A Study on a Load Combination Method for Multi-Supported Piping Systems*

(Fundamental Investigation Using Piping System with Two Inputs)

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This paper is an extended study of the practical combination method of dynamic loads proposed in the previous paper, in which problems of the maximum response calculation of piping systems to multiple-support excitations were discussed. In this paper, experimental and numerical investigations are carried out in order to confirm the validity and feasibility of the proposed method. Using a simple Z-beam-shaped piping model for shaking tests, the maximum response of the piping in the fundamental mode is measured, and the combination law is applied to various correlation cases among two-vibrational loads. From the experimental results, it has been made clear that the proposed method is reasonable in comparison with other combination laws, like the square root of the sum of the squares (SRSS) method.

Key Words: Vibration, Random Vibration, Seismic Design, Piping System, Dynamic Load Combination, Correlation, Multi-excitation Problem

1. Introduction

The response analysis methods and several calculation methods for the seismic design of piping systems, subjected to multiple-support excitations (i.e., the multiple support excitation problems) have been proposed^{(1)~(5)}. However, the quantitative evaluation of these methods and the advantages and the disadvantages depending on the various combination methods have not been sufficiently investigated. We have seen only a few reports about experimental studies on the multiple support excitation problems of piping systems⁽⁶⁾.

Concerning the seismic response analysis problems for multi-loading systems, the authors have proposed a conventional load combination method, as shown in Table 1, considering the statistical correlation among dynamic loads based on the random vibra-

tion theory⁽⁷⁾⁽⁸⁾. In the previous paper⁽⁹⁾, the proposed method was developed for calculating the maximum response of piping systems subjected to multiple-support excitations. The method has an advantage, such that cross-correlation among support excitations can be semi-automatically evaluated. As a basic example of its applicability to multiple support excitation problems, a single degree of freedom (SDOF) system to two-input excitations was analyzed and the validity and feasibility of the proposed method was

Table 1 Load combination methods

Method	Combination
Algebraic Summation	$X = \sum_{i=1}^n X_i(\max)$
Absolute Value Summation	$X = \sum_{i=1}^n X_i(\max) $
S R S S	$X = [\sum_{i=1}^n X_i^2(\max)]^{1/2}$
Proposed Method	$X = [\sum_{i=1}^n \sum_{j=1}^n \rho_{ij} X_i(\max) X_j(\max)]^{1/2}$

$X_i(\max)$: Maximum value of dynamic load component X_i
 ρ_{ij} : Correlation coefficient between X_i and X_j

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confirmed.

In this paper, an extended study of the practical combination method proposed in the previous paper is discussed. Accordingly, through the experimental investigation using a simple Z-beam-shaped piping model for the shaking tests, the proposed method is applied to various correlation cases among vibration loads. Furthermore, the correlation effect appearing among support excitations on the maximum acceleration response of piping systems is discussed.

2. A Shaking Test Using a Piping Model with Two Inputs

2.1 Experimental apparatus and procedure

In the previous paper dealing with the response in a fundamental mode of out-of-plane vibration of the piping system, the combination method was applied in order to calculate the maximum response in a fundamental mode. In this paper, the shaking test using a simple piping model with two correlational excitations is carried out, and an extended study of the practical combination method proposed in the previous paper is done. The material of the Z-beam-shaped piping model, as shown in Fig. 1, is mild steel of 2 mm diameter, and each free end is supported by a small electrical shaker. An overview of the experi-

Fig. 1 Schematic drawing of experimental apparatus

Fig. 2 Overview of experimental apparatus

mental apparatus is shown in Fig. 2.

The direction of excitation is out-of-plane, vertical to the piping model. Also, in order to deal with the response in the fundamental mode of piping model, an acceleration response is measured using a piezoelectric accelerometer at its mid-point, where a modal participation factor is expected to become dominant.

In order to evaluate the proposed method, the following three shaking tests are carried out:

- (1) A shaking test based on single excitation $\ddot{y}_1$;
- (2) A shaking test based on single excitation $\ddot{y}_2$;
- and
- (3) A shaking test based on simultaneous two-input excitations $\ddot{y}_1$ and $\ddot{y}_2$.

The maximum acceleration responses are first calculated with the aid of the combination method proposed here by using the results of shaking tests (1) and (2). Then, the results are compared with those obtained by the square root of the sum of the squares (SRSS) method and the absolute value summation (ABS) method. The values are finally compared with the measured maximum acceleration response through a two-inputs excitation test (3); that is, the "exact" solution to this two-inputs problem. Here, in order to simplify the above procedure, simultaneous shaking tests are carried out by letting both individual maximum acceleration responses, obtained through shaking tests (1) and (2), be at an identical level.

2.2 Dynamic characteristics of the piping model and input wave for the shaking test

First, in order to compare the experimental results obtained from the piping model with the numerically simulated results, we must know the natural frequency and damping ratio of a fundamental mode of out-of-plane vibration of piping. These modal dynamic parameters are obtained through the

Fig. 3 Resonance curve of piping model

sinusoidal sweep test, as shown in Fig. 3. In this figure, the symbols $\bigcirc$ and $\bullet$ represent the amplification factors obtained through single excitation tests. The solid curve corresponds to the calculated amplification factor based on an SDOF system with a natural frequency $f_p=16.2$ Hz at a dominant peak and a damping ratio $\zeta_p=0.0063$, which is given through the half-power method. The value of the damping ratio almost agrees with that estimated from the damped oscillation wave of the model. Furthermore, from this resonance curve, it is shown that the first mode of the piping model is more dominant than the second natural mode with a frequency of 28 Hz. These values of the first mode are used for numerical simulation, as reported in the next chapter.

As a combination of two input excitations, two kinds of correlational analog signals are given for shaking tests. The correlation among support excitations for the general case is given by assuming the following exponential-function-type cross-correlation, whereby two random excitations, $\ddot{y}_1$ and $\ddot{y}_2$ are simulated by the Monte Carlo method⁽¹⁰⁾.

$$R_{ij}(\tau) = \varepsilon_{ij} \sigma_i \sigma_j \exp(-\chi|\tau|) \quad ; i, j=1,2 \quad (1)$$

In this equation, σ_i is a standard deviation of $\ddot{y}_i$, ε_{ij} (when $i=j$, $\varepsilon_{ij}=1$) is a correlation factor between two excitations and χ is a constant value which gives the shape of the cross-correlation function.

Numerical values of ε_{12} , χ , etc. for this kind of combination of two-input excitations are summarized in Table 2. In this table, f_l and f_u represent the lower and upper frequencies in the frequency range of input analog signals, respectively.

Generally, the input excitations are significantly influenced by the dynamic response behavior of the support structure. With this in mind, the absolute acceleration responses of SDOF systems subjected to white noise excitation are simulated by combining the two-input excitations $\ddot{y}_1$ and $\ddot{y}_2$ with different, dynamic characteristics. Thus, the combination of the

two input excitations, which stand for the dynamic characteristics of the support system, is used for the shaking tests. Here, f_{si} and ζ_{si} mean, for example, the natural frequency and the damping ratio of the i -th support system.

Numerical values of f_{si} and ζ_{si} for simulating analog signals are shown in Table 3. In this table, Case 1 corresponds to the case where the piping systems are supported at different points on firm ground and soft ground, or on firm ground and in a building. In this case, the support excitations have relatively broad-band and narrow-band support motions. Case 2 corresponds to the case where the piping systems are supported by different buildings, or by different floors in the same building. In other words, piping systems are subjected to the relatively narrow-band support excitations with different dominant peaks. Furthermore, in this paper, from the point of the least feasible condition, the dominant frequencies of $\ddot{y}_1$ and $\ddot{y}_2$ for Case 1, and $\ddot{y}_1$ for Case 2 are identical to the natural frequency f_p of the piping model.

Also, in order to simplify the experimental procedure, the dynamic characteristic of input $\ddot{y}_1$ remains and that of input $\ddot{y}_2$ varies, depending on the degree of their correlation effects for two different combinations of the two inputs mentioned above.

3. Results of Shaking Test and Numerical Simulation

3.1 Relation between the response of piping and the cross-correlation among support excitations

The power spectral density functions of the acceleration response at the mid-point of the piping model and the cross-correlation functions between two-input excitations are shown in Fig. 4. This figure shows the results obtained by use of the combination of two-input excitations, under the assumption of an exponential-function-type cross-correlation with $\chi=10$ and $\varepsilon_{12}=0, 0.5$ and 1.0 . These cross-correlation

Table 2 Numerical values of parameters concerned with two-input excitations under the assumption of exponential-function-type cross-correlation

K	S.D. of $\ddot{y}_1, \ddot{y}_2$	Corr. Coeff. ε_{12}	Lower Freq.: $f_l = 14.0$ Upper Freq.: $f_u = 18.0$ (Hz)
1	$\sigma_1 = 146$	0.0	
	$\sigma_2 = 182$	0.25	
		0.5	
10	(cm/s ²)	0.75	
		1.0	

S.D.: Standard Deviation

Table 3 Numerical values of parameters concerned with two-input excitations considering support dynamic behavior

Case 1			Case 2		
$f_{s1}=f_{s2}$: Hz			$\zeta_{s1}=\zeta_{s2}$		
16.2			0.05		
ζ_{s1}	ζ_{s2}		f_{s1}	f_{s2} : Hz	
0.01	0.01, 0.05, 0.15		16.2	15.0, 15.5	
0.5	0.15, 0.5				

functions are normalized by the product of the standard deviations of two-input excitations. From this figure, it is shown that the peak amplitude and the area, or total power corresponding to the variance of response, in power spectral density functions grows large as the cross-correlation factor ε_{12} increases; that is, the correlation effect between the two-input excitations becomes stronger. Under the assumptions⁽¹¹⁾ that the acceleration amplification factor can be estimated by the ratio of the variances of the response and input excitations, and that the variances of $\ddot{y}_1$ and $\ddot{y}_2$ are kept constant in this experiment, it is emphasized that the cross-correlation has considerable effect on the response of the piping model.

In order to evaluate this effect, an example of the amplification factor corresponding to the maximum response versus the cross-correlation factor ε_{12} is plotted in Fig. 5. This figure is for a piping model with two-input excitations, based on the assumption of an exponential-function-type cross-correlation with parameters $\kappa=1$ and 10, and it summarizes the results for all six cases where the Monte Carlo method is utilized in order to determine the initial random numbers for each case. Also, these amplification factors are given through the use of the maximum value of $\ddot{y}_2$,

whose dynamic characteristics vary, depending on the selected value of ε_{12} . Even if small fluctuations of the amplification factors due to the initial values of their random numbers are seen, it is clarified that the amplification factor depends on the correlation effect among the two-input excitations by the fact that the maximum value of the amplification factor for $\varepsilon_{12}=1$ increases by about 1.7 times that for $\varepsilon_{12}=0$ in the case of $\kappa=10$, and by about 1.6 times in the case of $\kappa=1$.

3.2 Comparison of calculated maximum acceleration responses

In this section, the "exact" maximum acceleration responses measured through the simultaneous shaking tests are compared with those calculated by various methods, listed in Table 1, using the individual maximum acceleration responses.

The correlation coefficient ρ_{12} , which is introduced in the proposed method, is given by the following equation:

$$\rho_{12} = \frac{\text{Re}[G_{12}(f_p)]}{[G_{11}(f_p)G_{22}(f_p)]} \quad (2)$$

where $G_{ii}(f_p)$ and $G_{ij}(f_p)$ are the values of the one-sided power spectral density function of $\ddot{y}_i$ and the cross-spectral density function between $\ddot{y}_i$ and $\ddot{y}_j$ at $f=f_p$, respectively.

For two-input excitations under the assumption of an exponential-function-type cross-correlation, ρ_{12} in Eq. (2) is equal to ε_{12} , while for the two-input excitations which represent the dynamic characteristics of a support system, ρ_{12} is expressed as follows:

$$\rho_{12} = \frac{\text{Re}[H_1^*(f_p)H_2(f_p)]}{H_1(f_p)H_2(f_p)} \quad (3)$$

where $H_1(f_p)$ and $H_1^*(f_p)$ are the frequency response

Fig. 4 Relation between cross-correlation function and power spectral density function of acceleration response for $\kappa=10$

Fig. 5 Relation between cross-correlation factor and amplification factor (for a piping model with two-input excitations under the assumption of an exponential-function-type cross-correlation)

function of an SDOF support system and its complex conjugate at $f=f_p$, respectively.

The comparison of the calculated maximum acceleration responses of a piping model by use of the combination of two-input excitations under the assumption of an exponential-type correlation function is shown in Fig. 6. This figure is for $\kappa=10$, and the maximum acceleration responses are normalized by that for $\varepsilon_{12}=1$. In this figure, the solid line (Exact) represents the measured results through two-input shaking tests, and the dashed line represents those calculated by the proposed method. From this figure, it is shown that both results almost agree, irrespective of the degree of the correlation among the two-input excitations. Also, based on the results of the three cases for both $\kappa=1$ and 10, the average values of the absolute percent errors by various methods are summarized in Table 4. From this table, even if the proposed method results in a small error, it can be seen that this method is reasonable, compared with the SRSS method and ABS method.

On the other hand, for the piping model with two-input excitations, which stand for the support dynamic characteristics, the acceleration amplification factor and the absolute percent errors by various methods are shown in Table 5, whereby the value of the correlation coefficient ρ_{12} in Eq. (3) is given. This table corresponds to the results obtained by use of two-input excitations under the assumption of Case 1, as mentioned previously. In Table 5, the calculated values by the proposed method are shown. Due to coupling effects and the slight difference of the dynamic characteristics between two-vibration shakers, the proposed method considers the correlation coefficients ρ_{12} to be greater in value than the

exact solution. However, that error is small, compared with that of other methods.

Since the SRSS method, which is frequently introduced, underestimates results by about 30 percent, and the ABS method, which is utilized as a "conservative" design criteria, overestimates for all cases, these methods are not reasonable for the response analysis of multi-supported piping system.

3.3 Comparison of experimental results and numerical simulation results

Based on numerical simulation using the dynamic characteristics of the piping model estimated in Chapter 2 and two different combinations of two-input excitations $\ddot{y}_1, \ddot{y}_2$, the maximum acceleration responses calculated by the proposed method are compared with those by the SRSS and ABS methods.

Table 4 Comparison of percent errors (for a piping model with two-input excitations under the assumption of an exponential-function-type cross-correlation)

$f_{s1}=f_{s2}$ 16.2 Hz		Amplification Factor ($\bar{X}/\bar{Y}_2$) [Error(%)]				ρ_{12}
ζ_{s1}	ζ_{s2}	Exact	ABS	SRSS	Proposed Method	
0.01	0.01	9.61	10.5 (9.1)	7.43 (22.7)	10.5 (9.1)	1.0
	0.05	7.79	8.85 (10.9)	6.27 (21.6)	8.85 (10.8)	0.99
	0.15	5.53	6.26 (13.1)	4.42 (20.0)	6.20 (12.1)	0.96
0.5	0.15	1.75	2.11 (20.2)	1.50 (14.9)	2.05 (16.5)	0.88
	0.5	1.06	1.11 (4.6)	0.79 (25.8)	1.11 (4.6)	1.0

Fig. 6 Comparison of maximum acceleration responses (for a piping model with two-input excitations under the assumption of an exponential-function-type cross-correlation)

Table 5 Comparison of percent errors (for a piping model with two-input excitations considering support dynamic behavior)

κ	ε_{12}	ABS (%)	SRSS (%)	Proposed Method (%)
1	0.0	36.4	7.0	7.3
	0.25	21.1	13.7	7.1
	0.5	15.2	18.5	2.5
	0.75	5.8	25.0	3.2
	1.0	0.0	29.2	0.0
10	0.0	39.6	5.6	6.3
	0.25	35.6	8.5	12.3
	0.5	25.5	10.5	11.3
	0.75	12.9	20.3	5.3
	1.0	0.0	29.3	0.0

Finally, they are compared with the measured maximum acceleration responses obtained through simultaneous shaking tests.

Taking a fundamental mode of vibration, the equation of motion of dynamic displacement x^d of the piping model is given as follows,

$$\ddot{x}^d + 2\zeta_p\omega_p\dot{x}^d + \omega_p^2x^d = -\beta_1\ddot{y}_1 - \beta_2\ddot{y}_2 \quad (4)$$

where $\omega_p = 2\pi f_p$ is a natural circular frequency of a fundamental mode of vibration, and β_1 and β_2 are the participation factors for this mode of piping model due to $\ddot{y}_1$ and $\ddot{y}_2$, respectively. Since the individual maximum acceleration responses obtained from single shaking tests are identical in this case, $\beta_1 = \beta_2 = 0.5$ is given. Also, the acceleration response $\ddot{x}$ is given by the following equation,

$$\ddot{x} = \beta_1(\ddot{x}^d + \ddot{y}_1) + \beta_2(\ddot{x}^d + \ddot{y}_2) \quad (5)$$

In this equation, $(\ddot{x}^d + \ddot{y}_i)$ is the absolute acceleration response of an SDOF piping system to $\ddot{y}_i$, and its maximum value corresponds to the acceleration response spectrum. In this simulation procedure, the results calculated by the various methods using individual response spectra are compared with the measured experimental results obtained through the simultaneous shaking tests.

First, the time history of the acceleration response measured from the simultaneous shaking tests is compared with that obtained through the

Fig. 7 Comparison of time histories of acceleration response (for a piping model with two-input excitations under the assumption of an exponential-function-type cross-correlation)

numerical simulation using Eq. (5), as shown in Fig. 7. This figure represents the piping model with two-input excitations under the assumption of an exponential-function-type correlation with $\chi=1$ and $\varepsilon_{12}=0.75$. In this figure, the two upper time histories correspond to inputs $\ddot{y}_1$ and $\ddot{y}_2$, respectively, and the lower two correspond to the acceleration responses measured through experiment and simulation, respectively. From this figure, it is shown that both response-time histories approximately agree.

Next, the comparison of the maximum acceleration responses measured from the simultaneous shaking tests, and those calculated by the various methods using the individual response spectra are shown in Fig. 8. This figure represents the piping model with two-input excitations under the assumption of an exponential-function-type cross-correlation with $\chi=1$. In this figure, the exact solutions ($\triangle$) obtained from the simulated time history do not agree with the measured experimental results, which are represented by the solid line, due to the coupling effect between the two shakers. However, it becomes clear that the results represented by the dashed line and calculated by the proposed method match well with the measured, exact results, irrespective of the degree of correlation effect between the two-input excitations.

From the results based on both experiment and simulation, as mentioned above, it has been clarified that the proposed method introducing the correlation effect among support excitations is reasonable, compared with other methods.

Fig. 8 Comparison of maximum acceleration response (for a piping model with two-input excitations under the assumption of an exponential-function-type cross-correlation with $\chi=1$, $\varepsilon_{12}=0.75$)

4. Conclusions and Acknowledgements

Using a simple, Z-shaped piping model with two inputs, experimental and numerical investigations were carried out and the proposed method was applied to various correlation cases among vibration loads. The results are summarized as follows:

(1) The maximum acceleration response of a piping system can be calculated, depending on the degree of the correlation effect among support excitations; in other words, the stronger the correlation effect becomes, the larger these values grow.

(2) It has been clarified that the proposed method could be applied to a practical response calculation problem of multi-supported piping systems and it is reasonable, compared with the other laws which appear in the SRSS and ABS methods.

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