle and SDBS (sodium dodecylbenzen sulfonate) to form a second adsorption layer outside the oleate ion and obtain stable dispersion. Since oleic acid is easily held in solution in kerosine, oleate ion adsorbing magnetite particles should behave toward kerosine as if the particles do not exist in the solution. Thus, the influence of the adsorbed layer on flow behaviour is thought to be small, even in the case where the magnetic field causes magnetic particles to draw up toward the magnetic line of force. Therefore, introducing the effective diameter to the role of the surfactant layer, it is possible to describe the flow behaviour by means of a theoretical analysis. On the other hand, even in the case of W-35, the second layer surfactant, SDBS, and water show the same relation as oleate ion and kerosine in the case of HC-50. However, since the oleate ion in the adsorbed layer should represent the hydrophobic inclusions in water, the adsorbed layer offers resistance to the rotation of the magnetite particles, which form along the magnetic force line; thus, it results in a peculiar increase in the torque of the rotation. This is also confirmed from the fact that DES-40 shows characteristics similar to W-35, though the result for DES-40 shows poorer dependence on the parameters, in particular, shear rate, than that for W-35: the preparation procedure and composition, except for the solvent, are almost identical with each other, and the oleic acid is held somewhat in solution in the diester. Moreover, in the case of the hydrocarbon-based fluid (FN-40), the physical properties of the constituents of FN-40 are different from those of HC-50. However, it is known that the surfactant coalesces in the solvent. FN-40 is also believed to show the same behaviour as HC-50.

As regards theisotropic behaviour, the magnetic fluids used here did not show the characteristics of a thixotropic fluid with and without magnetic field.

4. Conclusion

Studies of the rheological characteristics of magnetic fluids are made with and without a magnetic field. The results obtained are summarized as follows:

1. Although the apparent viscosity of hydrocarbon-based magnetic fluids increases with an applied magnetic field, the influence of shear rate is not observed. In other words, the fluid shows the flow characteristics of a Newtonian fluid even in the case of an imposed magnetic field. The increase of the viscosity in this case can be described by the following equation based on theoretical considerations:

\[
\Delta \eta/\eta = (3/2) \phi \eta \xi (L(\xi))/[2 + \xi L(\xi)],
\]

\[
\xi = \mu_0 m H_0 k T
\]

2. Water-based and diester-based magnetic fluids show the flow characteristics of a Newtonian
fluid in the case of no magnetic field. However, they show the flow characteristics of a pseudo-plastic fluid in the case of an imposed magnetic field: the apparent viscosity is affected not only by the magnetic field, but also by shear rate. In particular, the viscosity increases extraordinarily due to the magnetic field in the case of water-based magnetic fluid in the region of small shear rate. The experimental results regarding the increase in the viscosity can be represented with the following empirical formulae individually. In the case of water-based magnetic fluid,

$$\Delta \eta/\eta = 1.218 \times 10^{-9} \left( \mu \mu m H_s/kT \right)^{-0.0265} \times (\gamma_0 D/\mu_0 MH_s)^{-0.4305}$$

and in the case of diester-based magnetic fluid,

$$\Delta \eta/\eta = 8.247 \times 10^{-5} \left( \mu \mu m H_s/kT \right)^{0.2099} \times (\gamma_0 D/\mu_0 MH_s)^{-0.1131}$$

(3) Thixotropic behaviour was not observed, neither with nor without the magnetic field in the case of the magnetic fluids measured here.

**Acknowledgements**

The authors wish to express their thanks to Mr. T. Oyama, Mr. T. Sato and Mr. H. Oizumi of the Institute of High Speed Mechanics, Tohoku University for their assistance. The authors are thankful to the staff of the Tokin Co. and the Matsumoto Yushi-Seiyaku Co. Ltd. for their kind cooperation.

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