A Real Time Method of Measuring Unsteady Flow Rate Employing Centerline Velocity in a Circular Pipe

Tong ZHAO**, Ato KITAGAWA** and Toshio TAKENAKA**

This paper presents a new real time method of measuring unsteady flow rate employing centerline velocity in a circular pipe. Transfer functions which relate variables in axially symmetrical laminar flow can be obtained by mathematical analysis. We applied an approximate function of transfer function between the centerline velocity and the mean flow velocity to the measurement of unsteady flow rate. An analog electronic circuit whose transfer characteristic is equivalent to the approximate transfer function can be made easily, since it is composed of a proportional element, two first order lag elements and an element based on the second order lag element. The real time measurement of unsteady flow rate can be realized with the analog electronic circuit by inputting a measured signal of the centerline velocity to it. The validity of this method is clarified experimentally by comparing it with another measuring method whose validity has already been confirmed.

Key Words:  Flow Measurement, Unsteady Laminar Flow, Flow Rate, Real Time Measurement, Laser Doppler Anemometer

1. Introduction

Development of a method for measuring unsteady flow rate is not only an important subject for industrial production, but it is also closely connected with the advancement of a lot of basic technology, because it is very difficult to measure unsteady flow rate in a pipe flow precisely with the present techniques, even though there are some current meters with excellent dynamic characteristics or with simple construction as a measuring instrument of local velocity at a specified point on the cross section of a pipe\(^{(1)}\). For example, Laser Doppler Anemometry and Hot Wire Anemeter give excellent results, and Pitot Tube is very simple in construction.

In the case of laminar and axially symmetrical flow, the relation between the flow rate which passes through a certain cross section, and the velocity at a specified point on the same cross section of a pipe has been defined in another report\(^{(2)}\). By employing the relation, we shall be able to obtain the unsteady flow rate from centerline velocity, which is measured with the above mentioned current meters. Uchiyama and Hakomori\(^{(3)}\) have developed a computerized flowmeter employing Kalman filtering techniques, and Nakano, Yokota and Ueyama\(^{(3)}\) have proposed a measuring system for the measurement of instantaneous flow rate which calculates a convolution integral. In both methods, unsteady flow rate is obtained from the computation of the measured value of the centerline velocity.

In another paper\(^{(2)}\), we presented transfer functions which relate the variables in an unsteady pipe flow, and derived the approximate transfer functions which can be applied briefly not only to the indirect measurement of the unsteady flow rate, but also to a high speed digital transformation calculus method of the unsteady variables\(^{(4)}\), and presented a real time method of measuring the unsteady flow rate, which utilize differential pressure in a circular pipe\(^{(5)}\).

The objective of this paper is to propose a new real time method of measuring the unsteady flow rate which employs an approximate transfer function.
between the mean velocity (flow rate) and the centerline velocity on the same cross section of the pipe. The validity is confirmed experimentally, that is, the unsteady flow rate can be measured easily and precisely in real time by making use of an analog electronic circuit, whose transfer characteristics are equivalent to the approximate transfer function.

**Nomenclature**

- $a$ : velocity of pressure wave
- $D$ : pipe diameter
- $G_s$ : transfer function from centerline velocity to mean velocity
- $g$ : acceleration of gravity
- $H$ : pressure head
- $H_s$ : pressure gradient $=-\Delta H/A_x$
- $J_0, J_1$ : Bessel function of first kind
- $j$ : the imaginary unit $= \sqrt{-1}$
- $L$ : pipe length
- $Q$ : flow rate $= V \cdot \pi D^2/4$
- $r$ : coordinate in radial direction
- $r_0$ : inside radius of pipe
- $r^* = r/r_0$
- $s$ : Laplace operator
- $s^* = s r_0^2/\nu$
- $t$ : time
- $\Delta t$ : sampling interval
- $u$ : flow velocity in axial direction
- $u_0$ : centerline velocity
- $u^* = u/(gr_0^2/\nu)$
- $V$ : mean flow velocity
- $V^* = V/(gr_0^2/\nu)$
- $V_{\text{in}}$ : initial mean flow velocity
- $V_{\text{out}}$ : final mean flow velocity
- $x$ : coordinate in axial direction
- $x^*$ : dimensionless distance from the upstream end of the pipe to the position of an object of the transformation calculus
- $\Delta x$ : distance over which differential pressure is taken
- $\nu$ : kinematic viscosity

**Subscripts**

- $\sim$ : Laplace variable
- $\sim^*$ : approximate equation
- $^*$ : dimensionless variable
- $\text{in}$ : input
- $\text{out}$ : output

### Table 1: Constants of approximate transfer function

<table>
<thead>
<tr>
<th>$\gamma_{21}$</th>
<th>$\gamma_{22}$</th>
<th>$m_{21}$</th>
<th>$m_{22}$</th>
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<td>148.724 340</td>
<td>2 439.001 3</td>
<td>-27.079 510</td>
<td>-771.404 46</td>
</tr>
<tr>
<td>$\zeta_2$</td>
<td>$\omega_{21}$</td>
<td>$K_{21}$</td>
<td>$K_{22}$</td>
</tr>
<tr>
<td>0.856 440 4</td>
<td>52.044 138</td>
<td>-0.010 915</td>
<td>-771.404 46</td>
</tr>
</tbody>
</table>

(mean flow velocity and the local velocity at a specified point on the same cross section in an unsteady pipe flow, and derived their approximate transfer functions. The transfer function from the local velocity to the flow rate will be used to measure unsteady flow rate in this paper. The function can be written as follows:

$$G_2(r^*, s^*) = \frac{V^*}{u^*}(r^*, s^*)$$

$$= \frac{J_0(j \sqrt{s^*})}{J_0(j \sqrt{r^*})}$$

We assume the centerline velocity to be an input in Eq. (1), the transfer function can be approximately expressed as follows:

$$\tilde{G}_2(0, s^*) = A_2 + \sum_{i=1}^{2} \left( \frac{m_{2i}}{s^* + n_{2i}} \right)$$

$$+ \frac{K_{21} s^* + K_{22}}{s^* + \frac{1}{2} \frac{m_{21}}{\omega_{21}} s^* + \omega_{21}^2}$$

where, $A_2 = 0.99$, individual constants of the Eq. (2) are shown in Table 1. The approximation is concise and easy to understand, and can easily be applied to indirect measurement of the flow rate. In addition, it is highly precise over a wide range of frequency domains.

### 2.2 High speed digital transformation calculus

Taking the step response of the transfer function as a weighting function, the output signal can be obtained by a convolution integral with the derivation of the input signal. The method is called “digital transformation calculus” in this paper. To use the approximate transfer function, as the Eq. (2), the digital transformation calculus for obtaining the unsteady flow rate from the centerline velocity can be carried out briefly by Eqs. (3) ~ (5).

$$Q(t + \Delta t) = [A_{2\text{in}} u(t + \Delta t) + \sum \frac{m_{2i}}{n_{2i}} y_i(t + \Delta t)]$$

$$+ K_{21} y_1(t + \Delta t) + K_{22} y_2(t + \Delta t)](\pi r_0^2/2)$$

$$\times \{u_0(t + \Delta t) - u_0(t)\}$$

$$\times \left[ \begin{array}{c}
\cos(I_2 \Delta t) \\
-\sin(I_2 \Delta t)
\end{array} \right]$$

$$\sin(I_2 \Delta t) \cos(I_2 \Delta t)$$

$$\times \left[ \begin{array}{c}
y_1(t + \Delta t)
\end{array} \right]$$

$$+ e^{-\zeta_2 \omega_{21} \Delta t/2} \left[ u_0(t + \Delta t) \right]$$

$$\times \left[ \begin{array}{c}
\cos(I_2 \Delta t/2)
\sin(I_2 \Delta t/2)
\end{array} \right]$$

$$- u_0(t)$$

$$\times \left[ \begin{array}{c}
\cos(I_2 \Delta t/2)
\sin(I_2 \Delta t/2)
\end{array} \right]$$

$$\times \left[ \begin{array}{c}
y_1(t + \Delta t)
\end{array} \right]$$

$$+ e^{-\zeta_2 \omega_{21} \Delta t/2} \left[ u_0(t + \Delta t) \right]$$

$$\times \left[ \begin{array}{c}
\cos(I_2 \Delta t/2)
\sin(I_2 \Delta t/2)
\end{array} \right]$$

$$- u_0(t)$$

$$\times \left[ \begin{array}{c}
\cos(I_2 \Delta t/2)
\sin(I_2 \Delta t/2)
\end{array} \right]$$

$$\times \left[ \begin{array}{c}
y_1(t + \Delta t)
\end{array} \right]$$

$$+ e^{-\zeta_2 \omega_{21} \Delta t/2} \left[ u_0(t + \Delta t) \right]$$

$$\times \left[ \begin{array}{c}
\cos(I_2 \Delta t/2)
\sin(I_2 \Delta t/2)
\end{array} \right]$$

$$- u_0(t)$$

$$\times \left[ \begin{array}{c}
\cos(I_2 \Delta t/2)
\sin(I_2 \Delta t/2)
\end{array} \right]$$

$$\times \left[ \begin{array}{c}
y_1(t + \Delta t)
\end{array} \right]$$
where

\[ A_0 = A_1 + \sum_{i=1}^{2} \left( \frac{m_{3i}}{n_{3i}} \right) + \frac{K_{23}}{\omega_{na}} \], \quad I_2 = \omega_{na} \sqrt{1 - \bar{q}^2}, \]

\[ \Delta t = \Delta t_0 / \sqrt{q}, \quad m_{3i} = -m_{3i}/n_{3i}, \quad K_{3i} = -K_{2i}/\omega_{na}, \]

\[ K_{23} = (K_{45} - \bar{q}_2 \omega_{na} K_{35})/I_2, \quad K_{35} = K_{32} = (2m_{32} K_{35} / \omega_{na}) \]

2.3 Real-time analog transformation

With an analog electronic circuit corresponding to the transfer function, transformation of the unsteady variables can be performed easily, that is, when the input signal (the centerline velocity) is fed to the analog electronic circuit, the output signal (unsteady flow rate) can be obtained as an electrical output signal in real time. We call it a "real-time analog transformation" in this paper.

The transfer function, as shown in Eq. (1), is a complicated function which consists of Bessel functions. On the other hand, the approximate function, as shown in Eq. (2), is very brief and clear. Therefore

![Fig. 1 Electronic circuit which transforms centerline velocity into unsteady flow rate](image)

![Fig. 2 Electronic circuit which transforms differential pressure into unsteady flow rate](image)

the analog electronic circuit corresponding to that can be made easily. The analog electronic circuit which transforms the centerline velocity into the flow rate is shown in Fig. 1. The unsteady flow rate can be measured indirectly from the measured value of the centerline velocity on the same cross section in real time, by the use of the electronic circuit. The circuit is composed of a proportional element, two first order lag elements, an element based on the second order lag element, and the summing and the subtracting circuit, and only nine general operational amplifiers (UA741) are used.

The unsteady flow rate which is obtained from the centerline velocity with the electronic circuit must be compared with that measured by another measuring method for examining its validity. However, there exists no proper flow meter for this purpose. For this reason, in the experiment, we also use another method for measuring unsteady flow rate, which employs differential pressure, and whose validity has been confirmed in another paper. In this method, the flow rate at the middle of two positions where the differential pressure is taken is obtained by feeding the differential pressure signal to an analog electronic circuit, which is republished as in Fig. 2.

3. Experimental Setup and Measuring Circuit

Experimental setup and measuring circuit is shown in Fig. 3. There are two kinds of pipes. One is a copper tube 10 mm in diameter and 50 m in length. The other is an acrylic tube 10 mm in diameter and 12 m in length. The ends of the pipe are connected with pneumatically pressurized constant pressure tanks. The test pipe can be turned into either model A or B, as shown in Fig. 4 by adjusting the pressure of the two tanks. Hydraulic operating fluid (ISO VG12) is used as the test fluid, and the temperature is kept constant during the experiments. The velocity of the pressure wave is 1250 m/s in the case of the copper tube, and 740 m/s in the case of the acrylic tube.

In this experiment, the fluid transients are caused by opening or closing a valve that is set at an end of the pipe. We measured the centerline velocity at a position 1 m from the end of the pipe by Laser Doppler Anemometry (LDA). For the purpose of examining the accuracy of the unsteady flow rate which is transformed from the centerline velocity, the obtained value is compared with that transformed from the differential pressure between the end of the pipe and a position 2 m from the end. Assuming that the pressure of the tank that is connected to the end of the pipe is constant, the differential pressure can be measured by a general pressure transducer, and is transformed into unsteady flow rate at a position 1 m from the end of
the pipe.

As shown in Fig. 3, the measured centerline velocity and the measured differential pressure are transformed into the unsteady flow rate through both of the transformation processes. One of the processes is a real time analog transformation process, that is, two measured signals are transformed into an unsteady flow rate in real time respectively by the use of the analog electronic circuits, as shown in Fig. 1 or Fig. 2, then they are fed to a A/D converter, and recorded in a computer memory. The other is a high speed digital transformation process, that is, two measured signals are fed to a computer memory through the A/D converter (sampling frequency 1 kHz), and are then transformed into an unsteady flow rate by the computer programs using Eqs. (3) ~ (5) respectively, or other transformation calculus equations which have been shown in the former paper 80.

4. Experimental Results

4.1 Measured results with the high speed transformation calculus

Figure 5 illustrates the value of unsteady flow rate measured by the high speed transformation calculus (Model B, copper tube) in the case of closing the valve at the end of the pipe. The experimental conditions and the parameters for the transformation calculus are shown in Table 2. \( H_{fin} \) (input 1) is the centerline velocity measured by LDA, and \( Q_{out} \) (output 1) is the unsteady flow rate that is estimated from \( H_{fin} \) (input 1) with the high speed transformation calculus using Eqs. (3) ~ (5). \( H_{fin} \) (input 2) and \( Q_{out} \) (output 2) represent the measured value of the differential pressure between two positions in the axial direction and the unsteady flow rate that is obtained from \( H_{fin} \) (input 2) with a high speed transformation calculus, which has been provided in another paper 80. The measured results of the unsteady flow rate which are obtained from different input signals by two kinds of transformation calculus agree well with each other, so that it is confirmed that the transformation calculus shown in this paper is effective.

4.2 Measured results with the real time analog transformation

Figure 6 illustrates a comparison of the unsteady flow rate that is obtained by the real time analog transformation with that estimated by the digital

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Fig. 3 Experimental setup and measuring circuit

Fig. 4 Pipe model

Fig. 5 Unsteady flow rate obtained by high speed digital transformation calculus
transformation calculus. The test pipe is the acrylic tube (Model B), and the fluid transients are caused by quickly opening and closing the valve irregularly. \(U_{\text{in}}\) is the centerline velocity measured by LDA, and \(Q_{\text{out}}\) (Analog) and \(Q_{\text{out}}\) (Digital) represent the unsteady flow rates which are obtained by the use of the analog electronic circuit as shown in Fig. 1 and those estimated by the high speed transformation calculus, respectively, both of whose input signal is the centerline velocity \(U_{\text{in}}\) measured LDA. The values measured by two different methods agree well with each other, so the validity of two measuring methods is proved.

The measured results of the unsteady flow rate with the real time analog transformation calculus are shown in Fig. 7~Fig. 12. The figures also illustrate the
Table. 2 Experimental conditions

<table>
<thead>
<tr>
<th>pipe model</th>
<th>Fig.5</th>
<th>Fig.6</th>
<th>Fig.7</th>
<th>Fig.8</th>
<th>Fig.9</th>
<th>Fig.10</th>
<th>Fig.11</th>
<th>Fig.12</th>
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<tr>
<td>Model B</td>
<td>Model B</td>
<td>Model B</td>
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<td>50</td>
<td>50</td>
<td>50</td>
<td>12</td>
<td>12</td>
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<tr>
<td>v (m/s)</td>
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<td>0.00017</td>
<td>0.00019</td>
<td>0.00019</td>
<td>0.00019</td>
<td>0.00017</td>
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<tr>
<td>a (m/s)</td>
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<td>740</td>
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<td>740</td>
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<tr>
<td>V&lt;sub&gt;in&lt;/sub&gt; (m/s)</td>
<td>0.134</td>
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<td>0.429</td>
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</tr>
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</table>

Fig. 12 Unsteady flow rate obtained by real time analog transformation

Results obtained by the previous real time analog transformation employing differential pressure in the pipe in addition to those obtained from the centerline velocity. Fig. 7~Fig. 9 are the results using a copper tube, and Fig. 10~Fig. 12 show the results using an acrylic tube as a test pipe.

The measured results for the test pipe model B are shown in Fig.7~Fig.9. Figure 7 is the case of the valve opening, Fig. 8 is the case of the valve opening and closing irregularly and Figure 9 is the case of valve closing. The measured values of the unsteady flow rate which are obtained by the two kinds of real time analog transformation agree well with each other. As shown in Fig. 9, the top of the wave-form of the flow rate is formed in projection of irregular conformation, since the oil column separation is generated at the position before the valve, and yet the value measured from the centerline velocity agrees well with that from the differential pressure because the measuring position is at contrary end of the pipe to the valve, and is kept in a laminar flow state.

In the next place, we discuss the experimental results using the acrylic tube. Figure 10 is the case of valve closing, and Fig. 11 and 12 are the cases of repeated valve opening and closing. As the valve is operated gradually, the wave-form of the flow rate as shown in Fig.10 is smoother than that as shown in Fig. 8. As in Fig.11, the amplitude of the fluctuation for the third period is attenuated for the foregoing reason, and yet the values of the flow rate measured by the two kinds of method agree well.

As described above, under various conditions, the values of the flow rate measured by the method proposed in this paper agree well with those by another measuring method whose validity has been confirmed, so that we are able to draw the conclusion that the real time measuring method is an effectual means for measuring the unsteady flow rate.

5. Conclusion

We proposed a new real time method for measuring unsteady flow rate, and proved the validity of the method experimentally. In this method, the unsteady flow rate is obtained from the measured value of local velocity at a specified point on the same cross section of a pipe, by the use of a simple analog electronic circuit, without using any special hydraulic resistance (choke type restriction, orifice type restriction, and so on). So the method can be applied not only to the measurement of flow rate, but can also be used as a
comparison standard for evaluating a new measuring method.

References
