Modeling and Control of Torsional Vibrations in a Flexible Structure*

Toshio FUKUDA**, Fumihito ARAI***,
Hidemi HOSOGAI** and Nobuyuki YAJIMA ***

This paper describes a modeling method of torsional vibrations of flexible, large, space structures, such as solar battery arrays, and a control method based on this model. The torsional vibrations are modelled by taking into account the flexibility of the solar array and the inertial moments of the supporting rigid body, based on the unconstrained mode method. Employing this model of the flexible structure, the system and the observation equations of the dynamics can be derived in the form of a state representation after an n mode decomposition. The torsional vibrations can be measured by using a newly developed differential-type sensor, which consists of a pair of neighboring solar cells. A vibration control method is shown by the state feedback based on the dynamics. Some of the experimental results employing the proposed control method are also shown.

Key Words: Flexibility Control, Torsional Vibration, Unconstrained Mode Method, Differential Solar Sensor

1. Introduction

Space structures tend to be larger and more flexible in the future, with the advancement of space development. In particular, solar battery arrays, which are considered to be one of the large space structures (LSS), will tend to be designed both extremely large and extremely light and will become structurally flexible in their use as a power supply system in space/1, 5/, so control in consideration of flexibility is inevitable/6/. For this reason, we proposed a method of flexibility control in the previous reports/6, 8/. Considering only the bending vibrations, we presented the effectuality of flexibility control to the bending vibrations by achieving the mode estimation/7/by means of a space sensor (differential solar sensor). We also reported a method of flexibility control with

a reaction wheel/8/, which is used as an actuator of the solar battery array. In this paper, considering only the torsional vibrations, we report the torsional vibrations and attitude control of the array.

Generally, an artificial satellite has flexible appendages such as solar battery arrays and antennas. Stability conditions of the attitude of such flexible satellites are more severe, compared with those of rigid ones. Especially in the case of a satellite with long, flexible appendages, the force produced by the gravitational gradient deforms the array and changes its moment of inertia at different times and places. This, in turn, affects its attitude/9/. So as one of the ways to reduce this interference; so, as one of the ways to reduce this interference, a reduction of the aspect ratio is considered. This leads to power consumption needed to rotate the arrays a reduction in the maximize in order to the exposure to light. This also leads to the tendency of the torsional vibrations to become easily excited. From this point of view, not only bending vibration control, but also torsional vibration control is necessary in the case of large, high-power solar arrays.

In this paper, the flexibility control of only the

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** Department of Mechanical Engineering, The Science University of Tokyo, 1-3 Kagurazaka, Shinjuku-ku, Tokyo, 162, Japan

*** Mechanical Engineering Laboratory, 1-2 Namiki, Sakura-mura, Niihari-gun, Ibaraki, 305, Japan

torsional vibrations of the array system is considered. Firstly the torsional vibrations are modelled by the unconstrained mode method, which treats both the array and the rigid body as one body, and the dynamic characteristic equations are derived. The torsional vibrations control method based on this model is then proposed. By estimating the mode using the outputs of the differential solar sensors, some experimental studies are carried out to determine the effectuality of this control method.

2. Mathematical Modeling of Torsional Vibrations

2.1 Torsional vibrations model and equations of motion

Figure 1 shows a model of a flexible solar battery array. We consider that a flexible cantilever is fixed to a rigid body, which simulates a spacecraft or a space station. The mathematical model is derived by the unconstrained mode method (vehicle mode method), which treats both the array and the rigid body as one body. This system is considered to be controlled by a DC motor simulating an actuator, such as a reaction wheel or a control moment gyroscope.

Figure 2 shows the coordinate system of this modelled solar array. We assume that the torsional vibrations are small, and vibrate only in torsion. Let θ_H be the angle of the rigid body, θ_p be the rotational angle generated by the elastic deformation of the array in torsion at an arbitrary point on the x -axis, and θ be the total rotational angle at the arbitrary point of the x -axis. Then, the relationship between the attitude of the array and the rigid body is represented as follows.

$$\theta(x, t) = \theta_H(t) + \theta_p(x, t) \quad (1)$$

where θ_p is small.

Assuming that this array is a continuous, homogeneous and symmetrical rectangular plate, and that the torsional rigidity D is constant while θ is small, we can treat this solar array as a one-dimensional continuous beam. The torsional frequency equation of this

array can then be written as follows.

$$\partial^2 \theta / \partial t^2 = D / \rho_p J_p \cdot \partial^2 \theta / \partial x^2 \quad (2)$$

where ρ_p and J_p represent the array's density and the polar 2nd moment of area, respectively. Considering the equilibrium of moments (all viscous terms are neglected here), we obtain the boundary conditions as follows.

$$(I_{Hx} + n^2 I_m) \partial^2 \theta / \partial t^2|_{x=0} = D \partial \theta / \partial x|_{x=0} + T(t) \quad (3)$$

$$\partial \theta / \partial x|_{x=l} = 0 \quad (4)$$

where I_{Hx} is the rigid hub's moment of inertia about the x -axis, I_m is the amature's moment of inertia about its center shaft, n is the reduction gear ratio and T is the control input. By letting $\psi_i(x)$ and $q_i(t)$ be the mode function and the time function of the i -th mode, θ can then be represented as follows.

$$\theta(x, t) = \sum_{i=0}^{\infty} \psi_i(x) q_i(t) \quad (5)$$

where ψ_0 is the rigid body mode. From Eqs. (2) ~ (5), the characteristic equations of this system are obtained as follows.

$$\omega_i^3 / \lambda \cdot I_D \cos(l / \lambda \cdot \omega_i) + \omega_i^2 / \lambda^2 \cdot D \sin(l / \lambda \cdot \omega_i) = 0$$

$$i = 1, 2, \dots \quad (6)$$

where

$$\lambda^2 = D / \rho_p J_p$$

$$I_D = I_{Hx} + n^2 I_m$$

where ω_i is the i -th natural frequency and $\omega_0 = 0$. The orthogonality relation is obtained as follows.

$$\int_0^l \psi_i(x) \psi_j(x) dx + \lambda^2 I_D / D \psi_i(0) \psi_j(0) = J \delta_{ij} \quad (7)$$

where J is arbitrary constant, given as follows.

$$J = I_{Hx} + I_{px} \quad (8)$$

where I_{px} is the array's moment of inertia about the x -axis.

Next, from the Eqs. (2) ~ (8), the following mode functions of this system are obtained.

(i) $i = 0$ (rigid body mode)

$$\psi_0(x) = \sqrt{J / (l + \lambda^2 I_D / D)} \quad (9)$$

(ii) $i = 1, 2, 3, \dots$

Fig. 1 Model

Fig. 2 Coordinate system

4. Differential Solar Sensor and Mode Estimation

4.1 Differential solar sensor

Figure 6 shows the differential solar sensor. Let S_A and S_B be the exposed area of each of the neighboring sensors, A and B , shown in Fig. 6. The angle θ^* can then be calculated as follows.

$$\theta^* = \tan^{-1}\{(w-c)/h \cdot (S_A - S_B)/(S_A + S_B)\} \quad (18)$$

This sensor is linear between $\theta = -10^\circ$ and $\theta = 10^\circ$ and its accuracy is about 1%. The dimensions of this prototype sensor is such that its height $\times$ length $\times$ width: 18 $\times$ 10 $\times$ 17 (mm), and its weight is about 2g. Figure 7 shows its performance.

4.2 Mode estimation of torsional vibrations

Now we consider the estimation of the modes of the torsional vibrations by using M differential solar sensors. There are other techniques of mode estimation, such as the linear optimal filtering technique, but most of them are complicated and require a long time for computation, so the following, simplified estimation method is employed.

Considering the n modes of torsional vibrations and using $(M+1)$ sensor outputs, we can represent the observation equation as follows.

$$y_m = C_m x_m \quad (19)$$

where

$$y_m^T = (\theta_H, \alpha_1, \alpha_2, \dots, \alpha_M)$$

$$C_m = \{\phi_i(x_j)\} \quad i=0, \dots, n$$

$$j=0, \dots, M$$

$$x_m^T = (q_0, q_1, q_2, \dots, q_n)$$

Fig. 5 Feedback control system

Fig. 6 Differential solar sensor

By letting $n=M$, C_m becomes a non-singular matrix. The time function x_m can be calculated as follows, by using an inverse matrix of C_m .

$$x_m = C_m^{-1} y_m \quad (20)$$

Then, $\dot{x}_m$ can be obtained by the differential calculus of each element of x_m . Figure 8 shows an experimental result of the estimation of the 0-th and the 1st torsional vibration modes while the array leaned 5 degrees from its basic attitude and was vibrating in torsion, without any external forces.

5. The Experimental Flexible Solar Array System

Figure 9 shows an outline of the experimental array system. One array (made of GFRP), which simulates a solar array, is attached to a rigid hub (made of iron). Bending stiffness is obtained by restricting the x -axis straight. So, while θ is small, the effects of gravity can be neglected. There are three differential solar sensors located on the axial center-line of the array depicted in Fig.9. Two pairs of strain

Fig. 7 Performance of the differential solar sensor

Fig. 8 Mode estimation result of the free vibration

gages are installed on both sides and at the same positions relative to the array for the reference of the evaluation of control performance. The rigid hub angle is measured by a potentiometer and the motor's angular velocity by an analog tachometer. Each measurement signal is taken into a CPU through an A/D converter, feedback control signals are computed in the CPU, and then a DC motor is driven by the outputs from a D/A converter based on the computed feedback signals. The dimensions of the array is such that the length $\times$ width $\times$ thickness : $700 \times 140 \times 0.5$ (mm). The 1st natural frequency is 5.5 Hz, and the damping ratio at this mode is 5 %.

6. Experimental Results

A light source simulating the sun was set in front of the array. Rotation of the array to a given angle (5 degree) was conducted by estimating the direction of the light source using the differential solar sensor outputs. This was a servo problem under a constant set-up value, which then become a regulator problem under the initial disturbances.

At first, mode estimation was conducted by using

Fig. 9 Outline of experimental array system

Fig. 10 Control result without consideration of flexibility

the differential solar sensors' outputs. Next, we performed some feedback control experiments described as follows:

- (i) position q_i feedback control (Fig.11)
- (ii) verosity $\dot{q}_i$ feedback control (Fig.12)
- (iii) perfect state feedback control (Fig.13)

For comparison, we also performed control experiments not considering the flexibility of the array, and depicted these results in Fig. 10. In this control system, the sampling period was 22.7 [ms], and we considered the 0-th, 1st and 2nd torsional vibration modes.

Comparing (i) and (ii), in the case of (ii), we

Fig. 11 Control result with consideration of flexibility (position feedback control)

Fig. 12 Control result with consideration of flexibility (velocity feedback control)

Fig. 13 Control result with consideration of flexibility (perfect state feedback control)

could give a sufficient damping to the system, and the torsional vibrations were damped faster. From (iii), we could compose a much more stable closed-loop system so that the overshoot is reduced. From these control results, we could obtain the effectuality of flexibility control on the torsional vibrations.

7. Conclusions

In this study about torsional vibration control, it was necessary to derive the mathematical model of the torsional vibrations. We derived the n -th mode dynamic model of the torsional vibrations by the unconstrained mode method in the case that the flexible appendage is attached to the rigid body. By expressing the dynamics of the flexible array in the

mode resion, we stated that this control problem results in the regulator problem, and so both the position and vibration control problems can be treated systematically. By using the differential solar sensor, the attitude measurement and mode estimation came to be practical. We demonstrated the effectuality of flexibility control on the torsional vibrations by experiment. In a future study, we will consider coupled bending-torsion vibrations.

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