Interfacial Instability of Magnetic Fluids in a Rectangular Container*

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An experimental and theoretical study is presented for the interfacial instability of magnetic fluids in rectangular containers subject to magnetic fields. Experiments were performed to determine the critical field intensity for incipient instability and to observe the growth of instability waves. Magnetic fluids used in the experiments were water-base ferricolloid W-35 and its dilution with distilled water. It was found that interfacial instability appeared when the field intensity exceeded the threshold value and grew in amplitude with the field intensity. It was also found that the wave number for the instability depended upon the field intensity and the dimensions of the container. The possible equilibrium configuration was analyzed theoretically by minimizing the total energy. The experimental results can be explained well by the theoretical analysis.

Key Words: Magnetic Fluid, Magnetohydrodynamics, Surface Instability, Static Waves, Labyrinth Instability

1. Introduction

During the development of the recent high technology, highquality magnetic fluids composed of solid magnetic particles of subdomain size dispersed colloidally in a liquid carrier have been developed. Several devices applying magnetic fluid have also been developed, such as rotary shut seals, bearings, actuators, dampers, ink jet printers, and so on⁽¹⁾. In regard to these applications, the fundamental responses of magnetic fluid by applied magnetic fields are very important and have been studied by a number of investigators⁽²⁾. In general, the interface behavior of magnetic fluid subject to applied magnetic fields exhibits various patterns according to the direction and intensity of the fields⁽³⁾, and extensive investigations on the interfacial instability of magnetic fluid have been conducted. Cowley and Rosensweig⁽⁴⁾ investigated the normal field surface instability. They showed the analysis and confirming experiments for a

nonlinearly magnetizable fluid forming a hexagonal array of peaks. The liquid magnetic labyrinth in thin horizontal layers in the presence of a homogeneous magnetic field normal to the layer was observed and analyzed by Romankiw et al.⁽⁵⁾ and analyzed theoretically by Rosensweig et al.⁽⁶⁾. Tsebers and Maiorov studied a series of the interfacial instability problem in connection with flat interfaces⁽⁷⁾, drops⁽⁸⁾, and bubbles⁽⁹⁾. In addition, the static stability of a ferrofluid seal⁽¹⁰⁾, parametric surface instability under a time-periodic magnetic field with a strong gradient⁽¹¹⁾, and so on were reported. However, the research data on the interfacial instability and the growth of instability waves on the magnetic fluid surface in wide rectangular containers subject to magnetic fields are insufficient, and there still remains a wide unexplored domain.

The present report, therefore, aims at obtaining more fundamental knowledge needed to provide technological applications of magnetic fluids. We studied experimentally the occurrence of the instability and the growth of the surface deflection of magnetic fluids under the magnetic field applied tangentially to the interface in rectangular containers of various dimen-

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sions. In addition, we discussed the interfacial instability theoretically and supported the experimental results rationally. The basic characteristics of the magnetic fluid surface were thus clarified.

Principal Nomenclature

- a : inner width of rectangular container (in longer direction)
 A : amplitude of stationary interfacial wave
 b : inner width of rectangular container (in shorter direction)
 B : magnetic flux density
 c : container height
 D : demagnetization coefficient
 E : energy
 g : gasvitational acceleration
 H : magnetic field strength
 I : electric current
 k : wave number ($=2\pi n/L$)
 L : container width ratio ($=a/b$)
 M : magnetization
 M_s : saturation magnetization
 n : number (positive integer)
 v : volume
 x, y, z : coordinates
 μ_0 : permeability of vacuum
 ξ : displacement from mean level
 ρ : magnetic liquid density
 χ : susceptibility
 σ : surface tension

2. Experimental Apparatus and Procedures

A water-cooled electromagnet with a 70 mm gap between magnetic poles was used in these tests. The magnetic field strength of the electromagnet was changed by the electric current. Figure 1 shows the distributions of magnetic flux densities along the x - and y -directions in the magnetic poles of the electromagnet used here. Several rectangular containers used in the experiment were made of transparent acrylic plates for the purpose of optical observations. All containers had walls 300 mm high and 5 mm thick, but the dimensions of the inner width varied. Table 1 shows the dimensions of the containers used in the experiment. Sample liquids were water-based ferri-colloid W-35 and its dilution with distilled water.

Each container was filled with magnetic fluid to a selected level of 165 mm and was subjected to magnetic fields applied tangentially to the free surface of the magnetic fluid; in other words, the direction of applied field was normal to the liquid layer. The relation between the direction of field and the coordinate system of rectangular containers is shown in Fig. 11, as described later. The strength of field was in-

creased gradually. The deflection of the magnetic fluid surface was observed by the photographic record using a reflection mirror.

3. Experimental Results

3.1 Occurrence and growth of instability waves

When the magnetic field H_0 applied to the surface of the magnetic fluid increases gradually from zero, the flat surface suddenly deflects at a certain field strength. The instability deformation on the surface appears as the stationary waveform deflection of the interface, and the amplitude of the wave is increased as the field strength H_0 is increased. A typical example of such surface configurations in the case of the container C 6 is shown in Fig. 2. These photographs were taken by aligning the camera in the y -direction. Such wavelike deflections are named instability waves in this paper. It can be seen from Fig. 2 that the instability wave shows a two-dimensional deflection and the wave crests on both wall sides grow with the increase of H_0 to become a scroll pattern. We define the

Table 1 Container dimensions

Container	Dimension (a×b×c mm)	Wall thickness (mm)
C1	100×10×300	5
C2	75×10×300	5
C3	50×20×300	5
C4	50×10×300	5
C5	50×7.5×300	5
C6	50×5×300	5
C7	50×2×300	5
C8	25×20×300	5
C9	25×10×300	5
C10	25×7.5×300	5
C11	25×5×300	5

Fig. 1 Distribution of magnetic flux density

amount of surface deflection by the parameters ξ_h and ξ_l . The former is the displacement of the wave crest at the middle part of the container and the latter is the displacement of the trough. Figure 3 shows the growth of instability waves (for the containers with $a=50$ mm). Both ξ_h and ξ_l were measured vertically from the plane level, without the magnetic field. It is obvious from Fig. 3 that the growth of such instabilities varies linearly with the applied magnetic field H_0 over the wide range of the field. In the range of a higher field, however, increases of ξ_l appear. This is caused

Fig. 2 Surface deflection of magnetic fluid

Fig. 3 Growth of instability waves for various b

by the rising deflection of the surface in the middle part of the container, as described later, or the gas entrainment into the magnetic fluid. Figure 4 shows the effect of the container dimension a on the growth of the instability waves for a constant b . The larger a is, the higher both ξ_h and ξ_l are. It is clear that the total amplitude $\xi_h - \xi_l$ of the instability waves does not increase, but rather decreases over the range of high H_0 . This fact is considered theoretically later.

Figure 5 shows the relation between the container dimension b and the critical magnetic field strength H_c at which the incipient instability waves are visibly observed. It can be seen from Fig. 5 that the critical field strength H_c increases with b ; that is, the smaller the wall gap of the container in the field direction, the more easily the instability waves occur. In Fig. 5, the container with larger b and $a=25$ mm has a higher

Fig. 4 Growth of instability waves for various a

Fig. 5 Relation of the critical field H_c to b

value of H_c . This seems to be caused by the fact that the occurrence of two-dimensional waves is limited because of the small a/b .

3.2 Wave number of instability waves

It was observed that the wave number of the instability waves on the surface of the magnetic fluid varied with the container dimension. We investigated the number n defined by the equation $k=2\pi n/L$ (n relates to the crest number of waves). Figure 6 shows the surface configurations at the state which slightly exceeds the critical field H_c for various b and for $a=50$ mm. It can be seen that the number n decreases as b increases.

In the case of $b=10$ mm and constant H_0 , the surface configurations for various a are shown in Fig. 7. It is clear that the number n increases as a increases. The wavelength of the instability waves,

Fig. 6 Surface deflection for various b

Fig. 7 Surface deflection for various a

however, is approximately the same value as b , and is kept constant for various a . It was observed that the increase of the wave number after sufficient growth of the instability waves was achieved by crest branching. Generally, the number n of the instability waves increases with the rise of H_0 , as shown in Fig. 8. Depending on the container dimension L , however, it was also observed that n decreased with the rise of H_0 . An example of such results for the container C1 is shown in Fig. 9. These facts are considered theoretically later.

3.3 Effect of Concentration of Magnetic Fluid

In the same way as described above, the experiments were performed for various concentrations of magnetic fluids. The trends of these results are similar

Fig. 8 Surface deflection for various H_0 (container C2)

Fig. 9 Surface deflection for various H_0 (container C1)

to those of the magnetic fluid of 33% mass concentration, as well. However, the critical field H_c corresponding to the incipience of the instability waves was not proportional to the mass concentration. An example for the container C7 is shown in Fig. 10. Magnetic fluid of mass concentration 27.8% showed the lowest H_c and that of 30% showed the highest H_c . Fluid of mass concentration 27.8% showed larger growth of the wave than that of the other concentration under the same magnetic field H_0 . In the same container and under the same magnetic field, the number of the instability waves for mass concentration 27.8% was also larger than that of the other.

4. Theoretical Consideration

We consider the surface instability of magnetic fluids in a rectangular container, subject to magnetic fields in the coordinate system as shown in Fig. 11. It is assumed that the magnetic fluids are homogeneous and incompressible. The instability appearing in the rectangular container with a long width and a thin depth is assumed to be two-dimensional, according to the experimental observations. The interface deflected by the magnetic field is defined by the function $\xi(x)$, so that the gravitational energy E_g derived from the interface deflection is described by Eq. (1).

$$E_g = \frac{\rho g b}{2} \int_0^a \xi^2 dx \quad (1)$$

Assuming very small deflection of the interface, the

Fig. 10 Effect of mass concentration on growth of instability waves

surface energy E_σ associated with the configuration is described by Eq. (2).

$$E_\sigma = \frac{\sigma b}{2} \int_0^a \left(\frac{d\xi}{dx} \right)^2 dx \quad (2)$$

Since the magnetic field in the magnetic fluid is decreased by the self-field effect, the magnetic energy E_m is written by Eq. (3) for the applied magnetic field H_0 .

$$E_m = -\mu_0 \int_v \left(\int_0^{H_0} \frac{\chi H_0}{1 + \chi D} dH_0 \right) dv \quad (3)$$

The total energy E is given by Eq. (4).

$$E = E_g + E_\sigma + E_m \quad (4)$$

We introduce the following dimensionless variables.

$$x^* = \frac{x}{b}, \quad z^* = \frac{z}{b}, \quad v^* = \frac{v}{b^3} \quad (5)$$

Substituting Eq. (5) into Eq. (4), the dimensionless Eq. (6) is obtained:

$$E^* = \lambda_1 \int_0^L \xi^{*2} dx^* + \int_0^L \left(\frac{d\xi^*}{dx^*} \right)^2 dx^* + \lambda_2 \int_{v^*} \left(\int_0^{H_0^*} \frac{\chi H_0^*}{1 + \chi D} dH_0^* \right) dv^* \quad (6)$$

where $L = a/b$, $\lambda_1 = \rho g b^2 / \sigma$, and $\lambda_2 = -2\mu_0 b M_s^2 / \sigma$. The interfacial configuration ξ is determined by minimizing Eq. (6). In general, it is very difficult to determine ξ exactly. Hence, we make a rough estimate. For simplification, we cancel the star sign. According to the experimental observations, the configuration of the interface can be taken approximately as follows:

$$\xi = A(k) \cos kx \quad (7)$$

where $k = 2\pi n/L$, and $A < 1$. Substituting Eq. (7) into Eq. (6), we obtain the following:

$$E = \frac{\lambda_1 A^2 L}{2} + \frac{k^2 L A^2}{2} + \frac{\lambda_2 H_0^2 K_1 L^3 e^{(-K_2 L H_0)} A^2}{2} \quad (8)$$

where K_1 and K_2 are constants. Then, taking account of the change of the susceptibility and the experimental results, the approximated equation $\chi/(1 + \chi D) \approx K_1 L^2 e^{(-K_2 L H_0)} A$ is used. Minimization of energy yields the following expression:

Fig. 11 Coordinates system

Fig. 12 Comparison of experimental with theoretical wave number

$$k^2 = -\lambda_1 - \lambda_2 K_1 L^2 e^{(-K_2 L H_0)} H_0^2 \quad (9)$$

Since the first term on the right side of Eq.(9) is negative at all times, the occurrence of the instability waves is suppressed, while the second term is positive. Hence these facts suggest that there exists a critical magnetic field H_c . Hence, based on Eq.(9), the following can be said: n increases as b decreases, n increases with H_0 if H_0 is limited within a specific range, and n decreases in the range of sufficiently high H_0 because of the decrease of $e^{(-K_2 L H_0)}$. These agree with the experimental results qualitatively. An example of the comparison of the experimental results with the calculated results from Eq.(9) is shown in Fig. 12.

Next, we consider the rise of the magnetic fluid surface at the middle of the container. The rise in the container with larger a is remarkable. The body force $\mu_0(M \cdot \nabla)H$ acts upon the magnetic fluid when the magnetic field is inhomogeneous in space. According to Neuringer and Rosensweig⁽¹²⁾, the liquid surface level difference between the position 1 and the position 2, selected arbitrarily in the x -direction, is described as follows:

$$\xi_{12} = \frac{\mu_0}{\rho g} (\bar{M}_1 H_1 - \bar{M}_2 H_2) \quad (10)$$

when

$$\bar{M} = \frac{1}{H} \int_0^H M dH$$

Figure 13 shows the comparison of the experimental results with the calculated results of the maximum surface difference obtained from Eq.(10) for the container C4. The magnetic fluids of mass concentrations 19.7% and 8.6%, which had sufficiently high H_c to avoid the influence of the instability waves, were used. In the calculations, the values of the magnetic field

Fig. 13 Comparison of experimental with theoretical maximum surface level difference

shown in Fig.1 were used. It can be seen that the experimental values qualitatively agree with the calculated values. Therefore, the uniformity of the distribution of the magnetic field appears as the electric current increases, and then the liquid level difference on the interface of the magnetic fluid occurs in correspondence with the field distribution.

5. Conclusions

An experimental study was performed on the interfacial instability of magnetic fluids in rectangular containers in the presence of a magnetic field tangential to the interface, and the concrete configurations of the interface in various containers were estimated by theoretical considerations. The results obtained are summarized as follows.

(1) The interfacial instability appearing in the rectangular containers was two-dimensional, and the amplitudes of the waves increased with the magnetic field. Some wave crests of the instability waves showed a scroll pattern.

(2) The incipient interfacial instability occurred when the field strength exceeded the critical value H_c . Also, H_c increased as the magnetic fluid layer b in the container increased.

(3) The number of instability waves, n , increased with the field strength H_0 , within a certain range of H_0 . On the contrary, depending on the container dimension, n decreased with H_0 in the range of sufficiently high fields. The increase of the wave number after sufficient growth of the instability waves was achieved by crest branching.

(4) In the range of this experiment, magnetic fluid of mass concentration 27.8% showed the lowest H_c . This magnetic fluid showed the larger growth of the instability waves and the higher wave number compared with the other concentration fluids.

(5) The theoretical analysis supported the experimental results for the critical magnetic field corresponding to the incipient instability and the wave number of the instability waves.

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